### 12.5 DISCOVERY PROJECT: VOLUMES OF HYPERSPHERES

This project can be completed anytime after you have studied Section I2.5 in the textbook.

In this project we find formulas for the volume enclosed by a hypersphere in $n$-dimensional space.
I. Use a double integral and the trigonometric substitution $y=r \sin \theta$, together with Formula 64 in the Table of Integrals, to find the area of a circle with radius $r$.
2. Use a triple integral and trigonometric substitution to find the volume of a sphere with radius $r$.
3. Use a quadruple integral to find the hypervolume enclosed by the hypersphere $x^{2}+y^{2}+z^{2}+w^{2}=r^{2}$ in $\mathbb{R}^{4}$. (Use only trigonometric substitution and the reduction formulas for $\int \sin ^{n} x d x$ or $\int \cos ^{n} x d x$.)
4. Use an $n$-tuple integral to find the volume enclosed by a hypersphere of radius $r$ in $n$-dimensional space $\mathbb{R}^{n}$. [Hint: The formulas are different for $n$ even and $n$ odd.]

