

10.6

CYLINDERS AND QUADRIC SURFACES

EXAMPLE A Identify and sketch the surface $4x^2 - y^2 + 2z^2 + 4 = 0$.

SOLUTION Dividing by -4 , we first put the equation in standard form:

$$-x^2 + \frac{y^2}{4} - \frac{z^2}{2} = 1$$

Comparing this equation with Table 1, we see that it represents a hyperboloid of two sheets, the only difference being that in this case the axis of the hyperboloid is the y -axis. The traces in the xy - and yz -planes are the hyperbolas

$$-x^2 + \frac{y^2}{4} = 1 \quad z = 0 \quad \text{and} \quad \frac{y^2}{4} - \frac{z^2}{2} = 1 \quad x = 0$$

The surface has no trace in the xz -plane, but traces in the vertical planes $y = k$ for $|k| > 2$ are the ellipses

$$x^2 + \frac{z^2}{2} = \frac{k^2}{4} - 1 \quad y = k$$

which can be written as

$$\frac{x^2}{\frac{k^2}{4} - 1} + \frac{z^2}{2\left(\frac{k^2}{4} - 1\right)} = 1 \quad y = k$$

These traces are used to make the sketch in Figure 1. ■

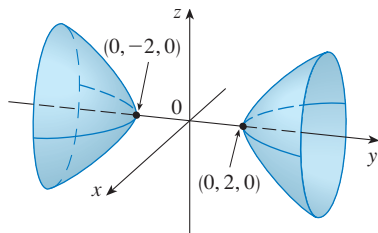


FIGURE 1
 $4x^2 - y^2 + 2z^2 + 4 = 0$