5.5 EXPONENTIAL GROWTH AND DECAY

EXAMPLE A A bacteria culture starts with 1000 bacteria, and after 2 h the population is 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 h.

SOLUTION Let y(t) be the number of bacteria after t hours. Then y(0) = 1000 and y(2) = 2500. Since we are assuming dy/dt = ky, Theorem 2 gives

$$y(t) = y(0)e^{kt} = 1000e^{kt}$$

 $y(2) = 1000e^{2k} = 2500$

 $e^{2k} = 2.5$ and $2k = \ln 2.5$

 $y(t) = 1000e^{\ln 2.5(t/2)}$

• Figure 1 shows a graph of the size of the bacteria population in Example A.



Substituting the value of $k = \frac{1}{2} \ln 2.5$ back into the expression for y(t), we have

Since $e^{\ln 2.5} = 2.5$, an alternative expression for Equation 1 is

 $y(t) = 1000(2.5)^{t/2}$

FIGURE I

and so

Therefore

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 $y(6) = 1000(2.5)^3 = 15,625$