## 8.8 APPLIED PROJECT: RADIATION FROM THE STARS

This project can be completed anytime after you have studied Section 8.8 in the textbook.

Any object emits radiation when heated. A *blackbody* is a system that absorbs all the radiation that falls on it. For instance, a matte black surface or a large cavity with a small hole in its wall (like a blastfurnace) is a blackbody and emits blackbody radiation. Even the radiation from the Sun is close to being blackbody radiation.

Proposed in the late 19th century, the Rayleigh-Jeans Law expresses the energy density of blackbody radiation of wavelength $\lambda$ as

$$f(\lambda) = \frac{8\pi kT}{\lambda^4}$$

where $\lambda$ is measured in meters, $T$ is the temperature in kelvins (K), and $k$ is Boltzmann's constant. The Rayleigh-Jeans Law agrees with experimental measurements for long wavelengths but disagrees drastically for short wavelengths. [The law predicts that $f(\lambda) \to \infty$ as $\lambda \to 0^+$ but experiments have shown that $f(\lambda) \to 0$.] This fact is known as the *ultraviolet catastrophe*.

In 1900 Max Planck found a better model (known now as Planck's Law) for blackbody radiation:

$$f(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/(\lambda kT)} - 1}$$

where $\lambda$ is measured in meters, $T$ is the temperature in kelvins, and

$$h = \text{Planck's constant} = 6.6262 \times 10^{-34}\ \text{J}\cdot\text{s}$$

$$c = \text{speed of light} = 2.997\,925 \times 10^{8}\ \text{m/s}$$

$$k = \text{Boltzmann's constant} = 1.3807 \times 10^{-23}\ \text{J/K}$$

**1.** Use l'Hospital's Rule to show that

$$\lim_{\lambda \to 0^+} f(\lambda) = 0 \qquad \text{and} \qquad \lim_{\lambda \to \infty} f(\lambda) = 0$$

for Planck's Law. So, for short wavelengths, this law models blackbody radiation better than the Rayleigh-Jeans Law.

**2.** Use a Taylor polynomial to show that, for large wavelengths, Planck's Law gives approximately the same values as the Rayleigh-Jeans Law.

**3.** Graph $f$ as given by both laws on the same screen and comment on the similarities and differences. Use $T = 5700$ K (the temperature of the Sun). (You may want to change from meters to the more convenient unit of micrometers: $1\ \mu\text{m} = 10^{-6}$ m.)

**4.** Use your graph in Problem 3 to estimate the value of $\lambda$ for which $f(\lambda)$ is a maximum under Planck's Law.

**5.** Investigate how the graph of $f$ changes as $T$ varies. (Use Planck's Law.) In particular, graph $f$ for the stars Betelgeuse ($T = 3400$ K), Procyon ($T = 6400$ K), and Sirius ($T = 9200$ K) as well as the Sun. How does the total radiation emitted (the area under the curve) vary with $T$? Use the graph to comment on why Sirius is known as a blue star and Betelgeuse as a red star.