12.1 DOUBLE INTEGRALS OVER RECTANGLES

EXAMPLE A The contour map in Figure 1 shows the snowfall, in inches, that fell on the state of Colorado on December 24, 1982. (The state is in the shape of a rectangle that measures 388 mi west to east and 276 mi south to north.) Use the contour map to estimate the average snowfall for Colorado as a whole on December 24.

The **average value** of a function f of two variables defined on a rectangle R is

$$f_{\text{ave}} = \frac{1}{A(R)} \iint_R f(x, y)\, dA$$

where $A(R)$ is the area of R.

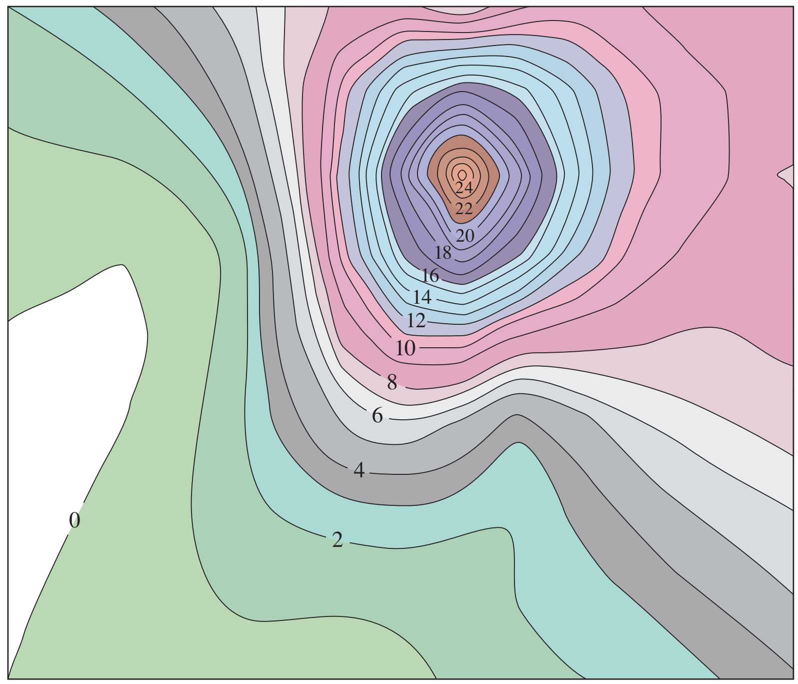

FIGURE 1

SOLUTION Let's place the origin at the southwest corner of the state. Then $0 \leqslant x \leqslant 388$, $0 \leqslant y \leqslant 276$, and $f(x, y)$ is the snowfall, in inches, at a location x miles to the east and y miles to the north of the origin. If R is the rectangle that represents Colorado, then the average snowfall for the state on December 24 was

$$f_{\text{ave}} = \frac{1}{A(R)} \iint_R f(x, y)\, dA$$

where $A(R) = 388 \cdot 276$. To estimate the value of this double integral let's use the Midpoint Rule with $m = n = 4$. In other words, we divide R into 16 subrectangles of equal size, as in Figure 2. The area of each subrectangle is

$$\Delta A = \tfrac{1}{16}(388)(276) = 6693 \text{ mi}^2$$

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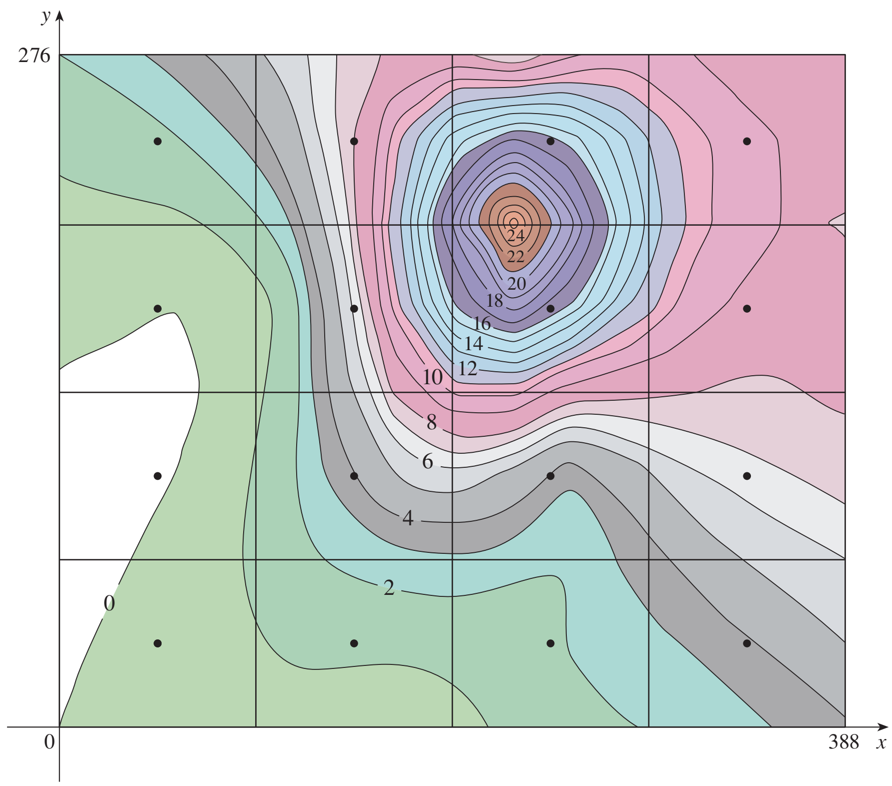

FIGURE 2

Using the contour map to estimate the value of f at the center of each subrectangle, we get

$$\iint_R f(x, y)\, dA \approx \sum_{i=1}^{4} \sum_{j=1}^{4} f(\overline{x}_i, \overline{y}_j)\, \Delta A$$

$$\approx \Delta A[0.4 + 1.2 + 1.8 + 3.9 + 0 + 3.9 + 4.0 + 6.5 + 0.1 + 6.1 + 16.5 + 8.8 + 1.8 + 8.0 + 16.2 + 9.4]$$

$$= (6693)(88.6)$$

Therefore $$f_{\text{ave}} \approx \frac{(6693)(88.6)}{(388)(276)} \approx 5.5$$

On December 24, 1982, Colorado received an average of approximately $5\frac{1}{2}$ inches of snow. ■

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