## 9.1 │ ⊞ **LABORATORY PROJECT:** RUNNING CIRCLES AROUND CIRCLES
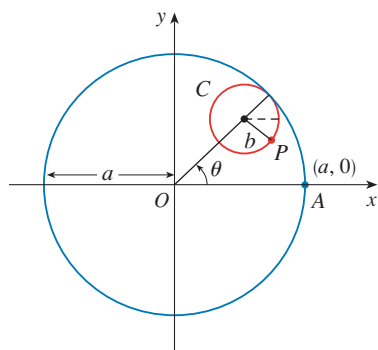
In this project we investigate families of curves, called *hypocycloids* and *epicycloids,* that are generated by the motion of a point on a circle that rolls inside or outside another circle.

**1.** A **hypocycloid** is a curve traced out by a fixed point $P$ on a circle $C$ of radius $b$ as $C$ rolls on the inside of a circle with center $O$ and radius $a$. Show that if the initial position of $P$ is $(a, 0)$ and the parameter $\theta$ is chosen as in the figure, then parametric equations of the hypocycloid are

$$x = (a - b) \cos \theta + b \cos\left(\frac{a - b}{b} \theta\right) \qquad y = (a - b) \sin \theta - b \sin\left(\frac{a - b}{b} \theta\right)$$

**2.** Use a graphing device to draw the graphs of hypocycloids with $a$ a positive integer and $b = 1$. How does the value of $a$ affect the graph? Show that if we take $a = 4$, then the parametric equations of the hypocycloid reduce to

$$x = 4 \cos^3\theta \qquad y = 4 \sin^3\theta$$

This curve is called a **hypocycloid of four cusps**, or an **astroid**.

**3.** Now try $b = 1$ and $a = n/d$, a fraction where $n$ and $d$ have no common factor. First let $n = 1$ and try to determine graphically the effect of the denominator $d$ on the shape of the graph. Then let $n$ vary while keeping $d$ constant. What happens when $n = d + 1$?

**4.** What happens if $b = 1$ and $a$ is irrational? Experiment with an irrational number like $\sqrt{2}$ or $e - 2$. Take larger and larger values for $\theta$ and speculate on what would happen if we were to graph the hypocycloid for all real values of $\theta$.

**5.** If the circle $C$ rolls on the *outside* of the fixed circle, the curve traced out by $P$ is called an **epicycloid**. Find parametric equations for the epicycloid.

**6.** Investigate the possible shapes for epicycloids. Use methods similar to Problems 2–4.