

5.6

INVERSE TRIGONOMETRIC FUNCTIONS
EXAMPLE A Evaluate:

$$(a) \sin(\sin^{-1} 0.6) \quad (b) \sin^{-1}\left(\sin \frac{\pi}{12}\right) \quad (c) \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$$

SOLUTION(a) Since 0.6 lies between -1 and 1 , the second cancellation equation in (2) gives

$$\sin(\sin^{-1} 0.6) = 0.6$$

(b) Since $\pi/12$ lies between $-\pi/2$ and $\pi/2$, the first cancellation equation gives

$$\sin^{-1}\left(\sin \frac{\pi}{12}\right) = \frac{\pi}{12}$$

(c) Since $2\pi/3$ does not lie in the interval $[-\pi/2, \pi/2]$, we can't use the cancellation equation. Instead we note that $\sin(2\pi/3) = \sqrt{3}/2$ and $\sin^{-1}(\sqrt{3}/2) = \pi/3$ because $\pi/3$ lies between $-\pi/2$ and $\pi/2$. Therefore

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \quad \blacksquare$$

EXAMPLE B Differentiate $y = \frac{1}{\sin^{-1}x}$.**SOLUTION**

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\sin^{-1}x)^{-1} = -(\sin^{-1}x)^{-2} \frac{d}{dx} (\sin^{-1}x) \\ &= -\frac{1}{(\sin^{-1}x)^2 \sqrt{1-x^2}} \quad \blacksquare \end{aligned}$$