

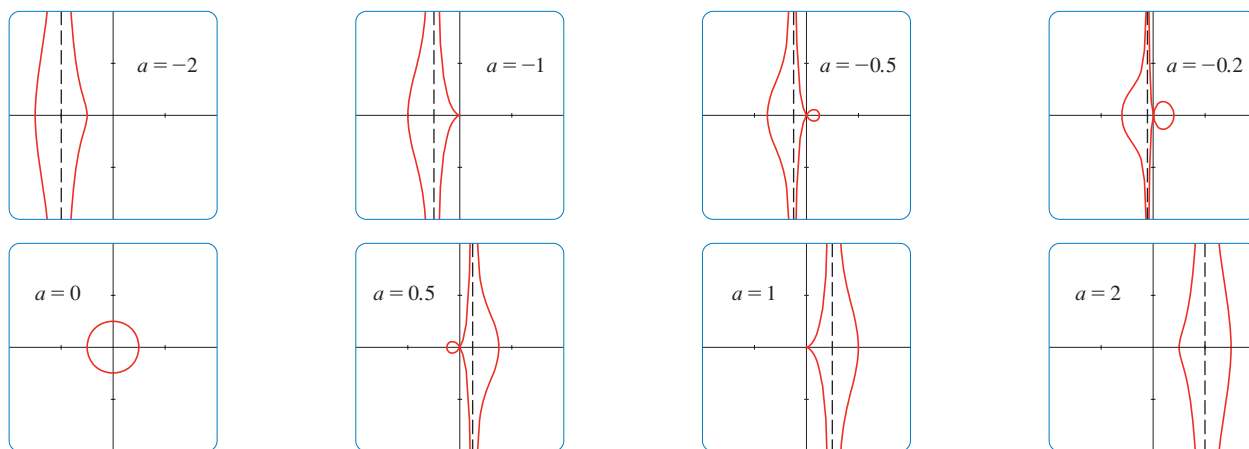
## 9.1 PARAMETRIC CURVES

**EXAMPLE A** Investigate the family of curves with parametric equations

$$x = a + \cos t \quad y = a \tan t + \sin t$$

What do these curves have in common? How does the shape change as  $a$  increases?

**SOLUTION** We use a graphing device to produce the graphs for the cases  $a = -2, -1, -0.5, -0.2, 0, 0.5, 1,$  and  $2$  shown in Figure 1. Notice that all of these curves (except the case  $a = 0$ ) have two branches, and both branches approach the vertical asymptote  $x = a$  as  $x$  approaches  $a$  from the left or right.



**FIGURE 1** Members of the family  $x = a + \cos t, y = a \tan t + \sin t$ , all graphed in the viewing rectangle  $[-4, 4]$  by  $[-4, 4]$

When  $a < -1$ , both branches are smooth; but when  $a$  reaches  $-1$ , the right branch acquires a sharp point, called a *cusp*. For  $a$  between  $-1$  and  $0$  the cusp turns into a loop, which becomes larger as  $a$  approaches  $0$ . When  $a = 0$ , both branches come together and form a circle (see Example 2). For  $a$  between  $0$  and  $1$ , the left branch has a loop, which shrinks to become a cusp when  $a = 1$ . For  $a > 1$ , the branches become smooth again, and as  $a$  increases further, they become less curved. Notice that the curves with  $a$  positive are reflections about the  $y$ -axis of the corresponding curves with  $a$  negative.

These curves are called **conchoids of Nicomedes** after the ancient Greek scholar Nicomedes. He called them conchoids because the shape of their outer branches resembles that of a conch shell or mussel shell. ■