2.8 LINEAR APPROXIMATIONS AND DIFFERENTIALS

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EXAMPLE A Suppose that after you stuff a turkey its temperature is 50°F and you then put it in a 325°F oven. After an hour the meat thermometer indicates that the temperature of the turkey is 93°F and after two hours it indicates 129°F. Predict the temperature of the turkey after three hours.

SOLUTION If $T(t)$ represents the temperature of the turkey after t hours, we are given that $T(0) = 50$, $T(1) = 93$, and $T(2) = 129$. In order to make a linear approximation with $a = 2$, we need an estimate for the derivative $T'(2)$. Because

$$T'(2) = \lim_{t \to 2} \frac{T(t) - T(2)}{t - 2}$$

we could estimate $T'(2)$ by the difference quotient with $t = 1$:

$$T'(2) \approx \frac{T(1) - T(2)}{1 - 2} = \frac{93 - 129}{-1} = 36$$

This amounts to approximating the instantaneous rate of temperature change by the average rate of change between $t = 1$ and $t = 2$, which is 36°F/h. With this estimate, the linear approximation (1) for the temperature after 3 h is

$$\begin{aligned} T(3) &\approx T(2) + T'(2)(3 - 2) \\ &\approx 129 + 36 \cdot 1 = 165 \end{aligned}$$

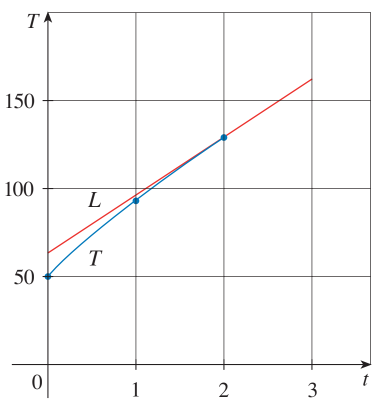

FIGURE 1

So the predicted temperature after three hours is 165°F.

We obtain a more accurate estimate for $T'(2)$ by plotting the given data, as in Figure 1, and estimating the slope of the tangent line at $t = 2$ to be

$$T'(2) \approx 33$$

Then our linear approximation becomes

$$T(3) \approx T(2) + T'(2) \cdot 1 \approx 129 + 33 = 162$$

and our improved estimate for the temperature is 162°F.

Because the temperature curve lies below the tangent line, it appears that the actual temperature after three hours will be somewhat less than 162°F, perhaps closer to 160°F. ■