3.6 NEWTON'S METHOD

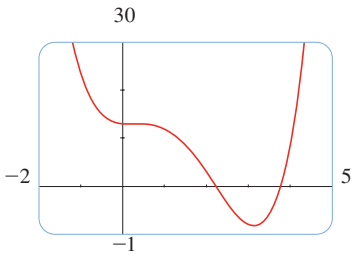

FIGURE 1

EXAMPLE A Find all roots of the equation $x^4 - 5x^3 + 4x^2 - x + 13 = 0$ correct to eight decimal places.

SOLUTION Figure 1 shows a graph of $f(x) = x^4 - 5x^3 + 4x^2 - x + 13$ and we see that the roots are near 2.2 and 3.8. The formula for Newton's Method is

$$x_{n+1} = x_n - \frac{x_n^4 - 5x_n^3 + 4x_n^2 - x_n + 13}{4x_n^3 - 15x_n^2 + 8x_n - 1}$$

Using Newton's Method with the initial approximations from the graph, we get

$$\begin{aligned} x_1 &= 2.2 & x_1 &= 3.8 \\ x_2 &\approx 2.22577566 & x_2 &\approx 3.76551041 \\ x_3 &\approx 2.22578253 & x_3 &\approx 3.76419061 \\ x_4 &\approx 2.22578253 & x_4 &\approx 3.76418872 \\ & & x_5 &\approx 376418872 \end{aligned}$$

The roots of the given equation, correct to eight decimal places, are 2.22578253 and 3.76418872. ■