Here we review the basic rules and procedures of algebra that you need to know in order to be successful in calculus.

**ARITHMETIC OPERATIONS**

The real numbers have the following properties:

\[
\begin{align*}
    a + b &= b + a &\text{(Commutative Law)} \\
    (a + b) + c &= a + (b + c) &\text{(Associative Law)} \\
    a(b + c) &= ab + ac &\text{(Distributive law)}
\end{align*}
\]

In particular, putting \( a = -1 \) in the Distributive Law, we get

\[-(b + c) = (-1)(b + c) = (-1)b + (-1)c\]

and so

\[-(b + c) = -b - c\]

**EXAMPLE 1**

(a) \((3xy)(-4x) = 3(-4)x^2y = -12x^2y\)
(b) \(2t(7x + 2x - 11) = 14tx + 4t^2x - 22t\)
(c) \(4 - 3(x - 2) = 4 - 3x + 6 = 10 - 3x\)

If we use the Distributive Law three times, we get

\[(a + b)(c + d) = (a + b)c + (a + b)d = ac + bc + ad + bd\]

This says that we multiply two factors by multiplying each term in one factor by each term in the other factor and adding the products. Schematically, we have

\[\overbrace{(a + b)(c + d)}\]

In the case where \( c = a \) and \( d = b \), we have

\[(a + b)^2 = a^2 + ba + ab + b^2\]

or

\[\overbrace{(a + b)^2 = a^2 + 2ab + b^2}\]

Similarly, we obtain

\[\overbrace{(a - b)^2 = a^2 - 2ab + b^2}\]

**EXAMPLE 2**

(a) \((2x + 1)(3x - 5) = 6x^2 + 3x - 10x - 5 = 6x^2 - 7x - 5\)
(b) \((x + 6)^2 = x^2 + 12x + 36\)
(c) \(3(x - 1)(4x + 3) - 2(x + 6) = 3(4x^2 - x - 3) - 2x - 12 = 12x^2 - 3x - 9 - 2x - 12 = 12x^2 - 5x - 21\)
FRACTIONS

To add two fractions with the same denominator, we use the Distributive Law:

\[
\frac{a}{b} + \frac{c}{b} = \frac{1}{b} \times a + \frac{1}{b} \times c = \frac{1}{b} (a + c) = \frac{a + c}{b}
\]

Thus, it is true that

\[
\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}
\]

But remember to avoid the following common error:

\[
\frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}
\]

(For instance, take \(a = b = c = 1\) to see the error.)

To add two fractions with different denominators, we use a common denominator:

\[
\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}
\]

We multiply such fractions as follows:

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

In particular, it is true that

\[
\frac{-a}{b} = \frac{-a}{b} = \frac{a}{-b}
\]

To divide two fractions, we invert and multiply:

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}
\]

EXAMPLE 3

(a) \(\frac{x + 3}{x} = \frac{x}{x} + \frac{3}{x} = 1 + \frac{3}{x}\)

(b) \(\frac{3}{x - 1} + \frac{x}{x + 2} = \frac{3(x + 2)}{(x - 1)(x + 2)} + \frac{x(x - 1)}{(x - 1)(x + 2)} = \frac{3x + 6 + x^2 - x}{x^2 + x - 2} = \frac{x^2 + 2x + 6}{x^2 + x - 2}\)

(c) \(\frac{s^2 t}{u} \div \frac{2}{-2u} = \frac{s^2 t^2}{u} \cdot \frac{-2u}{-2u} = \frac{s^2 t^2}{2}\)
FACToring

We have used the Distributive Law to expand certain algebraic expressions. We sometimes need to reverse this process (again using the Distributive Law) by factoring an expression as a product of simpler ones. The easiest situation occurs when the expression has a common factor as follows:

\[
\frac{x}{y} + 1 = \frac{x + y}{y} = \frac{x + y}{x - y} \times \frac{x}{y(x - y)} = \frac{x^2 + xy}{xy - y^2}
\]

To factor a quadratic of the form \(x^2 + bx + c\) we note that

\[(x + r)(x + s) = x^2 + (r + s)x + rs\]

so we need to choose numbers \(r\) and \(s\) so that \(r + s = b\) and \(rs = c\).

**Example 4** Factor \(x^2 + 5x - 24\).

**Solution** The two integers that add to give 5 and multiply to give -24 are -3 and 8. Therefore

\[x^2 + 5x - 24 = (x - 3)(x + 8)\]

**Example 5** Factor \(2x^2 - 7x - 4\).

**Solution** Even though the coefficient of \(x^2\) is not 1, we can still look for factors of the form \(2x + r\) and \(x + s\), where \(rs = -4\). Experimentation reveals that

\[2x^2 - 7x - 4 = (2x + 1)(x - 4)\]

Some special quadratics can be factored by using Equations 1 or 2 (from right to left) or by using the formula for a difference of squares:

\[a^2 - b^2 = (a - b)(a + b)\]

The analogous formula for a difference of cubes is

\[a^3 - b^3 = (a - b)(a^2 + ab + b^2)\]

which you can verify by expanding the right side. For a sum of cubes we have

\[a^3 + b^3 = (a + b)(a^2 - ab + b^2)\]

**Example 6**

(a) \(x^2 - 6x + 9 = (x - 3)^2\) (Equation 2; \(a = x, b = 3\))

(b) \(4x^2 - 25 = (2x - 5)(2x + 5)\) (Equation 3; \(a = 2x, b = 5\))

(c) \(x^3 + 8 = (x + 2)(x^2 - 2x + 4)\) (Equation 5; \(a = x, b = 2\))
EXAMPLE 7 Simplify \( \frac{x^2 - 16}{x^2 - 2x - 8} \).

SOLUTION Factoring numerator and denominator, we have

\[
\frac{x^2 - 16}{x^2 - 2x - 8} = \frac{(x - 4)(x + 4)}{(x - 4)(x + 2)} = \frac{x + 4}{x + 2}
\]

To factor polynomials of degree 3 or more, we sometimes use the following fact.

\[ \textbf{The Factor Theorem} \quad \text{If} \quad P \quad \text{is a polynomial and} \quad P(b) = 0, \quad \text{then} \quad x - b \quad \text{is a factor of} \quad P(x). \]

EXAMPLE 8 Factor \( x^3 - 3x^2 - 10x + 24 \).

SOLUTION Let \( P(x) = x^3 - 3x^2 - 10x + 24 \). If \( P(b) = 0 \), where \( b \) is an integer, then \( b \) is a factor of 24. Thus, the possibilities for \( b \) are \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \) and \( \pm 24 \).

We find that \( P(1) = 12, P(-1) = 30, P(2) = 0 \). By the Factor Theorem, \( x - 2 \) is a factor. Instead of substituting further, we use long division as follows:

\[
\begin{array}{r|llllll}
& & x^2 & - x & - 12 \\
\hline x - 2 & x^3 & - 3x^2 & - 10x & + 24 \\
& x^3 & - 2x^2 & & & \\
\hline
& -x^2 & - 10x & & & \\
& -x^2 & - 2x & & & \\
\hline
& & & -12x & + 24 & \\
& & & -12x & + 24 & \\
\hline
& & & & & 0
\end{array}
\]

Therefore \( x^3 - 3x^2 - 10x + 24 = (x - 2)(x^2 - x - 12) = (x - 2)(x + 3)(x - 4) \).

COMPLETING THE SQUARE

Completing the square is a useful technique for graphing parabolas or integrating rational functions. Completing the square means rewriting a quadratic \( ax^2 + bx + c \) in the form \( a(x + p)^2 + q \) and can be accomplished by:

1. Factoring the number \( a \) from the terms involving \( x \).
2. Adding and subtracting the square of half the coefficient of \( x \).

In general, we have

\[
ax^2 + bx + c = a \left[ x^2 + \frac{b}{a} x \right] + c
\]

\[
= a \left[ x^2 + \frac{b}{a} x + \left( \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 \right] + c
\]

\[
= a \left( x + \frac{b}{2a} \right)^2 + \left( c - \frac{b^2}{4a} \right)
\]

EXAMPLE 9 Rewrite \( x^2 + x + 1 \) by completing the square.

SOLUTION The square of half the coefficient of \( x \) is \( \frac{1}{4} \). Thus

\[
x^2 + x + 1 = x^2 + x + \frac{1}{4} - \frac{1}{4} + 1 = (x + \frac{1}{2})^2 + \frac{3}{4}
\]
EXAMPLE 10

\[2x^2 - 12x + 11 = 2[x^2 - 6x] + 11 = 2[x^2 - 6x + 9 - 9] + 11\]
\[= 2[(x - 3)^2 - 9] + 11 = 2(x - 3)^2 - 7\]

QUADRATIC FORMULA

By completing the square as above we can obtain the following formula for the roots of a quadratic equation.

The roots of the quadratic equation \(ax^2 + bx + c = 0\) are

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

EXAMPLE 11 Solve the equation \(5x^2 + 3x - 3 = 0\).

SOLUTION With \(a = 5\), \(b = 3\), \(c = -3\), the quadratic formula gives the solutions

\[x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-3)}}{2(5)} = \frac{-3 \pm \sqrt{69}}{10}\]

The quantity \(b^2 - 4ac\) that appears in the quadratic formula is called the discriminant. There are three possibilities:

1. If \(b^2 - 4ac > 0\), the equation has two real roots.
2. If \(b^2 - 4ac = 0\), the roots are equal.
3. If \(b^2 - 4ac < 0\), the equation has no real root. (The roots are complex.)

These three cases correspond to the fact that the number of times the parabola \(y = ax^2 + bx + c\) crosses the \(x\)-axis is 2, 1, or 0 (see Figure 1). In case (3) the quadratic \(ax^2 + bx + c\) can’t be factored and is called irreducible.

FIGURE 1
Possible graphs of \(y = ax^2 + bx + c\)

(a) \(b^2 - 4ac > 0\)  (b) \(b^2 - 4ac = 0\)  (c) \(b^2 - 4ac < 0\)

EXAMPLE 12 The quadratic \(x^2 + x + 2\) is irreducible because its discriminant is negative:

\[b^2 - 4ac = 1^2 - 4(1)(2) = -7 < 0\]

Therefore, it is impossible to factor \(x^2 + x + 2\).  

THE BINOMIAL THEOREM

Recall the binomial expression from Equation 1:

\[(a + b)^2 = a^2 + 2ab + b^2\]

If we multiply both sides by \((a + b)\) and simplify, we get the binomial expansion

\[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\]

Repeating this procedure, we get

\[(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\]

In general, we have the following formula.

**The Binomial Theorem**

If \(k\) is a positive integer, then

\[(a + b)^k = a^k + ka^{k-1}b + \frac{k(k-1)}{1 \cdot 2} a^{k-2}b^2 + \frac{k(k-1)(k-2)}{1 \cdot 2 \cdot 3} a^{k-3}b^3 + \cdots + \frac{k(k-1)\cdots(k-n+1)}{1 \cdot 2 \cdot 3 \cdots n} a^{k-n}b^n + \cdots + kab^{k-1} + b^k\]

**EXAMPLE 13** Expand \((x - 2)^5\).

**SOLUTION** Using the Binomial Theorem with \(a = x, b = -2, k = 5\), we have

\[(x - 2)^5 = x^5 + 5x^4(-2) + \frac{5 \cdot 4}{1 \cdot 2} x^3(-2)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} x^2(-2)^3 + 5x(-2)^4 + (-2)^5\]

\[= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32\]

RADICALS

The most commonly occurring radicals are square roots. The symbol \(\sqrt{}\) means “the positive square root of.” Thus

\[x = \sqrt{a} \quad \text{means} \quad x^2 = a \quad \text{and} \quad x \geq 0\]

Since \(a = x^2 \geq 0\), the symbol \(\sqrt{a}\) makes sense only when \(a \geq 0\). Here are two rules for working with square roots:

\[\sqrt{ab} = \sqrt{a} \sqrt{b} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\]

However, there is no similar rule for the square root of a sum. In fact, you should remember to avoid the following common error:

\[\sqrt{a + b} = \sqrt{a} + \sqrt{b}\]

(For instance, take \(a = 9\) and \(b = 16\) to see the error.)
EXAMPLE 14

(a) \( \frac{\sqrt{18}}{2} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3 \) \hspace{1cm} (b) \( \sqrt{x^2 y} = \sqrt{x^2} \sqrt{y} = |x| \sqrt{y} \)

Notice that \( \sqrt{x^2} = |x| \) because \( \sqrt{\cdot} \) indicates the positive square root. (See Absolute Value.)

In general, if \( n \) is a positive integer,

\[
x = \sqrt[n]{a} \quad \text{means} \quad x^n = a
\]

If \( n \) is even, then \( a \geq 0 \) and \( x \geq 0 \).

Thus \( \sqrt[3]{-8} = -2 \) because \((-2)^3 = -8\), but \( \sqrt[3]{-8} \) and \( \sqrt[3]{8} \) are not defined. The following rules are valid:

\[
\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b} \quad \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}
\]

EXAMPLE 15 \( \sqrt{x^2} = \sqrt[2]{x^2} = \sqrt[2]{x^2} \sqrt{x} = x \sqrt{x} \)

To rationalize a numerator or denominator that contains an expression such as \( \sqrt{a} - \sqrt{b} \), we multiply both the numerator and the denominator by the conjugate radical \( \sqrt{a} + \sqrt{b} \). Then we can take advantage of the formula for a difference of squares:

\[
(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b
\]

EXAMPLE 16 Rationalize the numerator in the expression \( \frac{\sqrt{x + 4} - 2}{x} \).

SOLUTION We multiply the numerator and the denominator by the conjugate radical \( \sqrt{x + 4} + 2 \):

\[
\frac{\sqrt{x + 4} - 2}{x} = \left( \frac{\sqrt{x + 4} - 2}{x} \right) \left( \frac{\sqrt{x + 4} + 2}{\sqrt{x + 4} + 2} \right) = \frac{(x + 4) - 4}{x(\sqrt{x + 4} + 2)}
\]

\[
= \frac{x}{x(\sqrt{x + 4} + 2)} = \frac{1}{\sqrt{x + 4} + 2}
\]

EXONENTS

Let \( a \) be any positive number and let \( n \) be a positive integer. Then, by definition,

1. \( a^n = a \cdot a \cdot \cdots \cdot a \) \( \quad \text{n factors} \)
2. \( a^0 = 1 \)
3. \( a^{-n} = \frac{1}{a^n} \)
4. \( a^{1/n} = \sqrt[n]{a} \)
   \( a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \) \( m \) is any integer
Laws of Exponents
Let $a$ and $b$ be positive numbers and let $r$ and $s$ be any rational numbers (that is, ratios of integers). Then

1. $a^r \times a^s = a^{r+s}$
2. $\frac{a^r}{a^s} = a^{r-s}$
3. $(a^r)^s = a^{rs}$
4. $(ab)^r = a^r b^r$
5. $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$ $\quad$ $b \neq 0$

In words, these five laws can be stated as follows:
1. To multiply two powers of the same number, we add the exponents.
2. To divide two powers of the same number, we subtract the exponents.
3. To raise a power to a new power, we multiply the exponents.
4. To raise a product to a power, we raise each factor to the power.
5. To raise a quotient to a power, we raise both numerator and denominator to the power.

**EXAMPLE 17**

(a) $2^8 \times 8^2 = 2^8 \times (2^3)^2 = 2^8 \times 2^6 = 2^{14}$

(b) $\frac{x^2 - y^2}{x^{-1} + y^{-1}} = \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{y^2}{y^2} - \frac{x^2}{x^2}}{\frac{y}{xy} + \frac{x}{xy}} = \frac{\frac{y^2 - x^2}{x^2 y^2}}{\frac{y + x}{xy}} = \frac{\frac{(y - x)(y + x)}{xy(y + x)}}{\frac{y + x}{xy}} = \frac{y - x}{xy}$

(c) $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$ $\quad$ Alternative solution: $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

(d) $\frac{1}{\sqrt{x^3}} = \frac{1}{x^{3/2}} = x^{-3/2}$

(e) $\left(\frac{x}{y}\right)^3 \left(\frac{y^2}{z}\right)^4 = \frac{x^3}{y^3} \cdot \frac{y^8}{z^4} = x^3 y^5 z^{-4}$

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**INEQUALITIES**

When working with inequalities, note the following rules.

**Rules for Inequalities**

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and $c < d$, then $a + c < b + d$.
3. If $a < b$ and $c > 0$, then $ac < bc$.
4. If $a < b$ and $c < 0$, then $ac > bc$.
5. If $0 < a < b$, then $1/a > 1/b$.

Rule 1 says that we can add any number to both sides of an inequality, and Rule 2 says that two inequalities can be added. However, we have to be careful with multiplication. Rule 3 says that we can multiply both sides of an inequality by a positive number, but Rule 4 says that if we multiply both sides of an inequality by a negative number, then we reverse the direction of the inequality. For example, if we take the inequality $a < b$, then $-a > -b$. Rule 5 says that if you divide both sides of an inequality by a positive number, then the inequality remains the same. If you divide both sides of an inequality by a negative number, then you reverse the direction of the inequality.
3 < 5 and multiply by 2, we get 6 < 10, but if we multiply by −2, we get −6 > −10.
Finally, Rule 5 says that if we take reciprocals, then we reverse the direction of an inequal-
ity (provided the numbers are positive).

**EXAMPLE 18** Solve the inequality $1 + x < 7x + 5$.

**SOLUTION** The given inequality is satisfied by some values of $x$ but not by others. To solve
an inequality means to determine the set of numbers $x$ for which the inequality is true.
This is called the solution set.

First we subtract 1 from each side of the inequality (using Rule 1 with $c = -1$):

$$x < 7x + 4$$

Then we subtract $7x$ from both sides (Rule 1 with $c = -7x$):

$$-6x < 4$$

Now we divide both sides by $-6$ (Rule 4 with $c = -\frac{1}{6}$):

$$x > -\frac{4}{6} = -\frac{2}{3}$$

These steps can all be reversed, so the solution set consists of all numbers greater
than $-\frac{2}{3}$. In other words, the solution of the inequality is the interval $(-\frac{2}{3}, \infty)$.

**EXAMPLE 19** Solve the inequality $x^2 - 5x + 6 \leq 0$.

**SOLUTION** First we factor the left side:

$$(x - 2)(x - 3) \leq 0$$

We know that the corresponding equation $(x - 2)(x - 3) = 0$ has the solutions 2 and 3.
The numbers 2 and 3 divide the real line into three intervals:

$$(-\infty, 2) \quad (2, 3) \quad (3, \infty)$$

On each of these intervals we determine the signs of the factors. For instance,

$$x \in (-\infty, 2) \Rightarrow x < 2 \Rightarrow x - 2 < 0$$

Then we record these signs in the following chart:

<table>
<thead>
<tr>
<th>Interval</th>
<th>$x - 2$</th>
<th>$x - 3$</th>
<th>$(x - 2)(x - 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 2$</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>$2 &lt; x &lt; 3$</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$x &gt; 3$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Another method for obtaining the information in the chart is to use test values. For
instance, if we use the test value $x = 1$ for the interval $(-\infty, 2)$, then substitution in
$x^2 - 5x + 6$ gives

$$1^2 - 5(1) + 6 = 2$$

The polynomial $x^2 - 5x + 6$ doesn’t change sign inside any of the three intervals, so we
conclude that it is positive on $(-\infty, 2)$.

Then we read from the chart that $(x - 2)(x - 3)$ is negative when $2 < x < 3$. Thus,
the solution of the inequality $(x - 2)(x - 3) \leq 0$ is

$$\{ x \mid 2 \leq x \leq 3 \} = [2, 3]$$
Notice that we have included the endpoints 2 and 3 because we are looking for values of \( x \) such that the product is either negative or zero. The solution is illustrated in Figure 3.

**EXAMPLE 20** Solve \( x^3 + 3x^2 > 4x \).

**SOLUTION** First we take all nonzero terms to one side of the inequality sign and factor the resulting expression:

\[
x^3 + 3x^2 - 4x > 0 \quad \text{or} \quad x(x - 1)(x + 4) > 0
\]

As in Example 19 we solve the corresponding equation \( x(x - 1)(x + 4) = 0 \) and use the solutions \( x = -4, x = 0, \) and \( x = 1 \) to divide the real line into four intervals \( (-\infty, -4), (-4, 0), (0, 1), \) and \( (1, \infty) \). On each interval the product keeps a constant sign as shown in the following chart.

<table>
<thead>
<tr>
<th>Interval</th>
<th>( x )</th>
<th>( x - 1 )</th>
<th>( x + 4 )</th>
<th>( x(x - 1)(x + 4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; -4 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( -4 &lt; x &lt; 0 )</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( 0 &lt; x &lt; 1 )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( x &gt; 1 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Then we read from the chart that the solution set is

\[
\{ x \mid -4 < x < 0 \text{ or } x > 1 \} = (-4, 0) \cup (1, \infty)
\]

The solution is illustrated in Figure 4.

**ABSOLUTE VALUE**

The **absolute value** of a number \( a \), denoted by \( |a| \), is the distance from \( a \) to 0 on the real number line. Distances are always positive or 0, so we have

\[
|a| \geq 0 \quad \text{for every number } a
\]

For example,

\[
|3| = 3 \quad |{-3}| = 3 \quad |0| = 0
\]

\[
|\sqrt{2} - 1| = \sqrt{2} - 1 \quad |3 - \pi| = \pi - 3
\]

In general, we have

\[
|a| = a \quad \text{if } a \geq 0
\]

\[
|a| = -a \quad \text{if } a < 0
\]

**EXAMPLE 21** Express \( |3x - 2| \) without using the absolute-value symbol.

**SOLUTION**

\[
|3x - 2| = \begin{cases} 
3x - 2 & \text{if } 3x - 2 \geq 0 \\
-(3x - 2) & \text{if } 3x - 2 < 0 
\end{cases}
\]

\[
= \begin{cases} 
3x - 2 & \text{if } x \geq \frac{2}{3} \\
2 - 3x & \text{if } x < \frac{2}{3}
\end{cases}
\]
Recall that the symbol \( \sqrt{} \) means “the positive square root of.” Thus, \( \sqrt{x^2} = x \) means \( x^2 = r \) and \( s \geq 0 \). Therefore, the equation \( \sqrt{a^2} = a \) is not always true. It is true only when \( a \geq 0 \). If \( a < 0 \), then \( -a > 0 \), so we have \( \sqrt{a^2} = -a \). In view of (12), we then have the equation

\[
\sqrt{a^2} = |a|
\]

which is true for all values of \( a \).

Hints for the proofs of the following properties are given in the exercises.

Properties of Absolute Values

Suppose \( a \) and \( b \) are any real numbers and \( n \) is an integer. Then

1. \(|ab| = |a||b|
2. \( \frac{|a|}{|b|} = \frac{|a|}{|b|} \quad (b \neq 0) \)
3. \( |a^n| = |a|^n \)

For solving equations or inequalities involving absolute values, it’s often very helpful to use the following statements.

Suppose \( a > 0 \). Then

4. \(|x| = a \) if and only if \( x = \pm a \)
5. \(|x| < a \) if and only if \( -a < x < a \)
6. \(|x| > a \) if and only if \( x > a \) or \( x < -a \)

For instance, the inequality \(|x| < a \) says that the distance from \( x \) to the origin is less than \( a \), and you can see from Figure 5 that this is true if and only if \( x \) lies between \( -a \) and \( a \).

If \( a \) and \( b \) are any real numbers, then the distance between \( a \) and \( b \) is the absolute value of the difference, namely, \(|a - b|\), which is also equal to \(|b - a|\). (See Figure 6.)

**EXAMPLE 22** Solve \(|2x - 5| = 3\).

**SOLUTION** By Property 4 of absolute values, \(|2x - 5| = 3\) is equivalent to

\[
2x - 5 = 3 \quad \text{or} \quad 2x - 5 = -3
\]

So \(2x = 8\) or \(2x = 2\). Thus, \(x = 4\) or \(x = 1\).

**EXAMPLE 23** Solve \(|x - 5| < 2\).

**SOLUTION 1** By Property 5 of absolute values, \(|x - 5| < 2\) is equivalent to

\[
-2 < x - 5 < 2
\]

Therefore, adding 5 to each side, we have

\[
3 < x < 7
\]

and the solution set is the open interval \((3, 7)\).

**SOLUTION 2** Geometrically, the solution set consists of all numbers \(x\) whose distance from 5 is less than 2. From Figure 7 we see that this is the interval \((3, 7)\).
EXAMPLE 24 Solve \(|3x + 2| \geq 4\).

**SOLUTION** By Properties 4 and 6 of absolute values, \(|3x + 2| \geq 4\) is equivalent to

\[
3x + 2 \geq 4 \quad \text{or} \quad 3x + 2 \leq -4
\]

In the first case, \(3x \geq 2\), which gives \(x \geq \frac{2}{3}\). In the second case, \(3x \leq -6\), which gives \(x \leq -2\). So the solution set is

\[
\left\{ x \mid x \leq -2 \text{ or } x \geq \frac{2}{3} \right\} = (-\infty, -2] \cup \left[\frac{2}{3}, \infty\right)
\]

### EXERCISES

1. \((-6ab)(0.5ac)\)
2. \(-(2x^2y)(-xy^4)\)
3. \(2x(x - 5)\)
4. \((4 - 3x)x\)
5. \(-2(4 - 3a)\)
6. \(8 - (4 + x)\)
7. \(4(x^2 - x + 2) - 5(x^2 - 2x + 1)\)
8. \(5(3t - 4) - (r^2 + 2) - 2(t - 3)\)
9. \((4x - 1)(3x + 7)\)
10. \(x(x - 1)(x + 2)\)
11. \((2x - 1)^2\)
12. \((2 + 3x)^2\)
13. \(y(6 - y)(5 + y)\)
14. \((r - 5)^2 - 2(r + 3)(8r - 1)\)
15. \((1 + 2x)(x^2 - 3x + 1)\)
16. \((1 + x - x^2)^2\)

17. \(\frac{2 + 8x}{2}\)
18. \(\frac{9b - 6}{3b}\)
19. \(\frac{1}{x + 5} + \frac{2}{x - 3}\)
20. \(\frac{1}{x + 1} + \frac{1}{x - 1}\)
21. \(u + 1 + \frac{u}{u + 1}\)
22. \(\frac{2}{a^2} - \frac{3}{ab} + \frac{4}{b^2}\)
23. \(\frac{x^2}{y}\)
24. \(\frac{x}{y}\)
25. \(\frac{-2r}{s}\left(\frac{x^2}{-6t}\right)\)
26. \(\frac{a}{bc} + \frac{b}{ac}\)
27. \(\frac{1 + \frac{1}{c - 1}}{1 - \frac{1}{c - 1}}\)
28. \(\frac{1}{1 + x}\)
29. \((x + 12a)^3\)
30. \(5ab - 8abc\)
31. \(x^2 + 7x + 6\)
32. \(x^2 - x - 6\)
33. \(x^2 - 2x - 8\)
34. \(2x^2 + 7x - 4\)
35. \(9x^2 - 36\)
36. \(8x^2 + 10x + 3\)
37. \(6x^2 - 5x - 6\)
38. \(x^2 + 10x + 25\)
39. \(x^3 + 1\)
40. \(4t^2 - 9s^2\)
41. \(4t^2 - 12t + 9\)
42. \(x^3 - 27\)
43. \(x^3 + 2x^2 + x\)
44. \(x^3 - 4x^2 + 5x - 2\)
45. \(x^3 + 3x^2 - x - 3\)
46. \(x^3 - 2x^2 - 23x + 60\)
47. \(x^3 + 5x^2 - 2x - 24\)
48. \(x^3 - 3x^2 - 4x + 12\)

49. \(x^2 + x - 2\)
50. \(\frac{2x^2 - 3x - 2}{x^2 - 4}\)
51. \(\frac{x^2 - 1}{x^2 - 9}\)
52. \(\frac{x^2 + 5x^2 + 6x}{x^2 - x - 12}\)
53. \(\frac{1}{x + 3} + \frac{1}{x - 9}\)
54. \(\frac{1}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4}\)

55. \(x^2 + 2x + 5\)
56. \(x^2 - 16x + 80\)
57. \(x^2 - 5x + 10\)
58. \(x^2 + 3x + 1\)
59. \(4x^2 + 4x - 2\)
60. \(3x^2 - 24x + 50\)

61. \(x^2 + 9x - 10 = 0\)
62. \(x^2 - 2x - 8 = 0\)
63. \(x^2 + 9x - 1 = 0\)
64. \(x^2 - 2x - 7 = 0\)
65. \(3x^2 + 5x + 1 = 0\)
66. \(2x^2 + 7x + 2 = 0\)
67. \(x^3 = 2x + 1 = 0\)
68. \(x^3 + 3x^2 + x - 1 = 0\)

69. \(2x^2 + 3x + 4\)
70. \(2x^2 + 9x + 4\)
71. \(3x^2 + x - 6\)
72. \(x^2 + 3x + 6\)

73. \((a + b)^6\)
74. \((a + b)^3\)
75. \((x^2 - 1)^4\)
76. \((3 + x^2)^5\)
77–82 Simplify the radicals.
77. \( \sqrt[3]{32} \sqrt{2} \)  
78. \( \sqrt{-\frac{2}{3}} \)  
79. \( \frac{\sqrt[3]{32} x^4}{\sqrt{2}} \)  
80. \( \sqrt{xy} \sqrt[3]{xy} \)  
81. \( \sqrt{15a^2b^3} \)  
82. \( \frac{\sqrt[3]{96a^6}}{\sqrt[3]{3a}} \)

83–100 Use the Laws of Exponents to rewrite and simplify the expression.
83. \( 3^{10} \times 9^8 \)  
84. \( 2^{16} \times 4^{10} \times 16^6 \)  
85. \( \frac{x^3(2x)^4}{x^3} \)  
86. \( a^n \times a^{2n+1} \)  
87. \( \frac{a^3b^4}{a^5b^3} \)  
88. \( \frac{x^{-1} + y^{-1}}{(x + y)^{-1}} \)  
89. \( 3^{1/2} \)  
90. \( 64^{4/3} \)  
91. \( (2x^2y^4)^{3/2} \)  
92. \( (x^{-3}y^{-2})^{-3/5} \)  
93. \( \sqrt[3]{y^6} \)  
94. \( \left( \sqrt[3]{a} \right)^3 \)  
95. \( \frac{1}{\sqrt[3]{y^5}} \)  
96. \( \sqrt[3]{\frac{1}{2^{3/2}}} \)  
97. \( \frac{\sqrt[3]{s^2}}{s^3} \)  
98. \( \sqrt{\frac{3^{1/2}}{\sqrt{3}}} \)  
99. \( \sqrt{\frac{1}{2^{2n+1}}} \times \sqrt{r^{-1}} \)

101–108 Rationalize the expression.
101. \( \frac{\sqrt{x} - 3}{x - 9} \)  
102. \( \frac{(1/\sqrt{x}) - 1}{x - 1} \)  
103. \( \frac{x\sqrt{x} - 8}{x - 4} \)  
104. \( \frac{\sqrt{2 + h} + \sqrt{2 - h}}{h} \)  
105. \( \frac{2}{3 - \sqrt{5}} \)  
106. \( \frac{1}{x - x \sqrt{x}} \)  
107. \( \sqrt{x^2 + 3x + 4} - x \)  
108. \( \sqrt{\frac{x^2 + x}{x} - \sqrt{x^2 - x}} \)

109–116 State whether or not the equation is true for all values of the variable.
109. \( \sqrt{x^2} = x \)  
110. \( \sqrt{x^2 + 4} = |x| + 2 \)  
111. \( \frac{16 + a}{16} = 1 + \frac{a}{16} \)  
112. \( \frac{x^{-1} + y^{-1}}{x + y} = x + y \)  
113. \( \frac{x}{x + y} = \frac{1}{1 + y} \)  
114. \( \frac{2}{4 + x} = \frac{1}{2} + \frac{2}{x} \)  
115. \( (x^3)^2 = x^7 \)  
116. \( 6 - 4(x + a) = 6 - 4x - 4a \)

117–126 Rewrite the expression without using the absolute value symbol.
117. \( 5 - 23 \)  
118. \( |\pi - 2| \)  
119. \( |\sqrt{5} - 5| \)  
120. \( ||-2| - |3|| \)  
121. \( |x - 2| \) if \( x < 2 \)  
122. \( |x - 2| \) if \( x > 2 \)  
123. \( |x + 1| \)  
124. \( |2x - 1| \)  
125. \( |x^2 + 1| \)  
126. \( |1 - 2x^2| \)

127–142 Solve the inequality in terms of intervals and illustrate the solution set on the real number line.
127. \( 2x + 7 > 3 \)  
128. \( 4 - 3x > 6 \)  
129. \( 1 - x \leq 2 \)  
130. \( 1 + 5x > 5 - 3x \)  
131. \( 0 \leq -1 - x < 1 \)  
132. \( 1 < 3x + 4 \leq 16 \)  
133. \( (x - 1)(x - 2) > 0 \)  
134. \( x^2 < 2x + 8 \)  
135. \( x^2 < 3 \)  
136. \( x^2 > 5 \)  
137. \( x^3 - x^2 \leq 0 \)  
138. \( (x + 1)(x - 2)(x + 3) \geq 0 \)  
139. \( x^3 > x \)  
140. \( x^3 + 3x < 4x^2 \)  
141. \( \frac{1}{x} < 4 \)  
142. \(-3 \leq \frac{1}{x} \leq 1 \)

143. The relationship between the Celsius and Fahrenheit temperature scales is given by \( C = \frac{5}{9}(F - 32) \), where \( C \) is the temperature in degrees Celsius and \( F \) is the temperature in degrees Fahrenheit. What interval on the Celsius scale corresponds to the temperature range \( 50 \leq F \leq 95 \)?

144. Use the relationship between \( C \) and \( F \) given in Exercise 143 to find the interval on the Fahrenheit scale corresponding to the temperature range \( 20 \leq C \leq 30 \).

145. As dry air moves upward, it expands and in so doing cools at a rate of about \( 1^\circ C \) for each 100-m rise, up to about 12 km. (a) If the ground temperature is \( 20^\circ C \), write a formula for the temperature at height \( h \). (b) What range of temperature can be expected if a plane takes off and reaches a maximum height of 5 km?

146. If a ball is thrown upward from the top of a building 128 ft high with an initial velocity of 16 ft/s, then the height \( h \) above the ground \( t \) seconds later will be

\[
h = 128 + 16t - 16t^2
\]

During what time interval will the ball be at least 32 ft above the ground?

147–148 Solve the equation for \( x \).
147. \( |x + 3| = 2x + 1 \)  
148. \( |3x + 5| = 1 \)

149–156 Solve the inequality.
149. \( |x| < 3 \)  
150. \( |x| \geq 3 \)  
151. \( |x - 4| < 1 \)  
152. \( |x - 6| < 0.1 \)  
153. \( |x + 5| \geq 2 \)  
154. \( |x + 1| \geq 3 \)  
155. \( |2x - 3| \leq 0.4 \)  
156. \( |5x - 2| < 6 \)

157. Solve the inequality \( a(bx - c) \geq bc \) for \( x \), assuming that \( a \), \( b \), and \( c \) are positive constants.

158. Solve the inequality \( ax + b < c \) for \( x \), assuming that \( a \), \( b \), and \( c \) are negative constants.

159 Prove that \( |ab| = |a| |b| \). [Hint: Use Equation 3.]

160. Show that if \( 0 < a < b \), then \( a^2 < b^2 \).
ANSWERS

1. \(-3a^2bc\)  2. \(2x^3y^3\)  3. \(2x^2 - 10x\)  4. \(4x^3 - 3x^2\)
5. \(-8 + 6a\)  6. \(4 - x\)  7. \(-x^2 + 6x + 3\)
8. \(-3t^2 + 21t - 22\)  9. \(12x^2 + 25x - 7\)
10. \(x^3 + x^2 - 2x\)  11. \(4x^4 - 4x + 1\)
12. \(9x^2 + 12x + 4\)  13. \(30y^4 + y^3 - y^6\)
14. \(-15x^2 + 56x + 31\)  15. \(2x^5 - 5x^4 - x + 1\)
16. \(x^4 - 2x^3 - 2x + 1\)  17. \(1 + 4x\)  18. \(3 - 2/b\)
19. \(\frac{3x + 7}{x^2 + 2x - 15}\)  20. \(\frac{2x}{x^2 - 1}\)  21. \(\frac{u^2 + 3u + 1}{u + 1}\)
22. \(\frac{a^2 - b^2}{a}\)  23. \(\frac{x}{y^2}\)  24. \(\frac{2x}{x^2 - 1}\)  25. \(\frac{2x^3 - 1}{x + 1}\)
26. \(\frac{a^2 + 4a^2}{b^2}\)  27. \(\frac{c}{c - 2}\)  28. \(\frac{3 + 2x}{2 + x}\)  29. \(2x(1 + 6x^2)\)
30. \(ab(5 - 8c)\)  31. \((x + 6)(x + 1)\)  32. \((x - 3)(x + 2)\)
33. \((x - 4)(x - 2)\)  34. \((2x - 1)(x + 4)\)
35. \(9(x - 2)(x + 2)\)  36. \((4x + 3)(2x + 1)\)
37. \(3(x + 2)(2x - 3)\)  38. \((x + 5)^2\)
39. \((t + 1)(t^2 - t + 1)\)  40. \((2t - 3s)(2t + 3s)\)
41. \((2t - 3)^2\)  42. \((x - 3)(x^2 + 3x + 9)\)
43. \(x(x + 1)^2\)  44. \((x - 1)^2(x - 2)\)
45. \((x - 1)(x + 1)(x + 3)\)  46. \((x - 3)(x^2 + 5)(x - 4)\)
47. \((x - 2)(x + 3)(x + 4)\)  48. \((x - 2)(x - 3)(x + 2)\)
49. \(\frac{2x + 1}{x + 2}\)  50. \(\frac{2x + 1}{x + 2}\)  51. \(\frac{x + 1}{x - 8}\)  52. \(\frac{x^2 + 2x}{x - 4}\)
53. \(\frac{x - 2}{x^2 - 9}\)  54. \(\frac{x^2 - 6x - 4}{(x - 1)(x + 2)(x - 4)}\)
55. \((x + 1)^2 + 4\)  56. \((x - 8)^2 + 16\)  57. \((x - \frac{1}{2})^2 + \frac{15}{4}\)
58. \((x + \frac{1}{2})^2 - \frac{1}{4}\)  59. \((2x + 1)^2 - 3\)
60. \(3(x - 4)^2 + 2\)  61. \(-10\)  62. \(-2, 4\)
63. \(-9 \pm \sqrt{85}\)  64. \(1 \pm 2\sqrt{3}\)  65. \(-5 \pm \sqrt{13}\)
66. \(-7 \pm \sqrt{33}\)  67. \(1 \pm \sqrt{5}\)  68. \(-1, -1 \pm \sqrt{2}\)
69. Irreducible  70. Not irreducible
71. Not irreducible (two real roots)  72. Irreducible
73. \(a^2 + 6a^2b + 15a^2b^2 + 20a^2b^3 + 15a^2b^4 + 6a^2b^5 + b^6\)
74. \(a^2 + 7a^2b + 21a^2b^2 + 35a^2b^3 + 35a^2b^4 + 21a^2b^5 + 7ab^6 + b^7\)
75. \(x^8 - 4x^6 + 6x^4 - 4x^2 + 1\)
76. \(243 + 405x^2 + 270x^4 + 90x^6 + 15x^8 + x^{10}\)
77. 8  78. \(-\frac{1}{3}\)  79. \(2 \{x\}\)  80. \(x^2 - 1\)
81. \(4a^2b\sqrt{5}\)  82. \(2a\)  83. \(3\sqrt{3}\)  84. \(2^{\sqrt{3}}\)  85. \(16x^{10}\)
86. \(a^{x+1}\)  87. \(a^2\)  88. \(\frac{xy}{y^2}\)  89. \(\sqrt{3}\)
90. \(\sqrt[3]{b}\)  91. 25  92. \(\sqrt{\pi}\)  93. \(2\sqrt{2}x\sqrt{y^5}\)
94. \(\frac{x^3}{y^{0.5}}\)  95. \(y^{6/5}\)  96. \(a^{3/4}\)  97. \(t^{-5/2}\)  98. \(\frac{1}{x^{1/2}}\)
**SOLUTIONS**

1. \((-6ab)(0.5ac) = (-6)(0.5)(a \cdot abc) = -3a^2bc\)
2. \(-2x^2y(-xy^4) = 2x^2xyy^4 = 2x^3y^5\)
3. \(2(x - 5) = 2x - 2 \cdot 5 = 2x^2 - 10x\)
4. \((4 - 3)x = 4 \cdot x - 3x \cdot x = 4x - 3x^2\)
5. \(-2(4 - 3a) = -2 \cdot 4 + 2 \cdot 3a = -8 + 6a\)
6. \(8 - (4 + x) = 8 - 4 - x = 4 - x\)
7. \(4(x^2 - x + 2) - 5(x^2 - 2x + 1) = 4x^2 - 4x + 8 - 5x^2 - 5(-2x) - 5\)
   \[= 4x^2 - 5x^2 - 4x + 10x + 8 - 5 = -x^2 + 6x + 3\]
8. \(5(3t - 4) - (t^2 + 2) = 2t(t - 3) = 15t - 20 - t^2 - 2 - 2t^2 + 6t\)
   \[= (-1 - 2)t^2 + (15 + 6)t - 20 - 2 = -3t^2 + 21t - 22\]
9. \((4x - 1)(3x + 7) = 4x(3x + 7) - (3x + 7) = 12x^2 + 28x - 3x - 7 = 12x^2 + 25x - 7\)
10. \(x(x - 1)(x + 2) = (x^2 - x)(x + 2) = x^2(x + 2) - x(x + 2) = x^2 + 2x^2 - x^2 - 2x\)
   \[= x^3 + x^2 - 2x\]
11. \((2x - 1)^2 = (2x)^2 - 2(2x)(1) + 1^2 = 4x^2 - 4x + 1\)
12. \((2 + 3x)^2 = 2^2 + 2(2)(3x) + (3x)^2 = 9x^2 + 12x + 4\)
13. \(y^4(6y)(5y) = y^4(6(5y) - y(5y)) = y^4(30 + 6y - 5y - y^2)\)
   \[= y^4(30 + y - y^2) = 30y^4 + y^5 - y^6\]
14. \((t - 5)^2 - 2(t + 3)(8t - 1) = t^2 - 2(5t) + 5^2 - 2(8t^2 - t + 24t - 3)\)
   \[= t^2 - 10t + 25 - 16t^2 + 2t - 48t + 6 = -15t^2 - 56t + 31\]
15. \((1 + 2x)(x^2 - 3x + 1) = 1(x^2 - 3x + 1) + 2x(x^2 - 3x + 1) = x^2 - 3x + 1 + 2x^3 - 6x^2 + 2x\)
   \[= 2x^3 - 5x^2 - x + 1\]
16. \((1 + x - x^2)^2 = (1 + x - x^2)(1 + x - x^2) = 1(1 + x - x^2) + x(1 + x - x^2) - x^2(1 + x - x^2)\)
   \[= 1 + x - x^2 + x^2 - x^3 - x^2 - x^3 + x^4 = x^4 - 2x^3 - x^2 + 2x + 1\]
17. \(\frac{2 + 8x}{2} = \frac{2}{2} + \frac{8x}{2} = 1 + 4x\)
18. \(\frac{9x - 6}{3b} = \frac{9b}{3b} - \frac{6}{3b} = 3 - \frac{2}{b}\)
19. \(\frac{1}{x + 5} + \frac{2}{x - 3} = \frac{(1)(x - 3) + 2(x + 5)}{(x + 5)(x - 3)} = \frac{x - 3 + 2x + 10}{(x + 5)(x - 3)} = \frac{3x + 7}{x^2 + 2x - 15}\)
20. \(\frac{1}{x + 1} + \frac{1}{x - 1} = \frac{1(x - 1) + 1(x + 1)}{(x + 1)(x - 1)} = \frac{x - 1 + x + 1}{x^2 - 1} = \frac{2x}{x^2 - 1}\)
21. \(u + 1 + \frac{u}{u + 1} = \frac{(u + 1)(u + 1) + u}{u + 1} = \frac{u^2 + 2u + 1 + u}{u + 1} = \frac{u^2 + 3u + 1}{u + 1}\)
22. \(\frac{2}{a^2} - \frac{3}{ab} + \frac{4}{b^2} = \frac{2b^2}{a^2b^2} - \frac{3ab}{a^2b^2} + \frac{4a^2}{a^2b^2} = \frac{2b^2 - 3ab + 4a^2}{a^2b^2}\)
23. \(\frac{x}{y} = \frac{x}{y} \cdot \frac{z}{z} = \frac{1}{z} \cdot \frac{x}{y} = \frac{x}{yz}\)
24. \(\frac{x}{y} = \frac{x}{y} \cdot \frac{1}{1} = \frac{z}{y} \cdot \frac{x}{z} = \frac{zx}{z}\)
25. \((\frac{-2r}{s})^2 = \frac{s^2}{-6t} = \frac{-2rs^2}{-6st} = \frac{rs}{3t}\)
26. \(\frac{a}{bc} \div \frac{b}{ac} = \frac{a}{bc} \times \frac{ac}{b} = \frac{a^2c}{b^2c} = \frac{a^2}{b^2}\)
27. \[ \frac{1 + \frac{1}{c}}{c - 1} = \frac{c - 1 + 1}{c - 1} = \frac{c}{c - 1} = \frac{c - 1}{c - 2} \]

28. \[ 1 + \frac{1}{1 + \frac{1}{x}} = 1 + \frac{1}{1 + x + \frac{1}{x + 1}} = 1 + \frac{1 + x}{2 + x} = \frac{2 + x + 1 + x}{2 + x} = \frac{3 + 2x}{2 + x} \]

29. \( 2x + 12x^3 = 2x \cdot 1 + 2x \cdot 6x^2 = 2x(1 + 6x^2) \)

30. \( 5ab - 8abc = ab \cdot 5 - ab \cdot 8c = ab(5 - 8c) \)

31. The two integers that add to give 7 and multiply to give 6 are 6 and 1. Therefore \( x^2 + 7x + 6 = (x + 6)(x + 1) \).

32. The two integers that add to give -1 and multiply to give -6 are -3 and 2.

Therefore \( x^2 - 2x - 6 = (x - 3)(x + 2) \).

33. The two integers that add to give -2 and multiply to give -8 are -4 and 2.

Therefore \( x^2 - 2x - 8 = (x - 4)(x + 2) \).

34. \( 2x^2 + 7x - 4 = (2x - 1)(x + 4) \)

35. \( 9x^2 - 36 = 9(x^2 - 4) = 9(x - 2)(x + 2) \) [Equation 3 with \( a = x, b = 2 \)]

36. \( 8x^2 + 10x + 3 = (4x + 3)(2x + 1) \)

37. \( 6x^2 - 5x - 6 = (3x + 2)(2x - 3) \)

38. \( x^2 + 10x + 25 = (x + 5)^2 \) [Equation 1 with \( a = -x, b = 5 \)]

39. \( t^2 + 1 = (t + 1)(t - t + 1) \) [Equation 5 with \( a = t, b = 1 \)]

40. \( 4t^2 - 9s^2 = (2t)^2 - (3s)^2 = (2t - 3s)(2t + 3s) \) [Equation 3 with \( a = 2t, b = 3s \)]

41. \( 4t^2 - 12t + 9 = (2t - 3)^2 \) [Equation 2 with \( a = 2t, b = 3 \)]

42. \( x^3 - 27 = (x - 3)(x^2 + 3x + 9) \) [Equation 4 with \( a = x, b = 3 \)]

43. \( x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)^2 \) [Equation 1 with \( a = x, b = 1 \)]

44. Let \( p(x) = x^3 - 4x^2 + 5x - 2 \), and notice that \( p(1) = 0 \), so by the Factor Theorem, \( x - 1 \) is a factor.

Use long division (as in Example 8):

\[
\begin{align*}
x^2 - 3x + 2 & \quad \overline{x - 1} \\
x^3 - 4x^2 + 5x - 2 & \\
- x^3 + x^2 & \\
- \frac{3x^2 + 5x}{- 3x^2 + 3x} & \\
2x - 2 & \\
2x - 2 & \\
\end{align*}
\]

Therefore \( x^3 - 4x^2 + 5x - 2 = (x - 1)(x^2 - 3x + 2) = (x - 1)(x - 2)(x - 1) = (x - 1)^2(x - 2) \).

45. Let \( p(x) = x^3 + 3x^2 - x - 3 \), and notice that \( p(1) = 0 \), so by the Factor Theorem, \( x - 1 \) is a factor.

Use long division (as in Example 8):

\[
\begin{align*}
x^2 + 4x + 3 & \quad \overline{x - 1} \\
x^3 + 3x^2 - x - 3 & \\
- x^3 + x^2 & \\
- \frac{4x^2 - x}{4x^2 - 4x} & \\
3x - 3 & \\
3x - 3 & \\
\end{align*}
\]

Therefore \( x^3 + 3x^2 - x - 3 = (x - 1)(x^2 + 4x + 3) = (x - 1)(x + 1)(x + 3) \).
46. Let \( p(x) = x^3 - 2x^2 - 23x + 60 \), and notice that \( p(3) = 0 \), so by the Factor Theorem, \( (x - 3) \) is a factor.

Use long division (as in Example 8):

\[
x^3 + x - 20 \\
x - 3 \overline{\phantom{x^3} x^3 - 2x^2 - 23x + 60} \\
\underline{x^3 - 3x^2} \\
x^2 - 23x \\
\underline{x^2 - 3x} \\
- 20x + 60 \\
- 20x + 60
\]

Therefore \( x^3 - 2x^2 - 23x + 60 = (x - 3)(x^2 + x - 20) = (x - 3)(x + 5)(x - 4) \).

47. Let \( p(x) = x^3 + 5x^2 - 2x - 24 \), and notice that \( p(2) = 2^3 + 5(2)^2 - 2(2) - 24 = 0 \), so by the Factor Theorem, \( (x - 2) \) is a factor.

Use long division (as in Example 8):

\[
x^3 + 7x + 12 \\
x - 2 \overline{\phantom{x^3} x^3 + 5x^2 - 2x - 24} \\
\underline{x^3 - 2x^2} \\
7x^2 - 2x \\
\underline{7x^2 - 14x} \\
12x - 24 \\
12x - 24
\]

Therefore \( x^3 + 5x^2 - 2x - 24 = (x - 2)(x^2 + 7x + 12) = (x - 2)(x + 3)(x + 4) \).

48. Let \( p(x) = x^3 - 3x^2 - 4x + 12 \), and notice that \( p(2) = 0 \), so by the Factor Theorem, \( (x - 2) \) is a factor.

Use long division (as in Example 8):

\[
x^2 - x - 6 \\
x - 2 \overline{\phantom{x^2} x^3 - 3x^2 - 4x + 12} \\
\underline{x^3 - 2x^2} \\
x^2 - 4x \\
\underline{x^2 - 2x} \\
- 6x + 12 \\
- 6x + 12
\]

Therefore \( x^3 - 3x^2 - 4x + 12 = (x - 2)(x^2 - x - 6) = (x - 2)(x - 3)(x + 2) \).

49. \[
\frac{x^2 + x - 2}{x^2 - 3x + 2} = \frac{(x + 2)(x - 1)}{(x - 2)(x - 1)} = \frac{x + 2}{x - 2}
\]

50. \[
\frac{2x^2 - 3x - 2}{x^2 - 4} = \frac{(2x + 1)(x - 2)}{(x - 2)(x + 2)} = \frac{2x + 1}{x + 2}
\]

51. \[
\frac{x^2 - 1}{x^2 - 9x + 8} = \frac{(x - 1)(x + 1)}{(x - 8)(x - 1)} = \frac{x + 1}{x - 8}
\]

52. \[
\frac{x^3 + 5x^2 + 6x}{x^2 - x - 12} = \frac{x(x^2 + 5x + 6)}{(x - 4)(x + 3)} = \frac{x(x + 3)(x + 2)}{(x - 4)(x + 3)} = \frac{x(x + 2)}{x - 4}
\]

53. \[
\frac{1}{x + 3} + \frac{1}{x^2 - 9} = \frac{1}{x + 3} + \frac{1}{(x - 3)(x + 3)} = \frac{1(x - 3) + 1}{(x - 3)(x + 3)} = \frac{x - 2}{x^2 - 9}
\]

54. \[
\frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} = \frac{x}{(x - 1)(x + 2)} - \frac{2}{(x - 1)(x + 2)(x - 4)} = \frac{x(x - 4) - 2(x + 2)}{(x - 1)(x + 2)(x - 4)} = \frac{x^2 - 4x - 2x - 4}{(x - 1)(x + 2)(x - 4)} = \frac{x^2 - 6x - 4}{(x - 1)(x + 2)(x - 4)}
\]

55. \[
x^2 + 2x + 5 = [x^2 + 2x] + 5 = [x^2 + 2x + (1)^2] - (1)^2 + 5 = (x + 1)^2 + 5 - 1 = (x + 1)^2 + 4
\]
56. \( x^2 - 16x + 80 = [x^2 - 16x] + 80 = [x^2 - 16x + (8)^2 - (8)^2] + 80 = (x - 8)^2 + 80 - 64 = (x - 8)^2 + 16 \)

57. \( x^2 - 5x + 10 = [x^2 - 5x + (-\frac{5}{2})^2 - (-\frac{5}{2})^2] + 10 = (x - \frac{5}{2})^2 + 10 - \frac{25}{4} = (x - \frac{5}{2})^2 + \frac{11}{4} \)

58. \( x^2 + 3x + 1 = [x^2 + 3x + (\frac{3}{2})^2 - (\frac{3}{2})^2] + 1 = (x + \frac{3}{2})^2 + 1 - (\frac{3}{2})^2 = (x + \frac{3}{2})^2 - \frac{5}{4} \)

59. \( 4x^2 + 4x - 2 = 4[x^2 + x + (\frac{1}{2})^2 - (\frac{1}{2})^2] - 2 = 4(x + \frac{1}{2})^2 - 2 - 4(\frac{1}{4}) = 4(x + \frac{1}{2})^2 - 3 \)

60. \( 3x^2 - 24x + 50 = 3[x^2 - 8x] + 50 = 3[x^2 - 8x + (-4)^2 - (-4)^2] + 50 = 3(x - 4)^2 + 50 - 3(-4)^2 = 3(x - 4)^2 + 2 \)

61. \( x^2 - 9x - 10 = 0 \iff (x + 10)(x - 1) = 0 \iff x + 10 = 0 \text{ or } x - 1 = 0 \iff x = -10 \text{ or } x = 1. \)

62. \( x^2 - 2x - 8 = 0 \iff (x - 4)(x + 2) = 0 \iff x - 4 = 0 \text{ or } x + 2 = 0 \iff x = 4 \text{ or } x = -2. \)

63. Using the quadratic formula, \( x^2 + 9x - 1 = 0 \iff x = \frac{-9 \pm \sqrt{9^2 - 4(1)(-1)}}{2(1)} = \frac{9 \pm \sqrt{85}}{2}. \)

64. Using the quadratic formula, \( x^2 - 2x - 7 = 0 \iff x = \frac{2 \pm \sqrt{4 - 4(1)(-7)}}{2} = \frac{2 \pm \sqrt{32}}{2} = 1 \pm \sqrt{2}. \)

65. Using the quadratic formula, \( 3x^2 + 5x + 1 = 0 \iff x = \frac{-5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)} = \frac{-5 \pm \sqrt{13}}{6}. \)

66. Using the quadratic formula, \( 2x^2 + 7x + 2 = 0 \iff x = \frac{-7 \pm \sqrt{49 - 4(2)(2)}}{2(2)} = \frac{-7 \pm \sqrt{33}}{4}. \)

67. Let \( p(x) = x^3 - 2x + 1, \) and notice that \( p(1) = 0, \) so by the Factor Theorem, \((x - 1)\) is a factor.

Use long division:

\[
\begin{array}{rrrr}
& x^2 & + x & - 1 \\
- & x & \hline
x^3 - x^2 &  & & - 1 \\
\hline
& x^2 & - 2x &  \\
\hline
& x^2 & \hline
\end{array}
\]

Therefore \( x^3 - 2x + 1 = (x - 1)(x^2 + x - 1) = 0 \iff x - 1 = 0 \text{ or } x^2 + x - 1 = 0 \iff x = 1 \text{ or } [\text{using the quadratic formula}] \ x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}. \)

68. Let \( p(x) = x^3 + 3x^2 + x - 1, \) and notice that \( p(-1) = 0, \) so by the Factor Theorem, \((x + 1)\) is a factor.

Use long division:

\[
\begin{array}{rrrr}
& x^2 & + 2x & - 1 \\
- & x & \hline
x^3 + x^2 &  & & - 1 \\
\hline
& 2x^2 & \hline
& 2x^2 & \hline
\end{array}
\]

Therefore \( x^3 + 3x^2 + x - 1 = (x + 1)(x^2 + 2x - 1) = 0 \iff x + 1 = 0 \text{ or } x^2 + 2x - 1 = 0 \iff x = -1 \text{ or } [\text{using the quadratic formula}] \ x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2} = -1 \pm \sqrt{2}. \)
69. $2x^2 + 3x + 4$ is irreducible because its discriminant is negative: $b^2 - 4ac = 9 - 4(2)(4) = -23 < 0$.

70. The quadratic $2x^2 + 9x + 4$ is not irreducible because $b^2 - 4ac = 9^2 - 4(2)(4) = 49 > 0$.

71. $3x^2 + x - 6$ is not irreducible because its discriminant is nonnegative: $b^2 - 4ac = 1 - 4(3)(-6) = 73 > 0$.

72. The quadratic $x^2 + 3x + 6$ is irreducible because $b^2 - 4ac = 3^2 - 4(1)(6) = -15 < 0$.

73. Using the Binomial Theorem with $k = 6$ we have

\[ (a + b)^6 = a^6 + 6a^5b + \frac{6 \cdot 5}{1 \cdot 2} a^4b^2 + \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} a^3b^3 + \frac{6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} a^2b^4 + 6ab^5 + b^6 \]

74. Using the Binomial Theorem with $k = 7$ we have

\[ (a + b)^7 = a^7 + 7a^6b + \frac{7 \cdot 6}{1 \cdot 2} a^5b^2 + \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3} a^4b^3 + \frac{7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4} a^3b^4 + 7a^2b^5 + 7ab^6 + b^7 \]

75. Using the Binomial Theorem with $a = x^2$, $b = -1$, $k = 4$ we have

\[ (x^2 - 1)^4 = [x^2 + (-1)]^4 = (x^2)^4 + 4(4)(x^2)^3(-1) + \frac{4 \cdot 3}{1 \cdot 2} (2)(x^2)(-1)^2 + 4(x^2)(-1)^3 + (-1)^4 \]

\[ = x^8 - 4x^6 + 6x^4 - 4x^2 + 1 \]

76. Using the Binomial Theorem with $a = 3$, $b = x^2$, $k = 5$ we have

\[ (3 + x^2)^5 = 3^5 + 5(3)^4(x^2)^1 + \frac{5 \cdot 4}{1 \cdot 2} (3)^3(x^2)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} (3)^2(x^2)^3 + 5(3)(x^2)^4 + (x^2)^5 \]

\[ = 243 + 405x^2 + 270x^4 + 90x^6 + 15x^8 + x^{10} \]

77. Using Equation 10, $\sqrt[3]{32} \cdot \sqrt{2} = \sqrt[3]{32 \cdot 2} = \sqrt[3]{64} = 8$.

78. $\frac{\sqrt[3]{-2}}{\sqrt[3]{54}} = \sqrt[3]{\frac{-2}{54}} = \sqrt[3]{\frac{-2}{27}} = \frac{-1}{3} = -\frac{1}{3}$

79. Using Equation 10, $\sqrt[4]{x^4} \cdot \sqrt[2]{x^2} = \sqrt[4]{x^4 \cdot x^2} = \sqrt[4]{x^6} = \sqrt[4]{x^6} \cdot |x| = 2 \cdot |x|$.

80. $\sqrt{xy} \cdot \sqrt{x^2} = \sqrt{(xy)(x^2)} = \sqrt{x^3y} = x^{3/2} \cdot |y|$

81. Using Equation 10, $\sqrt[10]{16a^4b^3} = \sqrt[10]{16a^4b^3} \cdot |x| = 4a^2b^{3/2} = 4a^2b^{3/2} = 4a^2b^{1/2}$

82. $\sqrt[3]{\sqrt[6]{a^6}} = \sqrt[3]{\sqrt[6]{a^6}} = \sqrt[3]{\sqrt[6]{a^6}} = 2a$

83. Using Laws 3 and 1 of Exponents respectively, $3^{10} \cdot 9^8 = 3^{10} \cdot (3^2)^8 = 3^{10} \cdot 3^{16} = 3^{10 + 16} = 3^{26}$.

84. Using Laws 3 and 1, $2^{16} \times 4^{10} = 2^{16} \times (2^2)^{10} = 2^{16} \times 2^{20} \times 2^{24} = 2^{60}$.

85. Using Laws 4, 1, and 2 of Exponents respectively,

\[ x^a (2x^b)^4 = \frac{x^a (2x^b)^4}{x^a} = \frac{16x^{a+4}}{x^a} = 16x^{a+4} = 16x^{10} \]

86. Using Laws 1 and 2, $a^n \times a^{n+1} = a^{n+2n+1} = a^{n+2n+1} = a^{3n+1-(n-2)} = a^{2n+3}$.

87. Using Law 2 of Exponents,

\[ a^{-n} b^m = a^{3} b^{-(n-m)} = a^{3} b^{-(n-2)} = a^{3n+1-(n-2)} = a^{2n+3} \]

88. $x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y} = (x + y) \left( \frac{1}{x} + \frac{1}{y} \right) = (x + y) \left( \frac{y + x}{xy} \right) = (y + x)^2 / xy$

89. By definitions 3 and 4 for exponents respectively, $3^{-1/2} = 1 \div 3^{-1/2} = \frac{1}{\sqrt{3}}$

90. $96^{1/3} = \sqrt[3]{96} = \sqrt[3]{32 \cdot 3} = \sqrt[3]{32} \cdot \sqrt[3]{3} = 2 \cdot \sqrt[3]{3}$

91. Using definition 4 for exponents, $125^{1/3} = \sqrt[3]{125} = 5^3 = 25$.

92. $64^{-4/3} = \frac{1}{64^{1/3}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4} = \frac{1}{256}$
93. \((2x^2y^4)^{3/2} = 2^{3/2}(x^2)^{3/2}(y^4)^{3/2} = 2 \cdot 2^{1/2} \left[ \sqrt{x^2} \right]^{3} \left[ \sqrt{y^4} \right]^{3} = 2 \sqrt{2} |x|^3 \left( y^2 \right)^3 = 2 \sqrt{2} |x|^3 y^6 \)

94. \((x^{-5}y^3 z^{10})^{-3/5} = (x^{-5})^{-3/5}(y^3)^{-3/5}(z^{10})^{-3/5} = x^{15/5}y^{-9/5}z^{-30/5} = \frac{x^3}{y^{9/5}z^6} \)

95. \(5^{\sqrt{3}} = y^{6/5}\) by definition for exponents.

96. \((\sqrt{a})^3 = (a^{1/2})^3 = a^{3/4} \)

97. \(\frac{1}{(\sqrt[6]{7})^5} = \frac{1}{(7^{1/6})^5} = \frac{1}{7^{5/6}} = t^{-5/2} \)

98. \(\sqrt[4]{x^5} = \sqrt[4]{x^{5/4}} = \sqrt[4]{x^{(5/4)-3/4}} = x^{1/8} = \frac{1}{x^{1/8}} \)

99. \(\sqrt[4]{\frac{t^{1/2} \sqrt{s.t}}{s^2}} = \left( \frac{t^{1/2} \sqrt{s \cdot t}}{s^2} \right)^{1/4} = \left( t^{(1/2)} \cdot (1/2) \sqrt{t} - (2/3) \right)^{1/4} = (t^{s^{-1/6}})^{1/4} \)

100. \(\sqrt[4]{x^n + t} \times \sqrt[4]{x^n + t} = \sqrt[4]{x^{2n} + 2t} = \sqrt[4]{x^{2n}} = (x^n)^{1/4} = r^{2n/4} = r^n/2 \)

101. \(\sqrt{x + 3} = \frac{\sqrt{x} - 3 \cdot \sqrt{x} + 3}{x - 9} \cdot \frac{x - 9}{(x - 9) (\sqrt{x} + 3)} = \frac{1}{\sqrt{x} + 3} \)

102. \(\sqrt{x} + 1 \times \frac{\sqrt{x} + 1}{x - 1} = \frac{1}{x - 1} \cdot \frac{x - 1}{\sqrt{x} + 1} = \frac{1 - x}{\sqrt{x} + 1} = \frac{1 - x}{x} \)

103. \(x \sqrt{x - 8} = \frac{x \sqrt{x - 8} \cdot x \sqrt{x - 8}}{x \sqrt{x - 8} \cdot x \sqrt{x - 8}} = \frac{x^3 - 64}{(x - 4)(x \sqrt{x - 8})} \)

[Equation 4 with a = x, b = 4] \(= \frac{x^2 + 4x + 16}{x \sqrt{x - 8}} \)

104. \(\frac{\sqrt{2} + h + \sqrt{2} - h}{h} = \frac{\sqrt{2} + h + \sqrt{2} - h}{h} \cdot \frac{\sqrt{2} + h - \sqrt{2} - h}{\sqrt{2} + h - \sqrt{2} - h} = \frac{2 + h - (2 - h)}{h \sqrt{2} + h - \sqrt{2} - h} \)

105. \(\frac{2}{3 - \sqrt{5}} = \frac{2}{3 - \sqrt{5}} \cdot \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{2(3 + \sqrt{5})}{3 - 5} = \frac{3 + \sqrt{5}}{2} \)

106. \(\frac{1}{\sqrt{x} - \sqrt{y}} = \frac{1}{\sqrt{x} - \sqrt{y}} \cdot \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \frac{\sqrt{x} + \sqrt{y}}{x - y} \)

107. \(\sqrt{x^2 + 3x + 4} - x = (\sqrt{x^2 + 3x + 4} - x) \cdot \sqrt{x^2 + 3x + 4} = \frac{x^2 + 3x + 4 - x^2}{\sqrt{x^2 + 3x + 4}} = \frac{3x + 4}{\sqrt{x^2 + 3x + 4}} \)

108. \(\sqrt{x^2 + x} - \sqrt{x^2 - x} = (\sqrt{x^2 + x} - \sqrt{x^2 - x}) \cdot \frac{\sqrt{x^2 + x} + \sqrt{x^2 - x}}{\sqrt{x^2 + x} + \sqrt{x^2 - x}} = \frac{x^2 + x - (x^2 - x)}{\sqrt{x^2 + x} + \sqrt{x^2 - x}} \)

109. False. See Example 14(b).

110. False. See the warning after Equation 10.

111. True: \(\frac{16 + a}{16} = \frac{16}{16} + \frac{a}{16} = 1 + \frac{a}{16} \)

112. False: \(\frac{1}{x - 1 + y - 1} = \frac{1}{x + y} \neq \frac{xy}{x + y} \)

113. False.

114. False. See the warning on page 2.
115. False. Using Law 3 of Exponents, \( (x^3)^4 = x^{3 \cdot 4} = x^{12} \neq x^7 \).

116. True.

117. \(|5 - 23| = |-18| = 18\)

118. \(|\pi - 2| = \pi - 2\) because \(\pi - 2 > 0\).

119. \(|\sqrt{5} - 5| = -(\sqrt{5} - 5) = 5 - \sqrt{5}\) because \(\sqrt{5} - 5 < 0\).

120. \(|-2| - |-3| = |2 - 3| = |-1| = 1\)

121. If \(x < 2\), \(x - 2 < 0\), so \(|x - 2| = -(x - 2) = 2 - x\).

122. If \(x > 2\), \(x - 2 > 0\), so \(|x - 2| = x - 2\).

123. \(|x + 1| = \begin{cases} 
  x + 1 & \text{if } x + 1 \geq 0 \\
  -(x + 1) & \text{if } x + 1 < 0
\end{cases}
= \begin{cases} 
  x + 1 & \text{if } x \geq -1 \\
  -x - 1 & \text{if } x < -1
\end{cases}

124. \(|2x - 1| = \begin{cases} 
  2x - 1 & \text{if } 2x - 1 \geq 0 \\
  -(2x - 1) & \text{if } 2x - 1 < 0
\end{cases}
= \begin{cases} 
  2x - 1 & \text{if } x \geq \frac{1}{2} \\
  1 - 2x & \text{if } x < \frac{1}{2}
\end{cases}

125. \(|x^2 + 1| = x^2 + 1\) (since \(x^2 + 1 \geq 0\) for all \(x\)).

126. Determine when \(1 - 2x^2 < 0 \iff 1 < 2x^2 \iff x^2 > \frac{1}{2} \iff \sqrt{x^2} > \sqrt{\frac{1}{2}} \iff |x| > \sqrt{\frac{1}{2}} \iff\)

\(x < -\sqrt{\frac{1}{2}}\) or \(x > \sqrt{\frac{1}{2}}\). Thus, \(|1 - 2x^2| = \begin{cases} 
  1 - 2x^2 & \text{if } -\sqrt{\frac{1}{2}} \leq x \leq \sqrt{\frac{1}{2}} \\
  2x^2 - 1 & \text{if } x < -\sqrt{\frac{1}{2}} \text{ or } x > \sqrt{\frac{1}{2}}
\end{cases}\)

127. \(2x + 7 > 3 \iff 2x > -4 \iff x > -2\), so \(x \in (-2, \infty)\).

128. \(4 - 3x \geq 6 \iff -3x \geq 2 \iff x \leq -\frac{2}{3}\), so \(x \in (-\infty, -\frac{2}{3})\).

129. \(1 - x \leq 2 \iff -x \leq 1 \iff x \geq -1\), so \(x \in [-1, \infty)\).

130. \(1 + 5x > 5 - 3x \iff 8x > 4 \iff x > \frac{1}{2}\), so \(x \in (\frac{1}{2}, \infty)\).

131. \(0 \leq 1 - x < 1 \iff -1 \leq -x < 0 \iff 1 \geq x > 0\), so \(x \in (0, 1]\).

132. \(1 < 3x + 4 \leq 16 \iff -3 < 3x \leq 12 \iff -1 < x \leq 4\), so \(x \in (-1, 4]\).

133. \((x - 1)(x - 2) > 0\). \text{Case 1:} (both factors are positive, so their product is positive)

\[x - 1 > 0 \iff x > 1, \text{ and } x - 2 > 0 \iff x > 2, \text{ so } x \in (2, \infty)\].

\text{Case 2:} (both factors are negative, so their product is positive)

\[x - 1 < 0 \iff x < 1, \text{ and } x - 2 < 0 \iff x < 2, \text{ so } x \in (-\infty, 1)\].

Thus, the solution set is \((-\infty, 1) \cup (2, \infty)\).

134. \(x^2 < 2x + 8 \iff x^2 - 2x - 8 < 0 \iff (x - 4)(x + 2) < 0\). \text{Case 1:} \(x > 4\) and \(x < -2\), which is impossible.

\text{Case 2:} \(x < 4\) and \(x > -2\). Thus, the solution set is \((-2, 4)\).

135. \(x^2 < 3 \iff x^2 - 3 < 0 \iff (x - \sqrt{3})(x + \sqrt{3}) < 0\). \text{Case 1:} \(x > \sqrt{3}\) and \(x < -\sqrt{3}\), which is impossible.

\text{Case 2:} \(x < \sqrt{3}\) and \(x > -\sqrt{3}\). Thus, the solution set is \((-\sqrt{3}, \sqrt{3})\).

\text{Another method:} \(x^2 < 3 \iff |x| < \sqrt{3} \iff -\sqrt{3} < x < \sqrt{3}\).
136. \( x^2 \geq 5 \iff x^2 - 5 \geq 0 \iff (x - \sqrt{5})(x + \sqrt{5}) \geq 0. \) \textit{Case 1:} \( x \geq \sqrt{5} \) and \( x \geq -\sqrt{5}, \) so \( x \in [\sqrt{5}, \infty). \) 
\textit{Case 2:} \( x \leq \sqrt{5} \) and \( x \leq -\sqrt{5}, \) so \( x \in (-\infty, -\sqrt{5}]. \) Thus, the solution set is \( (-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty). \)

\textit{Another method:} \( x^2 \geq 5 \iff |x| \geq \sqrt{5} \iff x \geq \sqrt{5} \) or \( x \leq -\sqrt{5}. \)

137. \( x^3 - x^2 \leq 0 \iff x(x-1) \leq 0. \) Since \( x^2 \geq 0 \) for all \( x, \) the inequality is satisfied when \( x-1 \leq 0 \iff x \leq 1. \) Thus, the solution set is \( (-\infty, 1]. \)

138. \((x+1)(x-2)(x+3) = 0 \iff x = -1, 2, \) or \(-3. \) Construct a chart:

<table>
<thead>
<tr>
<th>Interval</th>
<th>( x+1 )</th>
<th>( x-2 )</th>
<th>( x+3 )</th>
<th>((x+1)(x-2)(x+3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; -3 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(-3 &lt; x &lt; -1 )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(-1 &lt; x &lt; 2 )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( x &gt; 2 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Thus, \((x+1)(x-2)(x+3) \geq 0 \) on \([-3, -1]\) and \([2, \infty), \) and the solution set is \([-3, -1] \cup [2, \infty). \)

139. \( x^3 > x \iff x^3 - x > 0 \iff x(x^2 - 1) > 0 \iff x(x-1)(x+1) > 0. \) Construct a chart:

<table>
<thead>
<tr>
<th>Interval</th>
<th>( x )</th>
<th>( x-1 )</th>
<th>( x+1 )</th>
<th>( x(x-1)(x+1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; -1 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(-1 &lt; x &lt; 0 )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( 0 &lt; x &lt; 1 )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( x &gt; 1 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Since \( x^3 > x \) when the last column is positive, the solution set is \((-1, 0) \cup (1, \infty). \)

140. \( x^3 + 3x < 4x^2 \iff x^3 - 4x^2 + 3x < 0 \iff x(x^2 - 4x + 3) < 0 \iff x(x-1)(x-3) < 0. \) Construct a chart:

<table>
<thead>
<tr>
<th>Interval</th>
<th>( x )</th>
<th>( x-1 )</th>
<th>( x-3 )</th>
<th>( x(x-1)(x-3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 0 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( 0 &lt; x &lt; 1 )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( 1 &lt; x &lt; 3 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( x &gt; 3 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Thus, the solution set is \((-\infty, 0) \cup (1, 3). \)

141. \( 1/x < 4. \) This is clearly true for \( x < 0. \) So suppose \( x > 0. \) then \( 1/x < 4 \iff 1 < 4x \iff \frac{1}{4} < x. \) Thus, the solution set is \((-\infty, 0) \cup \left(\frac{1}{4}, \infty\right). \)
142. \(-3 < 1/x \leq 1\). We solve the two inequalities separately and take the intersection of the solution sets. First, 
\(-3 < 1/x\) is clearly true for \(x > 0\). So suppose \(x < 0\). Then \(-3 < 1/x\) \(\iff\) \(-3x > 1\) \(\iff\) \(x < -\frac{1}{3}\), so for this inequality, the solution set is \((-\infty, -\frac{1}{3})\) \(\cup\) \((0, \infty)\). Now \(1/x \leq 1\) is clearly true if \(x < 0\). So suppose \(x > 0\). Then 
\(1/x \leq 1\) \(\iff\) \(1 \leq x\), and the solution set here is \((-\infty, 0) \cup [1, \infty)\). Taking the intersection of the two solution sets gives the final solution set: \((-\infty, -\frac{1}{3}) \cup [1, \infty)\).

143. \(C = \frac{3}{5}(F - 32) \Rightarrow F = \frac{9}{5}C + 32\). So \(50 \leq F \leq 95 \Rightarrow 50 \leq \frac{9}{5}C + 32 \leq 95 \Rightarrow 18 \leq \frac{9}{5}C \leq 63 \Rightarrow 10 \leq C \leq 35\). So the interval is \([10, 35]\).

144. Since \(20 \leq C \leq 30\) and \(C = \frac{3}{5}(F - 32)\), we have \(20 \leq \frac{3}{5}(F - 32) \leq 30 \Rightarrow 36 \leq F - 32 \leq 54 \Rightarrow 68 \leq F \leq 86\). So the interval is \([68, 86]\).

145. (a) Let \(T\) represent the temperature in degrees Celsius and \(h\) the height in km. \(T = 20\) when \(h = 0\) and \(T\) decreases by \(10\)°C for every km (1°C for each 100-m rise). Thus, \(T = 20 - 10h\) when \(0 \leq h \leq 12\).

(b) From part (a), \(T = 20 - 10h \Rightarrow 10h = 20 - T \Rightarrow h = 2 - T/10\). So \(0 \leq h \leq 5 \Rightarrow 0 \leq 2 - T/10 \leq 5 \Rightarrow -2 \leq -T/10 \leq 3 \Rightarrow -20 \leq -T \leq 30 \Rightarrow 20 \geq T \geq -30 \Rightarrow -30 \leq T \leq 20\). Thus, the range of temperatures (in °C) to be expected is \([-30, 20]\).

146. The ball will be at least 32 ft above the ground if \(h \geq 32 \iff 128 + 16t - 16t^2 \geq 32 \iff 16t^2 - 16t - 96 \leq 0 \iff 16(t - 3)(t + 2) \leq 0, t = 3\) and \(t = -2\) are endpoints of the interval we’re looking for, and constructing a table gives \(-2 \leq t \leq 3\). But \(t \geq 0\), so the ball will be at least 32 ft above the ground in the time interval \([0, 3]\).

147. \(|x + 3| = |2x + 1| \iff\) either \(x + 3 = 2x + 1\) or \(x + 3 = -(2x + 1)\). In the first case, \(x = 2\), and in the second case, \(x + 3 = -2x - 1 \iff 3x = -4 \iff x = -\frac{4}{3}\). So the solutions are \(-\frac{4}{3}\) and 2.

148. \(|3x + 5| = 1 \iff\) either \(3x + 5 = 1\) or \(-1\). In the first case, \(3x = -4 \iff x = -\frac{4}{3}\), and in the second case, \(3x = -6 \iff x = -2\). So the solutions are \(-2\) and \(-\frac{4}{3}\).

149. By Property 5 of absolute values, \(|x| < 3 \iff -3 < x < 3\), so \(x \in (-3, 3)\).

150. By Properties 4 and 6 of absolute values, \(|x| \geq 3 \iff x \leq -3\) or \(x \geq 3\), so \(x \in (-\infty, -3] \cup [3, \infty)\).

151. \(|x - 4| < 1 \iff -1 < x - 4 < 1 \iff 3 < x < 5\), so \(x \in (3, 5)\).

152. \(|x - 6| < 0.1 \iff -0.1 < x - 6 < 0.1 \iff 5.9 < x < 6.1\), so \(x \in (5.9, 6.1)\).

153. \(|x + 5| \geq 2 \iff x + 5 \geq 2\) or \(x + 5 \leq -2 \iff x \geq -3\) or \(x \leq -7\), so \(x \in (-\infty, -7] \cup [-3, \infty)\).

154. \(|x + 1| \geq 3 \iff x + 1 \geq 3\) or \(x + 1 \leq -3 \iff x \geq 2\) or \(x \leq -4\), so \(x \in (-\infty, -4] \cup [2, \infty)\).

155. \(|2x - 3| \leq 0.4 \iff -0.4 \leq 2x - 3 \leq 0.4 \iff 2.6 \leq 2x \leq 3.4 \iff 1.3 \leq x \leq 1.7\), so \(x \in [1.3, 1.7]\).

156. \(|5x - 2| < 6 \iff -6 < 5x - 2 < 6 \iff -4 < 5x < 8 \iff \frac{-4}{5} < x < \frac{8}{5}\), so \(x \in (-\frac{4}{5}, \frac{8}{5})\).

157. \(a(bx - c) \geq bc \iff bx - c \geq \frac{bc}{a} \iff bx \geq \frac{bc}{a} + c = \frac{bc + ac}{a} \iff x \geq \frac{bc + ac}{ab}\).

158. \(ax + b < c \iff ax < c - b \iff x > \frac{c - b}{a}\) (since \(a < 0\)).

159. \(|ab| = \sqrt{(ab)^2} = \sqrt{a^2b^2} = \sqrt{a^2} \sqrt{b^2} = |a| |b|\).

160. If \(0 < a < b\), then \(a \cdot a < a \cdot b\) and \(a \cdot b < b \cdot b\) [using Rule 3 of Inequalities]. So \(a^2 < ab < b^2\) and hence \(a^2 < b^2\).