3.5 INVERSE TRIGONOMETRIC FUNCTIONS

EXAMPLE A Evaluate:

(a) $\sin(\sin^{-1}0.6)$ (b) $\sin^{-1}\left(\sin\frac{\pi}{12}\right)$ (c) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

SOLUTION

(a) Since 0.6 lies between -1 and 1, the second cancellation equation in (2) gives

$$\sin(\sin^{-1}0.6) = 0.6$$

(b) Since $\pi/12$ lies between $-\pi/2$ and $\pi/2$, the first cancellation equation gives

$$\sin^{-1}\left(\sin\frac{\pi}{12}\right) = \frac{\pi}{12}$$

(c) Since $2\pi/3$ does not lie in the interval $[-\pi/2, \pi/2]$, we can't use the cancellation equation. Instead we note that $\sin(2\pi/3) = \sqrt{3}/2$ and $\sin^{-1}(\sqrt{3}/2) = \pi/3$ because $\pi/3$ lies between $-\pi/2$ and $\pi/2$. Therefore

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$ ■

EXAMPLE B Differentiate $y = \dfrac{1}{\sin^{-1}x}$.

SOLUTION

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}x)^{-1} = -(\sin^{-1}x)^{-2}\frac{d}{dx}(\sin^{-1}x)$$

$$= -\frac{1}{(\sin^{-1}x)^2\sqrt{1-x^2}}$$ ■