12.5 DISCOVERY PROJECT: VOLUMES OF HYPERSPHERES

This project can be completed anytime after you have studied Section 12.5 in the textbook.

In this project we find formulas for the volume enclosed by a hypersphere in n-dimensional space.

1. Use a double integral and the trigonometric substitution $y = r \sin\theta$, together with Formula 64 in the Table of Integrals, to find the area of a circle with radius r.
2. Use a triple integral and trigonometric substitution to find the volume of a sphere with radius r.
3. Use a quadruple integral to find the hypervolume enclosed by the hypersphere $x^2 + y^2 + z^2 + w^2 = r^2$ in $\mathbb{R}^4$. (Use only trigonometric substitution and the reduction formulas for $\int \sin^n x\,dx$ or $\int \cos^n x\,dx$.)
4. Use an n-tuple integral to find the volume enclosed by a hypersphere of radius r in n-dimensional space $\mathbb{R}^n$. [*Hint:* The formulas are different for n even and n odd.]