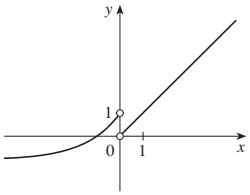
49.



51. The rate at which the number of US $20 bills in circulation is changing with respect to time; 0.156 billion bills per year

53. 0

PROBLEMS PLUS ■ PAGE 169

1. $\frac{2}{3}$ **3.** -4 **5.** (a) Does not exist (b) 1

7. $a = \frac{1}{2} \pm \frac{1}{2}\sqrt{5}$ **9.** $\frac{3}{4}$ **11.** (b) Yes (c) Yes; no

13. (a) 0 (b) 1 (c) $f'(x) = x^2 + 1$

CHAPTER 3

EXERCISES 3.1 ■ PAGE 180

1. (a) e is the number such that $\lim_{h \to 0} \dfrac{e^h - 1}{h} = 1$.

(b) 0.99, 1.03; $2.7 < e < 2.8$

3. $f'(x) = 0$ **5.** $f'(x) = 5.2$ **7.** $f'(t) = 6t^2 - 6t - 4$

9. $g'(x) = 2x - 6x^2$ **11.** $g'(t) = -\frac{3}{2}t^{-7/4}$

13. $F'(r) = -15/r^4$ **15.** $R'(a) = 18a + 6$

17. $S'(p) = \frac{1}{2}p^{-1/2} - 1$ **19.** $y' = 3e^x - \frac{4}{3}x^{-4/3}$

21. $h'(u) = 3Au^2 + 2Bu + C$

23. $y' = \frac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x\sqrt{x}}$ **25.** $j'(x) = 2.4x^{1.4}$

27. $G'(q) = -2q^{-2} - 2q^{-3}$ **29.** $f'(v) = -\frac{2}{3}v^{-5/3} - 2e^v$

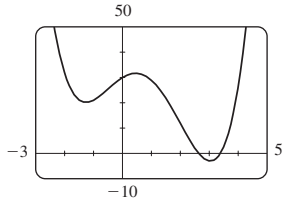
31. $z' = -10A/y^{11} + Be^y$ **33.** $y = 4x - 1$

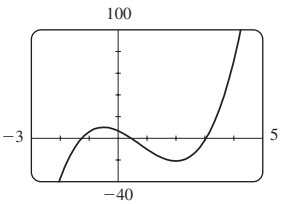
35. $y = \frac{1}{2}x + 2$

37. Tangent: $y = 2x + 2$; normal: $y = -\frac{1}{2}x + 2$

39. $y = 3x - 1$ **41.** $f'(x) = 4x^3 - 6x^2 + 2x$

43. (a) (c) $4x^3 - 9x^2 - 12x + 7$





45. $f'(x) = 0.005x^4 - 0.06x^2$, $f''(x) = 0.02x^3 - 0.12x$

47. $f'(x) = 2 - \frac{15}{4}x^{-1/4}$, $f''(x) = \frac{15}{16}x^{-5/4}$

49. (a) $v(t) = 3t^2 - 3$, $a(t) = 6t$ (b) 12 m/s^2

(c) $a(1) = 6$ m/s^2

51. 1.718; at 12 years, the length of the fish is increasing at a rate of 1.718 in/year

53. (a) $V = 5.3/P$

(b) -0.00212; instantaneous rate of change of the volume with respect to the pressure at 25°C; m^3/kPa

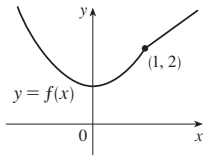
55. $(-2, 21)$, $(1, -6)$ **59.** $y = 3x - 3$, $y = 3x - 7$

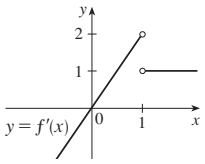
61. $y = -2x + 3$

63. $(\pm 2, 4)$ **67.** $P(x) = x^2 - x + 3$

69. $y = \frac{3}{16}x^3 - \frac{9}{4}x + 3$

71. No

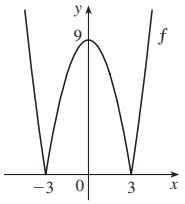


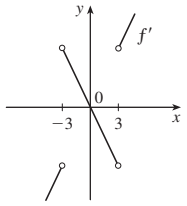


73. (a) Not differentiable at 3 or -3

$$f'(x) = \begin{cases} 2x & \text{if } |x| > 3 \\ -2x & \text{if } |x| < 3 \end{cases}$$

(b)





75. $y = 2x^2 - x$ **77.** $a = -\frac{1}{2}$, $b = 2$ **79.** $-\frac{1}{3}$

81. $m = 4$, $b = -4$ **83.** 1000 **85.** 3; 1

EXERCISES 3.2 ■ PAGE 188

1. $1 - 2x + 6x^2 - 8x^3$ **3.** $f'(x) = e^x(3x^2 + x - 5)$

5. $y' = \dfrac{1 - x}{e^x}$ **7.** $g'(x) = \dfrac{10}{(3 - 4x)^2}$ **9.** $H'(u) = 2u - 1$

11. $F'(y) = 5 + \dfrac{14}{y^2} + \dfrac{9}{y^4}$ **13.** $y' = \dfrac{x(-x^3 - 3x - 2)}{(x^3 - 1)^2}$

15. $y' = \dfrac{t^4 - 8t^3 + 6t^2 + 9}{(t^2 - 4t + 3)^2}$

17. $y' = e^p\left(1 + \frac{3}{2}\sqrt{p} + p + p\sqrt{p}\right)$ **19.** $y' = \dfrac{3 - 2\sqrt{s}}{2s^{5/2}}$

21. $f'(t) = \dfrac{-2t - 3}{3t^{2/3}(t - 3)^2}$ **23.** $f'(x) = \dfrac{xe^x(x^3 + 2e^x)}{(x^2 + e^x)^2}$

25. $f'(x) = \dfrac{2cx}{(x^2 + c)^2}$

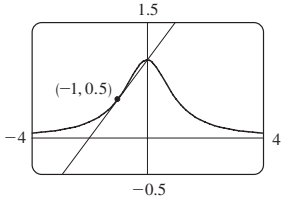
27. $(x^3 + 3x^2 + 1)e^x$; $(x^3 + 6x^2 + 6x + 1)e^x$

29. $\dfrac{x(2 + 2e^x - xe^x)}{(1 + e^x)^2}$;

$$\frac{2 + 4e^x - 4xe^x - x^2e^x + 2e^{2x} - 4xe^{2x} + x^2e^{2x}}{(1 + e^x)^3}$$

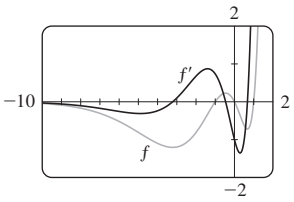
31. $y = \frac{2}{3}x - \frac{2}{3}$ **33.** $y = 2x$; $y = -\frac{1}{2}x$

35. (a) $y = \frac{1}{2}x + 1$ (b)



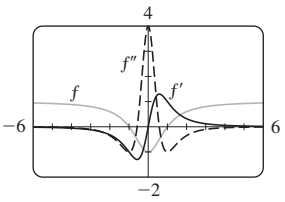
37. (a) $e^x(x^3 + 3x^2 - x - 1)$

(b)



39. (a) $f'(x) = \dfrac{4x}{(x^2+1)^2}$; $f''(x) = \dfrac{4(1-3x^2)}{(x^2+1)^3}$

(b)



41. $\frac{1}{4}$ **43.** (a) -16 (b) $-\frac{20}{9}$ (c) 20 **45.** 7

47. $y = -2x + 18$ **49.** (a) 0 (b) $-\frac{2}{3}$

51. (a) $y' = xg'(x) + g(x)$ (b) $y' = \dfrac{g(x) - xg'(x)}{[g(x)]^2}$

(c) $y' = \dfrac{xg'(x) - g(x)}{x^2}$

53. Two, $\left(-2 \pm \sqrt{3}, \frac{1}{2}\left(1 \mp \sqrt{3}\right)\right)$ **55.** 1

57. \$1.627 billion/year

59. $\dfrac{0.0021}{(0.015 + [S])^2}$;

The rate of change of the rate of an enzymatic reaction with respect to the concentration of a substrate S.

61. (c) $3e^{3x}$

63. $f'(x) = (x^2 + 2x)e^x$, $f''(x) = (x^2 + 4x + 2)e^x$, $f'''(x) = (x^2 + 6x + 6)e^x$, $f^{(4)}(x) = (x^2 + 8x + 12)e^x$, $f^{(5)}(x) = (x^2 + 10x + 20)e^x$; $f^{(n)}(x) = [x^2 + 2nx + n(n-1)]e^x$

EXERCISES 3.3 ■ PAGE 196

1. $f'(x) = x^2 \cos x + 2x \sin x$ **3.** $f'(x) = e^x(\cos x - \sin x)$

5. $y' = \sec\theta\,(\sec^2\theta + \tan^2\theta)$

7. $y' = -c \sin t + t(t \cos t + 2 \sin t)$

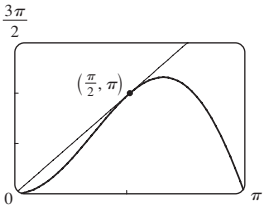
9. $y' = \dfrac{2 - \tan x + x \sec^2 x}{(2 - \tan x)^2}$ **11.** $f'(\theta) = \dfrac{1}{1 + \cos\theta}$

13. $y' = \dfrac{(t^2 + t)\cos t + \sin t}{(1+t)^2}$

15. $f'(\theta) = \frac{1}{2}\sin 2\theta + \theta \cos 2\theta$

21. $y = x + 1$ **23.** $y = x - \pi - 1$

25. (a) $y = 2x$ (b)



27. (a) $\sec x \tan x - 1$

29. $\theta \cos\theta + \sin\theta$; $2\cos\theta - \theta\sin\theta$

31. (a) $f'(x) = (1 + \tan x)/\sec x$ (b) $f'(x) = \cos x + \sin x$

33. $(2n+1)\pi \pm \frac{1}{3}\pi$, n an integer

35. (a) $v(t) = 8\cos t$, $a(t) = -8\sin t$

(b) $4\sqrt{3}$, -4, $-4\sqrt{3}$; to the left

37. 5 ft/rad **39.** $\frac{5}{3}$ **41.** 3 **43.** $-\frac{3}{4}$

45. $\frac{1}{2}$ **47.** $-\frac{1}{4}$ **49.** $-\sqrt{2}$ **51.** $-\cos x$

53. $A = -\frac{3}{10}$, $B = -\frac{1}{10}$

55. (a) $\sec^2 x = \dfrac{1}{\cos^2 x}$ (b) $\sec x \tan x = \dfrac{\sin x}{\cos^2 x}$

(c) $\cos x - \sin x = \dfrac{\cot x - 1}{\csc x}$

57. 1

EXERCISES 3.4 ■ PAGE 204

1. $\dfrac{4}{3\sqrt[3]{(1+4x)^2}}$ **3.** $\pi \sec^2 \pi x$ **5.** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

7. $F'(x) = 24x^{11}(5x^3 + 2)^3(5x^3 + 1)$

9. $f'(x) = \dfrac{5}{2\sqrt{5x+1}}$ **11.** $f'(\theta) = -2\theta \sin(\theta^2)$

13. $y' = xe^{-3x}(2 - 3x)$ **15.** $f'(t) = e^{at}(b\cos bt + a \sin bt)$

17. $f'(x) = (2x-3)^3(x^2 + x + 1)^4(28x^2 - 12x - 7)$

19. $h'(t) = \frac{2}{3}(t+1)^{-1/3}(2t^2 - 1)^2(20t^2 + 18t - 1)$

21. $y' = \dfrac{1}{2\sqrt{x}\,(x+1)^{3/2}}$ **23.** $y' = (\sec^2\theta)\, e^{\tan\theta}$

25. $g'(u) = \dfrac{48u^2(u^3 - 1)^7}{(u^3 + 1)^9}$ **27.** $r'(t) = \dfrac{(\ln 10)10^{2\sqrt{t}}}{\sqrt{t}}$

29. $H'(r) = \dfrac{2(r^2 - 1)^2(r^2 + 3r + 5)}{(2r+1)^6}$

31. $F'(t) = e^{t \sin 2t}(2t \cos 2t + \sin 2t)$

33. $G'(x) = -C(\ln 4)\dfrac{4^{C/x}}{x^2}$

35. $y' = \dfrac{4e^{2x}}{(1 + e^{2x})^2} \sin\dfrac{1 - e^{2x}}{1 + e^{2x}}$

37. $y' = -2\cos\theta \cot(\sin\theta)\csc^2(\sin\theta)$

39. $f'(t) = -\sec^2(\sec(\cos t))\sec(\cos t)\tan(\cos t)\sin t$

41. $f'(t) = 4\sin(e^{\sin^2 t})\cos(e^{\sin^2 t})\, e^{\sin^2 t} \sin t \cos t$

43. $g'(x) = 2r^2 p(\ln a)(2ra^{rx} + n)^{p-1} a^{rx}$

45. $y' = \dfrac{-\pi \cos(\tan \pi x)\sec^2(\pi x)\sin\sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}}$

47. $y' = -3\cos 3\theta \sin(\sin 3\theta)$;

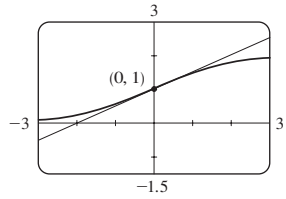
$y'' = -9\cos^2(3\theta)\cos(\sin 3\theta) + 9(\sin 3\theta)\sin(\sin 3\theta)$

49. $y' = \dfrac{-\sec t \tan t}{2\sqrt{1 - \sec t}}$;

$y'' = \dfrac{\sec t\,(3\sec^3 t - 4\sec^2 t - \sec t + 2)}{4(1 - \sec t)^{3/2}}$

51. $y = (\ln 2)x + 1$ **53.** $y = -x + \pi$

55. (a) $y = \frac{1}{2}x + 1$ (b)



57. (a) $f'(x) = \dfrac{2 - 2x^2}{\sqrt{2 - x^2}}$

59. $((\pi/2) + 2n\pi, 3), ((3\pi/2) + 2n\pi, -1)$, n an integer

61. 24 **63.** (a) 30 (b) 36

65. (a) $\frac{3}{4}$ (b) Does not exist (c) -2 **67.** $-\frac{1}{6}\sqrt{2}$

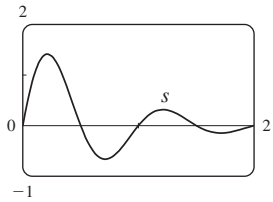
69. (a) $F'(x) = e^x f'(e^x)$ (b) $G'(x) = e^{f(x)} f'(x)$

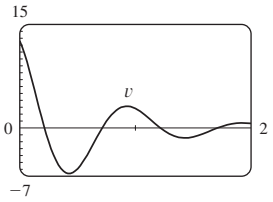
71. 120 **73.** 96

77. $-2^{50} \cos 2x$ **79.** $v(t) = \frac{5}{2}\pi \cos(10\pi t)$ cm/s

81. (a) $\dfrac{dB}{dt} = \dfrac{7\pi}{54} \cos \dfrac{2\pi t}{5.4}$ (b) 0.16

83. $v(t) = 2e^{-1.5t}(2\pi \cos 2\pi t - 1.5 \sin 2\pi t)$





85. (a) 0.0075 (mg/mL)/min (b) 0.0030 (mg/mL)/min

87. dv/dt is the rate of change of velocity with respect to time; dv/ds is the rate of change of velocity with respect to displacement

89. (a) $Q = ab^t$ where $a \approx 100.01244$ and $b \approx 0.000045146$
(b) $-670.63\ \mu A$

91. (b) The factored form **95.** (b) $-n \cos^{n-1}x \sin[(n + 1)x]$

EXERCISES 3.5 ■ PAGE 215

1. (a) $y' = 9x/y$ (b) $y = \pm\sqrt{9x^2 - 1}$, $y' = \pm 9x/\sqrt{9x^2 - 1}$

3. (a) $y' = -\sqrt{y}/\sqrt{x}$ (b) $y = \left(1 - \sqrt{x}\right)^2$, $y' = 1 - 1/\sqrt{x}$

5. $y' = \dfrac{2y - x}{y - 2x}$ **7.** $y' = -\dfrac{2x(2x^2 + y^2)}{y(2x^2 + 3y)}$

9. $y' = \dfrac{x(x + 2y)}{2x^2y + 4xy^2 + 2y^3 + x^2}$ **11.** $y' = \dfrac{2x + y \sin x}{\cos x - 2y}$

13. $y' = \dfrac{1 - 8x^3\sqrt{x + y}}{8y^3\sqrt{x + y} - 1}$ **15.** $y' = \dfrac{y(y - e^{x/y})}{y^2 - xe^{x/y}}$

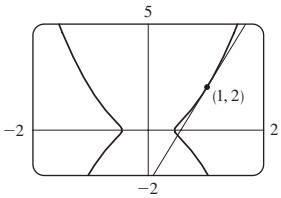
17. $y' = \dfrac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$

19. $y' = -\dfrac{y \cos(xy) + \sin(x + y)}{x \cos(xy) + \sin(x + y)}$ **21.** $-\frac{16}{13}$

23. $x' = \dfrac{-2x^4y + x^3 - 6xy^2}{4x^3y^2 - 3x^2y + 2y^3}$ **25.** $y = \frac{1}{2}x$

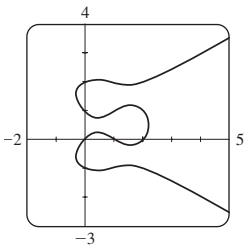
27. $y = \frac{3}{4}x - \frac{1}{2}$ **29.** $y = x + \frac{1}{2}$ **31.** $y = -\frac{9}{13}x + \frac{40}{13}$

33. (a) $y = \frac{9}{2}x - \frac{5}{2}$ (b)



35. $-1/(4y^3)$ **37.** $\dfrac{\cos^2y \cos x + \sin^2x \sin y}{\cos^3y}$ **39.** $1/e^2$

41. (a)



Eight; $x \approx 0.42, 1.58$

(b) $y = -x + 1$, $y = \frac{1}{3}x + 2$ (c) $1 \mp \frac{1}{3}\sqrt{3}$

43. $\left(\pm\frac{5}{4}\sqrt{3}, \pm\frac{5}{4}\right)$ **45.** $(x_0x/a^2) - (y_0y/b^2) = 1$

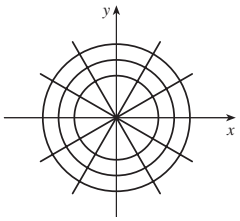
49. $y' = \dfrac{2 \tan^{-1}x}{1 + x^2}$ **51.** $y' = \dfrac{1}{\sqrt{-x^2 - x}}$

53. $F'(x) = \dfrac{3}{\sqrt{x^6 - 1}} + \sec^{-1}(x^3)$ **55.** $h'(t) = 0$

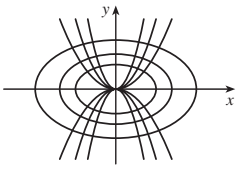
57. $y' = \sin^{-1}x$ **59.** $y' = \dfrac{\sqrt{a^2 - b^2}}{a + b \cos x}$

61. $1 - \dfrac{x \arcsin x}{\sqrt{1 - x^2}}$

65.



67.



71. (a) $\dfrac{V^3(nb - V)}{PV^3 - n^2aV + 2n^3ab}$ (b) ≈ -4.04 L/atm

73. $\left(\pm\sqrt{3}, 0\right)$ **75.** $(-1, -1), (1, 1)$ **77.** (b) $\frac{3}{2}$

79. (a) 0 (b) $-\frac{1}{2}$

EXERCISES 3.6 ■ PAGE 223

1. The differentiation formula is simplest.

3. $f'(x) = \dfrac{\cos(\ln x)}{x}$ **5.** $f'(x) = -\dfrac{1}{x}$

7. $f'(x) = \dfrac{-\sin x}{(1 + \cos x) \ln 10}$ **9.** $g'(x) = \dfrac{1}{x} - 2$

11. $F'(t) = \ln t \left(\ln t \cos t + \dfrac{2 \sin t}{t}\right)$

13. $G'(y) = \dfrac{10}{2y + 1} - \dfrac{y}{y^2 + 1}$ **15.** $F'(s) = \dfrac{1}{s \ln s}$

17. $T'(z) = 2^z\left(\dfrac{1}{z \ln 2} + \ln z\right)$

19. $y' = \dfrac{-x}{1 + x}$ **21.** $y' = \sec^2[\ln(ax + b)]\dfrac{a}{ax + b}$

23. $y' = (2 + \ln x)/(2\sqrt{x})$; $y'' = -\ln x/(4x\sqrt{x})$

25. $y' = \tan x$; $y'' = \sec^2x$

27. $f'(x) = \dfrac{2x - 1 - (x - 1)\ln(x - 1)}{(x - 1)[1 - \ln(x - 1)]^2}$;
$(1, 1 + e) \cup (1 + e, \infty)$

29. $f'(x) = \dfrac{2(x - 1)}{x(x - 2)}$; $(-\infty, 0) \cup (2, \infty)$ **31.** 2

33. $y = 3x - 9$ **35.** $\cos x + 1/x$ **37.** 7

39. $y' = (x^2 + 2)^2(x^4 + 4)^4\left(\dfrac{4x}{x^2 + 2} + \dfrac{16x^3}{x^4 + 4}\right)$

41. $y' = \sqrt{\dfrac{x - 1}{x^4 + 1}}\left(\dfrac{1}{2x - 2} - \dfrac{2x^3}{x^4 + 1}\right)$

43. $y' = x^x(1 + \ln x)$

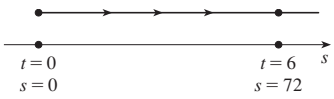
45. $y' = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x \ln x\right)$

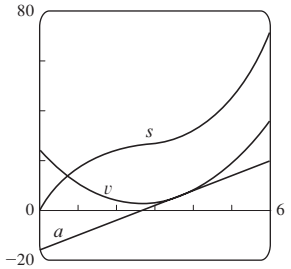
47. $y' = (\cos x)^x(-x \tan x + \ln \cos x)$

49. $y' = (\tan x)^{1/x}\left(\dfrac{\sec^2 x}{x \tan x} - \dfrac{\ln \tan x}{x^2}\right)$

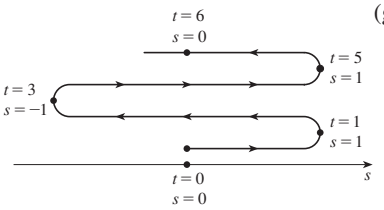
51. $y' = \dfrac{2x}{x^2 + y^2 - 2y}$ **53.** $f^{(n)}(x) = \dfrac{(-1)^{n-1}(n - 1)!}{(x - 1)^n}$

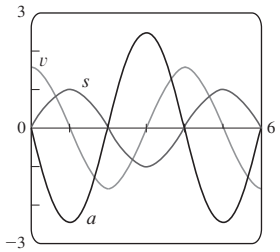
EXERCISES 3.7 ■ PAGE 233

1. (a) $3t^2 - 16t + 24$ (b) 11 ft/s (c) Never (d) Always
(e) 72 ft
(f) 

(g) $6t - 16$; -10 ft/s^2
(h) 

(i) Speeding up when $t > \frac{8}{3}$; slowing down when $0 \leqslant t < \frac{8}{3}$

3. (a) $(\pi/2)\cos(\pi t/2)$ (b) 0 ft/s
(c) $t = 2n + 1$, t a nonnegative integer
(d) $0 < t < 1$, $3 < t < 5$, $7 < t < 9$, and so on (e) 6 ft
(f) 

(g) $(-\pi^2/4)\sin(\pi t/2)$; $-\pi^2/4$ ft/s^2
(h) 

(i) Speeding up when $1 < t < 2$, $3 < t < 4$, and $5 < t < 6$; slowing down when $0 < t < 1$, $2 < t < 3$, and $4 < t < 5$
5. (a) Speeding up when $0 < t < 1$ or $2 < t < 3$; slowing down when $1 < t < 2$
(b) Speeding up when $1 < t < 2$ or $3 < t < 4$; slowing down when $0 < t < 1$ or $2 < t < 3$
7. (a) 4.9 m/s; -14.7 m/s (b) After 2.5 s (c) $32\frac{5}{8}$ m
(d) ≈ 5.08 s (e) ≈ -25.3 m/s
9. (a) 7.56 m/s (b) ≈ 6.24 m/s; ≈ -6.24 m/s
11. (a) 30 mm^2/mm; the rate at which the area is increasing with respect to side length as x reaches 15 mm
(b) $\Delta A \approx 2x\,\Delta x$
13. (a) (i) 5π (ii) 4.5π (iii) 4.1π
(b) 4π (c) $\Delta A \approx 2\pi r\,\Delta r$
15. (a) 8π ft^2/ft (b) 16π ft^2/ft (c) 24π ft^2/ft
The rate increases as the radius increases.
17. (a) 6 kg/m (b) 12 kg/m (c) 18 kg/m
At the right end; at the left end
19. (a) 4.75 A (b) 5 A; $t = \frac{2}{3}$ s
23. (a) $dV/dP = -C/P^2$ (b) At the beginning
25. $400(3^t)\ln 3$; ≈ 6850 bacteria/h
27. (a) 16 million/year; 78.5 million/year
(b) $P(t) = at^3 + bt^2 + ct + d$, where $a \approx -0.0002849$, $b \approx 0.5224331$, $c \approx -6.395641$, $d \approx 1720.586$
(c) $P'(t) = 3at^2 + 2bt + c$
(d) 14.16 million/year (smaller); 71.72 million/year (smaller)
(e) $f'(t) = (1.43653 \times 10^9) \cdot (1.01395)^t \ln 1.01395$
(f) 26.25 million/year (larger); 60.28 million/year (smaller)
(g) $P'(85) \approx 76.24$ million/year, $f'(85) = 64.61$ million/year
29. (a) 0.926 cm/s; 0.694 cm/s; 0
(b) 0; -92.6 (cm/s)/cm; -185.2 (cm/s)/cm
(c) At the center; at the edge
31. (a) $C'(x) = 3 + 0.02x + 0.0006x^2$
(b) \$11/pair; the rate at which the cost is changing as the 100th pair of jeans is being produced; the cost of the 101st pair
(c) \$11.07
33. (a) $[xp'(x) - p(x)]/x^2$; the average productivity increases as new workers are added.
35. $\dfrac{dt}{dc} = \dfrac{3\sqrt{9c^2 - 8c} + 9c - 4}{\sqrt{9c^2 - 8c}\,(3c + \sqrt{9c^2 - 8c})}$; the rate of change of duration of dialysis required with respect to the initial urea concentration
37. -0.2436 K/min
39. (a) 0 and 0 (b) $C = 0$
(c) (0, 0), (500, 50); it is possible for the species to coexist.

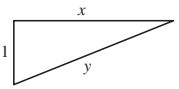
EXERCISES 3.8 ■ PAGE 242

1. About 235
3. (a) $100(4.2)^t$ (b) ≈ 7409 (c) $\approx 10{,}632$ bacteria/h
(d) $(\ln 100)/(\ln 4.2) \approx 3.2$ h
5. (a) 1508 million, 1871 million (b) 2161 million
(c) 3972 million; wars in the first half of century, increased life expectancy in second half
7. (a) $Ce^{-0.0005t}$ (b) $-2000 \ln 0.9 \approx 211$ s
9. (a) $100 \times 2^{-t/30}$ mg (b) ≈ 9.92 mg (c) ≈ 199.3 years
11. ≈ 2500 years **13.** Yes; 12.5 billion years
15. (a) $\approx 137°$F (b) ≈ 116 min
17. (a) $13.\overline{3}°$C (b) ≈ 67.74 min
19. (a) ≈ 64.5 kPa (b) ≈ 39.9 kPa

21. (a) (i) \$3828.84 (ii) \$3840.25 (iii) \$3850.08
(iv) \$3851.61 (v) \$3852.01 (vi) \$3852.08
(b) $dA/dt = 0.05A$, $A(0) = 3000$

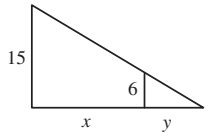
EXERCISES 3.9 ■ PAGE 249

1. $dV/dt = 3x^2\,dx/dt$ **3.** 48 cm²/s **5.** $3/(25\pi)$ m/min
7. 128π cm²/min **9.** (a) 1 (b) 25 **11.** -18
13. (a) The plane's altitude is 1 mi and its speed is 500 mi/h.
(b) The rate at which the distance from the plane to the station is increasing when the plane is 2 mi from the station
(c)



(d) $y^2 = x^2 + 1$
(e) $250\sqrt{3}$ mi/h

15. (a) The height of the pole (15 ft), the height of the man (6 ft), and the speed of the man (5 ft/s)
(b) The rate at which the tip of the man's shadow is moving when he is 40 ft from the pole
(c)

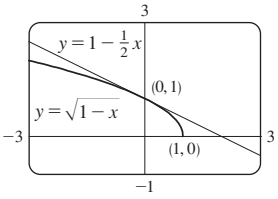


(d) $\dfrac{15}{6} = \dfrac{x+y}{y}$ (e) $\frac{25}{3}$ ft/s

17. 65 mi/h **19.** $837/\sqrt{8674} \approx 8.99$ ft/s
21. -1.6 cm/min **23.** $\frac{720}{13} \approx 55.4$ km/h
25. $(10{,}000 + 800{,}000\pi/9) \approx 2.89 \times 10^5$ cm³/min
27. $\frac{10}{3}$ cm/min **29.** $6/(5\pi) \approx 0.38$ ft/min
31. $150\sqrt{3}$ cm²/min **33.** 5 m **35.** πr^2 cm²/h
37. 80 cm³/min **39.** $\frac{107}{810} \approx 0.132\ \Omega$/s
41. $\sqrt{7}\,\pi/21 \approx 0.396$ m/min
43. (a) 360 ft/s (b) 0.096 rad/s
45. $\frac{10}{9}\pi$ km/min **47.** $1650/\sqrt{31} \approx 296$ km/h
49. $\frac{7}{4}\sqrt{15} \approx 6.78$ m/s

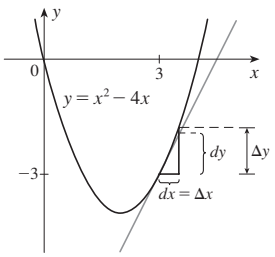
EXERCISES 3.10 ■ PAGE 256

1. $L(x) = 16x + 23$ **3.** $L(x) = \frac{1}{4}x + 1$
5. $\sqrt{1-x} \approx 1 - \frac{1}{2}x$; $\sqrt{0.9} \approx 0.95$, $\sqrt{0.99} \approx 0.995$

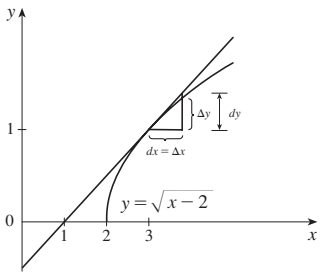


7. $-0.383 < x < 0.516$ **9.** $-0.368 < x < 0.677$
11. (a) $dy = (1 - 4x)e^{-4x}\,dx$ (b) $dy = -\dfrac{2t^3}{\sqrt{1-t^4}}\,dt$
13. (a) $dy = \dfrac{\sec^2\sqrt{t}}{2\sqrt{t}}\,dt$ (b) $dy = \dfrac{-4v}{(1+v^2)^2}\,dv$
15. (a) $dy = \frac{1}{10}e^{x/10}\,dx$ (b) 0.01
17. (a) $dy = \dfrac{x}{\sqrt{3+x^2}}\,dx$ (b) -0.05

19. $\Delta y = 1.25$, $dy = 1$



21. $\Delta y \approx 0.34$, $dy = 0.4$



23. 15.968 **25.** $10.00\overline{3}$ **27.** 1.1
33. (a) 270 cm³, 0.01, 1% (b) 36 cm², $0.00\overline{6}$, $0.\overline{6}$%
35. (a) $84/\pi \approx 27$ cm²; $\frac{1}{84} \approx 0.012 = 1.2\%$
(b) $1764/\pi^2 \approx 179$ cm³; $\frac{1}{56} \approx 0.018 = 1.8\%$
37. (a) $2\pi rh\,\Delta r$ (b) $\pi(\Delta r)^2 h$
43. (a) 4.8, 5.2 (b) Too large

EXERCISES 3.11 ■ PAGE 264

1. (a) 0 (b) 1 **3.** (a) $\frac{13}{5}$ (b) $\frac{1}{2}(e^5 + e^{-5}) \approx 74.20995$
5. (a) 1 (b) 0
21. $\operatorname{sech} x = \frac{3}{5}$, $\sinh x = \frac{4}{3}$, $\operatorname{csch} x = \frac{3}{4}$, $\tanh x = \frac{4}{5}$, $\coth x = \frac{5}{4}$
23. (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) 0 (f) 1
(g) ∞ (h) $-\infty$ (i) 0 (j) $\frac{1}{2}$
31. $f'(x) = \dfrac{\operatorname{sech}^2\sqrt{x}}{2\sqrt{x}}$ **33.** $h'(x) = 2x\cosh(x^2)$
35. $G'(t) = \dfrac{t^2+1}{2t^2}$
37. $y' = 3e^{\cosh 3x}\sinh 3x$
39. $g'(t) = \coth\sqrt{t^2+1} - \dfrac{t^2}{\sqrt{t^2+1}}\operatorname{csch}^2\sqrt{t^2+1}$
41. $y' = \dfrac{1}{2\sqrt{x(x-1)}}$
43. $y' = \sinh^{-1}(x/3)$ **45.** $y' = -\csc x$
51. (a) 0.3572 (b) 70.34°
53. (a) 164.50 m (b) 120 m; 164.13 m
55. (b) $y = 2\sinh 3x - 4\cosh 3x$
57. $\left(\ln\left(1+\sqrt{2}\right), \sqrt{2}\right)$

CHAPTER 3 REVIEW ■ PAGE 266

True-False Quiz

1. True **3.** True **5.** False **7.** False **9.** True
11. True **13.** True **15.** True

Exercises

1. $4x^7(x+1)^3(3x+2)$ **3.** $\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x^3}}$

5. $x(\pi x\cos\pi x + 2\sin\pi x)$

7. $\dfrac{8t^3}{(t^4+1)^2}$ **9.** $\dfrac{1+\ln x}{x\ln x}$ **11.** $\dfrac{\cos\sqrt{x} - \sqrt{x}\sin\sqrt{x}}{2\sqrt{x}}$

13. $-\dfrac{e^{1/x}(1+2x)}{x^4}$ **15.** $\dfrac{2xy - \cos y}{1 - x\sin y - x^2}$

17. $\dfrac{1}{2\sqrt{\arctan x}(1+x^2)}$ **19.** $\dfrac{1-t^2}{(1+t^2)^2}\sec^2\left(\dfrac{t}{1+t^2}\right)$

21. $3^{x\ln x}(\ln 3)(1+\ln x)$ **23.** $-(x-1)^{-2}$

25. $\dfrac{2x - y\cos(xy)}{x\cos(xy)+1}$ **27.** $\dfrac{2}{(1+2x)\ln 5}$

29. $\cot x - \sin x\cos x$ **31.** $\dfrac{4x}{1+16x^2} + \tan^{-1}(4x)$

33. $5\sec 5x$ **35.** $-6x\csc^2(3x^2+5)$

37. $\cos\left(\tan\sqrt{1+x^3}\right)\left(\sec^2\sqrt{1+x^3}\right)\dfrac{3x^2}{2\sqrt{1+x^3}}$

39. $2\cos\theta\tan(\sin\theta)\sec^2(\sin\theta)$

41. $\dfrac{(2-x)^4(3x^2-55x-52)}{2\sqrt{x+1}\,(x+3)^8}$ **43.** $2x^2\cosh(x^2) + \sinh(x^2)$

45. $3\tanh 3x$ **47.** $\dfrac{\cosh x}{\sqrt{\sinh^2 x - 1}}$

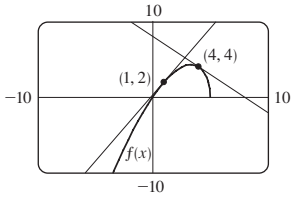
49. $\dfrac{-3\sin\left(e^{\sqrt{\tan 3x}}\right)e^{\sqrt{\tan 3x}}\sec^2(3x)}{2\sqrt{\tan 3x}}$ **51.** $-\frac{4}{27}$

53. $-5x^4/y^{11}$ **57.** $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$

59. $y = 2x + 1$ **61.** $y = -x + 2$; $y = x + 2$

63. (a) $\dfrac{10-3x}{2\sqrt{5-x}}$ (b) $y = \frac{7}{4}x + \frac{1}{4}$, $y = -x + 8$

(c)



65. $(\pi/4, \sqrt{2}), (5\pi/4, -\sqrt{2})$

69. (a) 4 (b) 6 (c) $\frac{7}{9}$ (d) 12

71. $2xg(x) + x^2g'(x)$ **73.** $2g(x)g'(x)$

75. $g'(e^x)e^x$ **77.** $g'(x)/g(x)$

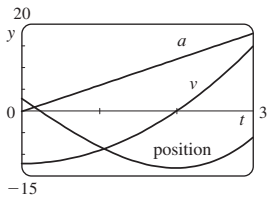
79. $\dfrac{f'(x)[g(x)]^2 + g'(x)[f(x)]^2}{[f(x)+g(x)]^2}$

81. $f'(g(\sin 4x))g'(\sin 4x)(\cos 4x)(4)$

83. $(-3, 0)$ **85.** $y = -\frac{2}{3}x^2 + \frac{14}{3}x$

87. $v(t) = -Ae^{-ct}[c\cos(\omega t+\delta) + \omega\sin(\omega t+\delta)]$,
$a(t) = Ae^{-ct}[(c^2-\omega^2)\cos(\omega t+\delta) + 2c\omega\sin(\omega t+\delta)]$

89. (a) $v(t) = 3t^2 - 12$; $a(t) = 6t$ (b) $t > 2$; $0 \leqslant t < 2$
(c) 23 (d)



(e) $t > 2$; $0 < t < 2$

91. 4 kg/m

93. (a) $200(3.24)^t$ (b) ≈22,040
(c) ≈25,910 bacteria/h (d) $(\ln 50)/(\ln 3.24) \approx 3.33$ h

95. (a) C_0e^{-kt} (b) ≈100 h **97.** $\frac{4}{3}$ cm²/min

99. 13 ft/s **101.** 400 ft/h

103. (a) $L(x) = 1 + x$; $\sqrt[3]{1+3x} \approx 1 + x$; $\sqrt[3]{1.03} \approx 1.01$
(b) $-0.23 < x < 0.40$

105. $12 + \frac{3}{2}\pi \approx 16.7$ cm² **107.** $\left[\dfrac{d}{dx}\sqrt[4]{x}\right]_{x=16} = \frac{1}{32}$

109. $\frac{1}{4}$ **111.** $\frac{1}{8}x^2$

PROBLEMS PLUS ■ PAGE 271

1. $\left(\pm\sqrt{3}/2, \frac{1}{4}\right)$ **5.** $3\sqrt{2}$ **11.** $\left(0, \frac{5}{4}\right)$

13. 3 lines; (0, 2), $\left(\frac{4}{3}\sqrt{2}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}\sqrt{2}, \frac{10}{3}\right)$, $\left(-\frac{4}{3}\sqrt{2}, \frac{2}{3}\right)$ and $\left(-\frac{2}{3}\sqrt{2}, \frac{10}{3}\right)$

15. (a) $4\pi\sqrt{3}/\sqrt{11}$ rad/s (b) $40\left(\cos\theta + \sqrt{8+\cos^2\theta}\right)$ cm
(c) $-480\pi\sin\theta\left(1 + \cos\theta/\sqrt{8+\cos^2\theta}\right)$ cm/s

19. $x_T \in (3, \infty)$, $y_T \in (2, \infty)$, $x_N \in \left(0, \frac{5}{3}\right)$, $y_N \in \left(-\frac{5}{2}, 0\right)$

21. (b) (i) 53° (or 127°) (ii) 63° (or 117°)

23. R approaches the midpoint of the radius AO.

25. $-\sin a$ **27.** $2\sqrt{e}$ **31.** $(1, -2)$, $(-1, 0)$

33. $\sqrt{29}/58$ **35.** $2 + \frac{375}{128}\pi \approx 11.204$ cm³/min

CHAPTER 4

EXERCISES 4.1 ■ PAGE 283

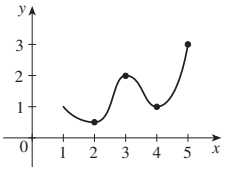
Abbreviations: abs, absolute; loc, local; max, maximum; min, minimum

1. Abs min: smallest function value on the entire domain of the function; loc min at c: smallest function value when x is near c

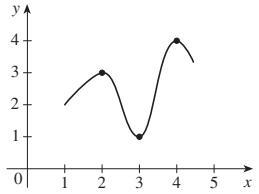
3. Abs max at s, abs min at r, loc max at c, loc min at b and r, neither a max nor a min at a and d

5. Abs max $f(4) = 5$, loc max $f(4) = 5$ and $f(6) = 4$, loc min $f(2) = 2$ and $f(1) = f(5) = 3$

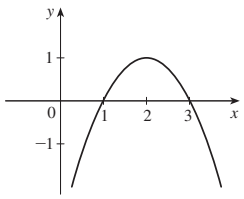
7.



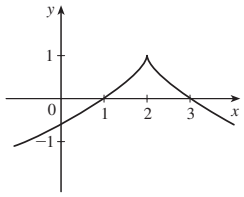
9.



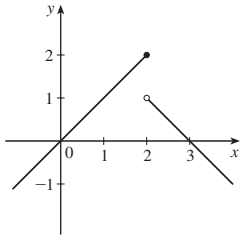
11. (a)



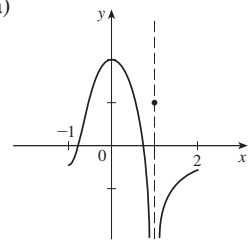
(b)



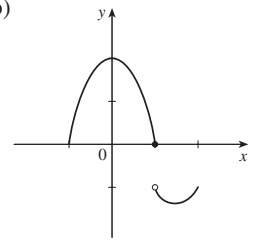
(c)



13. (a)

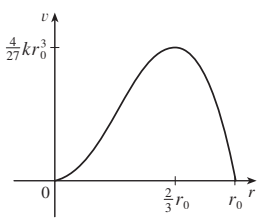


(b)



15. Abs max $f(3) = 4$ **17.** Abs max $f(1) = 1$
19. Abs min $f(0) = 0$
21. Abs max $f(\pi/2) = 1$; abs min $f(-\pi/2) = -1$
23. Abs max $f(2) = \ln 2$ **25.** Abs max $f(0) = 1$
27. Abs min $f(1) = -1$ **29.** $\frac{1}{3}$ **31.** $-2, 3$ **33.** 0
35. $0, 2$ **37.** $0, \frac{4}{9}$ **39.** $0, \frac{8}{7}, 4$ **41.** $n\pi$ (n an integer)
43. $0, \frac{2}{3}$ **45.** 10 **47.** $f(2) = 16, f(5) = 7$
49. $f(-1) = 8, f(2) = -19$ **51.** $f(-2) = 33, f(2) = -31$
53. $f(0.2) = 5.2, f(1) = 2$
55. $f(4) = 4 - \sqrt[3]{4}, f(\sqrt{3}/9) = -2\sqrt{3}/9$
57. $f(\pi/6) = \frac{3}{2}\sqrt{3}, f(\pi/2) = 0$
59. $f(e^{1/2}) = 1/(2e), f(\frac{1}{2}) = -4 \ln 2$
61. $f(1) = \ln 3, f(-\frac{1}{2}) = \ln \frac{3}{4}$
63. $f\left(\dfrac{a}{a+b}\right) = \dfrac{a^a b^b}{(a+b)^{a+b}}$
65. (a) 2.19, 1.81 (b) $\frac{6}{25}\sqrt{\frac{3}{5}} + 2, -\frac{6}{25}\sqrt{\frac{3}{5}} + 2$
67. (a) 0.32, 0.00 (b) $\frac{3}{16}\sqrt{3}, 0$
69. 0.177 mg/mL; 21.4 min **71.** ≈3.9665°C
73. About 4.1 months after Jan. 1
75. (a) $r = \frac{2}{3}r_0$ (b) $v = \frac{4}{27}kr_0^3$

(c)

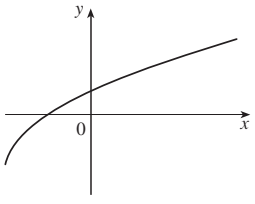


EXERCISES 4.2 ■ PAGE 291

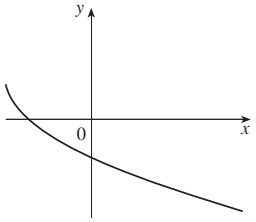
1. 1, 5
3. (a) g is continuous on $[0, 8]$ and differentiable on $(0, 8)$.
(b) 2.2, 6.4 (c) 3.7, 5.5
5. 1 **7.** π
9. f is not differentiable on $(-1, 1)$ **11.** 1
13. $3/\ln 4$ **15.** 1; yes **17.** f is not continous at 3
25. 16 **27.** No **33.** No

EXERCISES 4.3 ■ PAGE 300

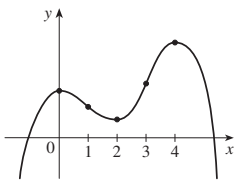
1. (a) (1, 3), (4, 6) (b) (0, 1), (3, 4) (c) (0, 2)
(d) (2, 4), (4, 6) (e) (2, 3)
3. (a) I/D Test (b) Concavity Test
(c) Find points at which the concavity changes.
5. (a) Inc on (1, 5); dec on (0, 1) and (5, 6)
(b) Loc max at $x = 5$, loc min at $x = 1$
7. (a) 3, 5 (b) 2, 4, 6 (c) 1, 7
9. (a) Inc on $(-\infty, -1)$, $(3, \infty)$; dec on $(-1, 3)$
(b) Loc max $f(-1) = 9$; loc min $f(3) = -23$
(c) CU on $(1, \infty)$, CD on $(-\infty, 1)$; IP $(1, -7)$
11. (a) Inc on $(-1, 0)$, $(1, \infty)$; dec on $(-\infty, -1)$, $(0, 1)$
(b) Loc max $f(0) = 3$; loc min $f(\pm 1) = 2$
(c) CU on $\left(-\infty, -\sqrt{3}/3\right), \left(\sqrt{3}/3, \infty\right)$;
CD on $\left(-\sqrt{3}/3, \sqrt{3}/3\right)$; IP $\left(\pm\sqrt{3}/3, \frac{22}{9}\right)$
13. (a) Inc on $(0, \pi/4)$, $(5\pi/4, 2\pi)$; dec on $(\pi/4, 5\pi/4)$
(b) Loc max $f(\pi/4) = \sqrt{2}$; loc min $f(5\pi/4) = -\sqrt{2}$
(c) CU on $(3\pi/4, 7\pi/4)$; CD on $(0, 3\pi/4)$, $(7\pi/4, 2\pi)$;
IP $(3\pi/4, 0)$, $(7\pi/4, 0)$
15. (a) Inc on $\left(-\frac{1}{3}\ln 2, \infty\right)$; dec on $\left(-\infty, -\frac{1}{3}\ln 2\right)$
(b) Loc min $f\left(-\frac{1}{3}\ln 2\right) = 2^{-2/3} + 2^{1/3}$ (c) CU on $(-\infty, \infty)$
17. (a) Inc on $(1, \infty)$; dec on $(0, 1)$ (b) Loc min $f(1) = 0$
(c) CU on $(0, \infty)$; No IP
19. Loc max $f(1) = 2$; loc min $f(0) = 1$
21. Loc min $f\left(\frac{1}{16}\right) = -\frac{1}{4}$
23. (a) f has a local maximum at 2.
(b) f has a horizontal tangent at 6.
25. (a)



(b)



27.



29.