3.3 DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

EXAMPLE A

$$\frac{d}{dx}(\pi^{x^3}) = \pi^{x^3} \ln \pi \frac{d}{dx}(x^3) = (3 \ln \pi)x^2\pi^{x^3}$$ ■

EXAMPLE B If $y = xe^{x^3}$, then

$$y' = 1 \cdot e^{x^3} + x \cdot e^{x^3} \frac{d}{dx}(x^3)$$
$$= e^{x^3} + xe^{x^3}(3x^2) = e^{x^3}(1 + 3x^3)$$ ■

EXAMPLE C
(a) If $f(x) = xe^x$, find $f'(x)$.
(b) Find the nth derivative, $f^{(n)}(x)$.

SOLUTION
(a) By the Product Rule, we have

$$f'(x) = \frac{d}{dx}(xe^x) = x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x)$$
$$= xe^x + e^x \cdot 1 = (x + 1)e^x$$

■ Figure 1 shows the graphs of the function f of Example C and its derivative f'. Notice that $f'(x)$ is positive when f is increasing and negative when f is decreasing.

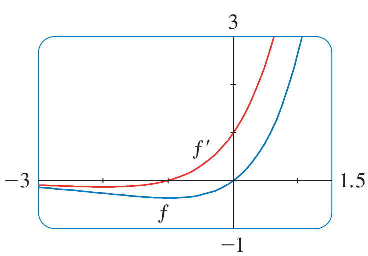

FIGURE 1

(b) Using the Product Rule a second time, we get

$$f''(x) = \frac{d}{dx}[(x + 1)e^x] = (x + 1)\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x + 1)$$
$$= (x + 1)e^x + e^x \cdot 1 = (x + 2)e^x$$

Further applications of the Product Rule give

$$f'''(x) = (x + 3)e^x \qquad f^{(4)}(x) = (x + 4)e^x$$

In fact, each successive differentiation adds another term e^x, so

$$f^{(n)}(x) = (x + n)e^x$$ ■