EXAMPLE A Find all roots of the equation \( x^4 - 5x^3 + 4x^2 - x + 13 = 0 \) correct to eight decimal places.

SOLUTION Figure 1 shows a graph of \( f(x) = x^4 - 5x^3 + 4x^2 - x + 13 \) and we see that the roots are near 2.2 and 3.8. The formula for Newton’s Method is

\[
x_{n+1} = x_n - \frac{x_n^4 - 5x_n^3 + 4x_n^2 - x_n + 13}{4x_n^3 - 15x_n^2 + 8x_n - 1}
\]

Using Newton’s Method with the initial approximations from the graph, we get

\[
\begin{align*}
x_1 &= 2.2 & x_1 &= 3.8 \\
x_2 &\approx 2.22577566 & x_2 &\approx 3.76551041 \\
x_3 &\approx 2.22578253 & x_3 &\approx 3.76419061 \\
x_4 &\approx 2.22578253 & x_4 &\approx 3.76418872 \\
x_5 &\approx 3.76418872
\end{align*}
\]

The roots of the given equation, correct to eight decimal places, are 2.22578253 and 3.76418872.

\[\blacksquare\]