

1.1 FUNCTIONS AND THEIR REPRESENTATIONS

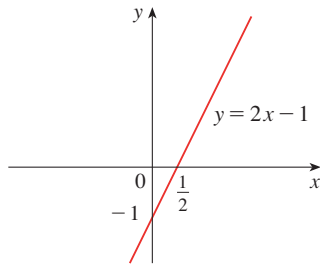


FIGURE 1

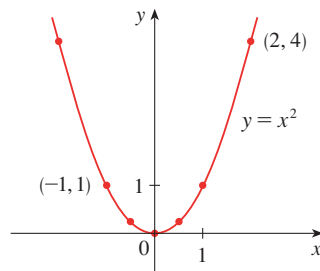


FIGURE 2

EXAMPLE A Sketch the graph and find the domain and range of each function.

(a) $f(x) = 2x - 1$

(b) $g(x) = x^2$

SOLUTION

(a) The equation of the graph is $y = 2x - 1$, and we recognize this as being the equation of a line with slope 2 and y -intercept -1 . (Recall the slope-intercept form of the equation of a line: $y = mx + b$.) This enables us to sketch the graph of f in Figure 1. The expression $2x - 1$ is defined for all real numbers, so the domain of f is the set of all real numbers, which we denote by \mathbb{R} . The graph shows that the range is also \mathbb{R} .

(b) Since $g(2) = 2^2 = 4$ and $g(-1) = (-1)^2 = 1$, we could plot the points $(2, 4)$ and $(-1, 1)$, together with a few other points on the graph, and join them to produce the graph (Figure 2). The equation of the graph is $y = x^2$, which represents a parabola. The domain of g is \mathbb{R} . The range of g consists of all values of $g(x)$, that is, all numbers of the form x^2 . But $x^2 \geq 0$ for all numbers x and any positive number y is a square. So the range of g is $\{y \mid y \geq 0\} = [0, \infty)$. This can also be seen from Figure 2. ■

EXAMPLE B If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\frac{f(a+h) - f(a)}{h}$.

SOLUTION We first evaluate $f(a+h)$ by replacing x by $a+h$ in the expression for $f(x)$:

$$\begin{aligned} f(a+h) &= 2(a+h)^2 - 5(a+h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 5(a+h) + 1 \\ &= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1 \end{aligned}$$

Then we substitute into the given expression and simplify:

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5 \end{aligned}$$

[▶ Play the Video](#)

[▶ Don't have Flash? Click here](#)

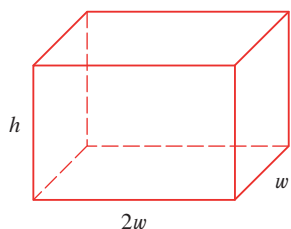


FIGURE 3

EXAMPLE C A rectangular storage container with an open top has a volume of 10 m^3 . The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

SOLUTION We draw a diagram as in Figure 3 and introduce notation by letting w and $2w$ be the width and length of the base, respectively, and h be the height.

The area of the base is $(2w)w = 2w^2$, so the cost, in dollars, of the material for the base is $10(2w^2)$. Two of the sides have area wh and the other two have area $2wh$, so the cost of the material for the sides is $6[2(wh) + 2(2wh)]$. The total cost is therefore

$$C = 10(2w^2) + 6[2(wh) + 2(2wh)] = 20w^2 + 36wh$$

To express C as a function of w alone, we need to eliminate h and we do so by using the fact that the volume is 10 m^3 . Thus

$$w(2w)h = 10$$

which gives

$$h = \frac{10}{2w^2} = \frac{5}{w^2}$$

Substituting this into the expression for C , we have

$$C = 20w^2 + 36w\left(\frac{5}{w^2}\right) = 20w^2 + \frac{180}{w}$$

Therefore, the equation

$$C(w) = 20w^2 + \frac{180}{w} \quad w > 0$$

expresses C as a function of w . ■

EXAMPLE D Find a formula for the function f graphed in Figure 4.

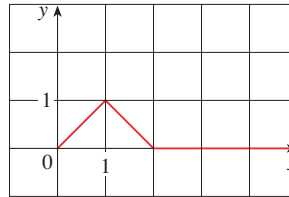


FIGURE 4

SOLUTION The line through $(0, 0)$ and $(1, 1)$ has slope $m = 1$ and y -intercept $b = 0$, so its equation is $y = x$. Thus, for the part of the graph of f that joins $(0, 0)$ to $(1, 1)$, we have

$$f(x) = x \quad \text{if } 0 \leq x \leq 1$$

The line through $(1, 1)$ and $(2, 0)$ has slope $m = -1$, so its point-slope form is

- Point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = (-1)(x - 2) \quad \text{or} \quad y = 2 - x$$

So we have

$$f(x) = 2 - x \quad \text{if } 1 < x \leq 2$$

We also see that the graph of f coincides with the x -axis for $x > 2$. Putting this information together, we have the following three-piece formula for f :

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } 1 < x \leq 2 \\ 0 & \text{if } x > 2 \end{cases} \quad \blacksquare$$