2.6 IMPLICIT DIFFERENTIATION

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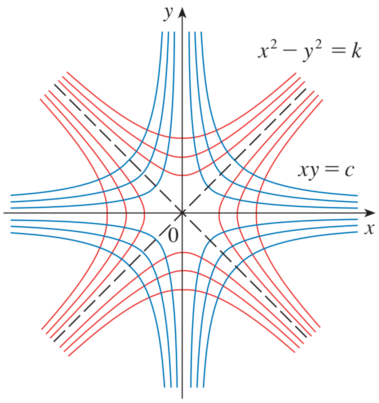

FIGURE 1

EXAMPLE A The equation

$$xy = c \qquad c \neq 0 \tag{1}$$

represents a family of hyperbolas. (Different values of the constant c give different hyperbolas. See Figure 1.) The equation

$$x^2 - y^2 = k \qquad k \neq 0 \tag{2}$$

represents another family of hyperbolas with asymptotes $y = \pm x$. Show that every curve in the family (1) is orthogonal to every curve in the family (2); that is, the families are orthogonal trajectories of each other.

SOLUTION Implicit differentiation of Equation 1 gives

$$x\frac{dy}{dx} + y = 0 \qquad \text{so} \qquad \frac{dy}{dx} = -\frac{y}{x} \tag{3}$$

Implicit differentiation of Equation 2 gives

$$2x - 2y\frac{dy}{dx} = 0 \qquad \text{so} \qquad \frac{dy}{dx} = \frac{x}{y} \tag{4}$$

From (3) and (4) we see that at any point of intersection of curves from each family, the slopes of the tangents are negative reciprocals of each other. Therefore, the curves intersect at right angles; that is, they are orthogonal. ■