3.2 INVERSE FUNCTIONS AND LOGARITHMS

EXAMPLE A Although the function $y = x^2$, $x \in \mathbb{R}$, is not one-to-one and therefore does not have an inverse function, we can turn it into a one-to-one function by restricting its domain. For instance, the function $f(x) = x^2$, $0 \leqslant x \leqslant 2$, is one-to-one (by the Horizontal Line Test) and has domain $[0, 2]$ and range $[0, 4]$. (See Figure 1.) Thus f has an inverse function f^{-1} with domain $[0, 4]$ and range $[0, 2]$.

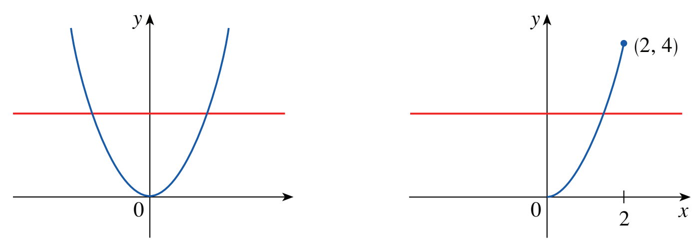

FIGURE 1 (a) $y = x^2$, $x \in \mathbb{R}$ (b) $f(x) = x^2$, $0 \leqslant x \leqslant 2$

Without computing a formula for $(f^{-1})'$ we can still calculate $(f^{-1})'(1)$. Since $f(1) = 1$, we have $f^{-1}(1) = 1$. Also $f'(x) = 2x$. So by Theorem 7 we have

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(1)} = \frac{1}{2}$$

In this case it is easy to find f^{-1} explicitly. In fact, $f^{-1}(x) = \sqrt{x}$, $0 \leqslant x \leqslant 4$. [In general, we could use the method given by (5).] Then $(f^{-1})'(x) = 1/(2\sqrt{x})$, so $(f^{-1})'(1) = \frac{1}{2}$, which agrees with the preceding computation. The functions f and f^{-1} are graphed in Figure 2. ■

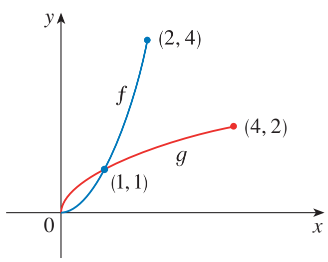

FIGURE 2