| **3.6** | **NEWTON'S METHOD** |

EXAMPLE A Find all roots of the equation $x^4 - 5x^3 + 4x^2 - x + 13 = 0$ correct to eight decimal places.

SOLUTION Figure 1 shows a graph of $f(x) = x^4 - 5x^3 + 4x^2 - x + 13$ and we see that the roots are near 2.2 and 3.8. The formula for Newton's Method is

$$x_{n+1} = x_n - \frac{x_n^4 - 5x_n^3 + 4x_n^2 - x_n + 13}{4x_n^3 - 15x_n^2 + 8x_n - 1}$$

Using Newton's Method with the initial approximations from the graph, we get

$$x_1 = 2.2 \qquad\qquad x_1 = 3.8$$

$$x_2 \approx 2.22577566 \qquad\qquad x_2 \approx 3.76551041$$

$$x_3 \approx 2.22578253 \qquad\qquad x_3 \approx 3.76419061$$

$$x_4 \approx 2.22578253 \qquad\qquad x_4 \approx 3.76418872$$

$$x_5 \approx 376418872$$

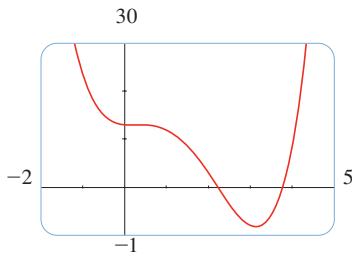The roots of the given equation, correct to eight decimal places, are 2.22578253 and 3.76418872. ■

30

−2

5

−1

FIGURE 1