

<b>5.6</b>
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**INVERSE TRIGONOMETRIC FUNCTIONS**
**EXAMPLE A** Evaluate:

$$(a) \sin(\sin^{-1} 0.6) \quad (b) \sin^{-1}\left(\sin \frac{\pi}{12}\right) \quad (c) \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$$

**SOLUTION**(a) Since 0.6 lies between  $-1$  and  $1$ , the second cancellation equation in (2) gives

$$\sin(\sin^{-1} 0.6) = 0.6$$

(b) Since  $\pi/12$  lies between  $-\pi/2$  and  $\pi/2$ , the first cancellation equation gives

$$\sin^{-1}\left(\sin \frac{\pi}{12}\right) = \frac{\pi}{12}$$

(c) Since  $2\pi/3$  does not lie in the interval  $[-\pi/2, \pi/2]$ , we can't use the cancellation equation. Instead we note that  $\sin(2\pi/3) = \sqrt{3}/2$  and  $\sin^{-1}(\sqrt{3}/2) = \pi/3$  because  $\pi/3$  lies between  $-\pi/2$  and  $\pi/2$ . Therefore

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \quad \blacksquare$$

**EXAMPLE B** Differentiate  $y = \frac{1}{\sin^{-1}x}$ .**SOLUTION**

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\sin^{-1}x)^{-1} = -(\sin^{-1}x)^{-2} \frac{d}{dx} (\sin^{-1}x) \\ &= -\frac{1}{(\sin^{-1}x)^2 \sqrt{1-x^2}} \quad \blacksquare \end{aligned}$$