## 5.5 EXPONENTIAL GROWTH AND DECAY

**EXAMPLE A** A bacteria culture starts with 1000 bacteria, and after 2 h the population is 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 h.

**SOLUTION** Let y(t) be the number of bacteria after t hours. Then y(0) = 1000 and y(2) = 2500. Since we are assuming dy/dt = ky, Theorem 2 gives

$$y(t) = y(0)e^{kt} = 1000e^{kt}$$
  
 $y(2) = 1000e^{2k} = 2500$ 

 $e^{2k} = 2.5$  and  $2k = \ln 2.5$ 

 $y(t) = 1000e^{\ln 2.5(t/2)}$ 

• Figure 1 shows a graph of the size of the bacteria population in Example A.



Substituting the value of  $k = \frac{1}{2} \ln 2.5$  back into the expression for y(t), we have

Since  $e^{\ln 2.5} = 2.5$ , an alternative expression for Equation 1 is

 $y(t) = 1000(2.5)^{t/2}$ 

**FIGURE I** 

and so

Therefore

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 $y(6) = 1000(2.5)^3 = 15,625$