

**4.2** THE MEAN VALUE THEOREM

**EXAMPLE A** Prove the identity  $\tan^{-1}x + \cot^{-1}x = \pi/2$ .

**SOLUTION** Although calculus isn't needed to prove this identity, the proof using calculus is quite simple. If  $f(x) = \tan^{-1}x + \cot^{-1}x$ , then

$$f'(x) = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0$$

for all values of  $x$ . Therefore,  $f(x) = C$ , a constant. To determine the value of  $C$ , we put  $x = 1$  [because we can evaluate  $f(1)$  exactly]. Then

$$C = f(1) = \tan^{-1}1 + \cot^{-1}1 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Thus,  $\tan^{-1}x + \cot^{-1}x = \pi/2$ . ■