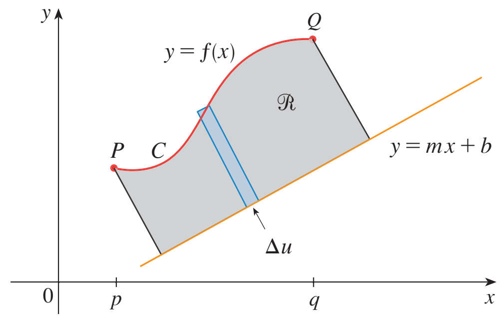
7.2 DISCOVERY PROJECT: ROTATING ON A SLANT

This project can be completed anytime after you have studied Section 7.2 in the textbook.

We know how to find the volume of a solid of revolution obtained by rotating a region about a horizontal or vertical line (see Section 7.2). But what if we rotate about a slanted line, that is, a line that is neither horizontal nor vertical? In this project you are asked to discover a formula for the volume of a solid of revolution when the axis of rotation is a slanted line.

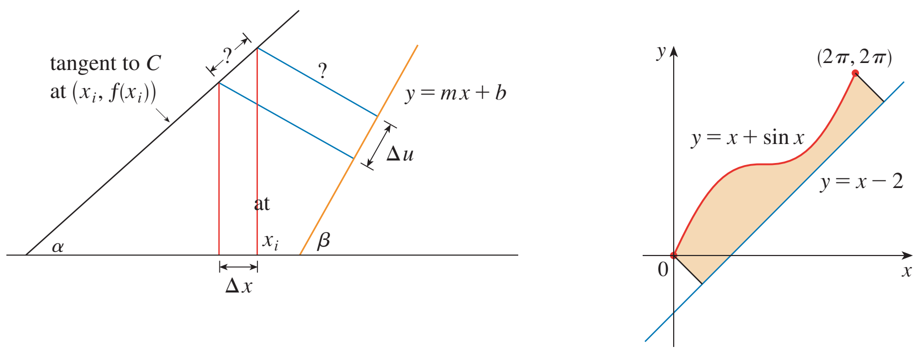
Let C be the arc of the curve $y = f(x)$ between the points $P(p, f(p))$ and $Q(q, f(q))$ and let $\mathcal{R}$ be the region bounded by C, by the line $y = mx + b$ (which lies entirely below C), and by the perpendiculars to the line from P and Q.



1. Show that the area of $\mathcal{R}$ is

$$\frac{1}{1 + m^2} \int_p^q [f(x) - mx - b][1 + mf'(x)]\, dx$$

[*Hint:* This formula can be verified by subtracting areas, but it will be helpful throughout the project to derive it by first approximating the area using rectangles perpendicular to the line, as shown in the figure. Use part (a) of the following figure to help express Δu in terms of Δx.]



(a) (b)

2. Find the area of the region shown in part (b) of the figure.

3. Find a formula (similar to the one in Problem 1) for the volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = mx + b$.

4. Find the volume of the solid obtained by rotating the region of Problem 2 about the line $y = x - 2$.