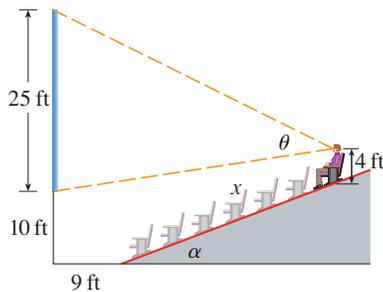


**5.6** APPLIED PROJECT: CAS WHERE TO SIT AT THE MOVIES

This project can be completed anytime after you have studied Section 5.6 in the textbook.

A movie theater has a screen that is positioned 10 ft off the floor and is 25 ft high. The first row of seats is placed 9 ft from the screen and the rows are set 3 ft apart. The floor of the seating area is inclined at an angle of  $\alpha = 20^\circ$  above the horizontal and the distance up the incline that you sit is  $x$ . The theater has 21 rows of seats, so  $0 \leq x \leq 60$ . Suppose you decide that the best place to sit is in the row where the angle  $\theta$  subtended by the screen at your eyes is a maximum. Let's also suppose that your eyes are 4 ft above the floor, as shown in the figure.



1. Show that

$$\theta = \arccos\left(\frac{a^2 + b^2 - 625}{2ab}\right)$$

where

$$a^2 = (9 + x \cos \alpha)^2 + (31 - x \sin \alpha)^2$$

and

$$b^2 = (9 + x \cos \alpha)^2 + (x \sin \alpha - 6)^2$$

2. Use a graph of  $\theta$  as a function of  $x$  to estimate the value of  $x$  that maximizes  $\theta$ . In which row should you sit? What is the viewing angle  $\theta$  in this row?
3. Use your computer algebra system to differentiate  $\theta$  and find a numerical value for the root of the equation  $d\theta/dx = 0$ . Does this value confirm your result in Problem 2?
4. Use the graph of  $\theta$  to estimate the average value of  $\theta$  on the interval  $0 \leq x \leq 60$ . Then use your CAS to compute the average value. Compare with the maximum and minimum values of  $\theta$ .