EXAMPLE A Find the centroid of the region bounded by the curves $y = \cos x$, $y = 0$, $x = 0$, and $x = \pi/2$.

SOLUTION The area of the region is

$$A = \int_0^{\pi/2} \cos x \, dx = \sin x \Big]_0^{\pi/2} = 1$$

so Formulas 12 give

$$\bar{x} = \frac{1}{A} \int_0^{\pi/2} x f(x) \, dx = \int_0^{\pi/2} x \cos x \, dx$$

$$= x \sin x \Big]_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx \qquad \text{(by integration by parts)}$$

$$= \frac{\pi}{2} - 1$$

$$\bar{y} = \frac{1}{A} \int_0^{\pi/2} \tfrac{1}{2}[f(x)]^2 \, dx = \tfrac{1}{2} \int_0^{\pi/2} \cos^2 x \, dx$$

$$= \tfrac{1}{4} \int_0^{\pi/2} (1 + \cos 2x) \, dx = \tfrac{1}{4}\big[x + \tfrac{1}{2} \sin 2x\big]_0^{\pi/2}$$

$$= \frac{\pi}{8}$$

$y = x$

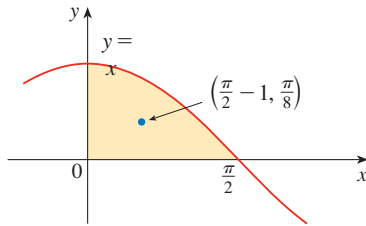$\left(\frac{\pi}{2} - 1, \frac{\pi}{8}\right)$

FIGURE I

The centroid is $((\pi/2) - 1, \pi/8)$ and is shown in Figure 1. ∎