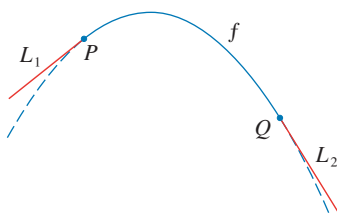


2.3 APPLIED PROJECT: BUILDING A BETTER ROLLER COASTER

This project can be completed anytime after you have studied Section 2.3 in the textbook.



Suppose you are asked to design the first ascent and drop for a new roller coaster. By studying photographs of your favorite coasters, you decide to make the slope of the ascent 0.8 and the slope of the drop -1.6 . You decide to connect these two straight stretches $y = L_1(x)$ and $y = L_2(x)$ with part of a parabola $y = f(x) = ax^2 + bx + c$, where x and $f(x)$ are measured in feet. For the track to be smooth there can't be abrupt changes in direction, so you want the linear segments L_1 and L_2 to be tangent to the parabola at the transition points P and Q . (See the figure.) To simplify the equations you decide to place the origin at P .

1. (a) Suppose the horizontal distance between P and Q is 100 ft. Write equations in a , b , and c that will ensure that the track is smooth at the transition points.
 - (b) Solve the equations in part (a) for a , b , and c to find a formula for $f(x)$.
 - (c) Plot L_1 , f , and L_2 to verify graphically that the transitions are smooth.
 - (d) Find the difference in elevation between P and Q .
2. The solution in Problem 1 might *look* smooth, but it might not *feel* smooth because the piecewise defined function [consisting of $L_1(x)$ for $x < 0$, $f(x)$ for $0 \leq x \leq 100$, and $L_2(x)$ for $x > 100$] doesn't have a continuous second derivative. So you decide to improve the design by using a quadratic function $q(x) = ax^2 + bx + c$ only on the interval $10 \leq x \leq 90$ and connecting it to the linear functions by means of two cubic functions:

$$g(x) = kx^3 + lx^2 + mx + n \quad 0 \leq x < 10$$

$$h(x) = px^3 + qx^2 + rx + s \quad 90 < x \leq 100$$

- (a) Write a system of equations in 11 unknowns that ensure that the functions and their first two derivatives agree at the transition points.
- (b) Solve the equations in part (a) with a computer algebra system to find formulas for $q(x)$, $g(x)$, and $h(x)$.
- (c) Plot L_1 , g , q , h , and L_2 , and compare with the plot in Problem 1(c).