1.3 THE LIMIT OF A FUNCTION

x	$x^3 + \dfrac{\cos 5x}{10{,}000}$
1	1.000028
0.5	0.124920
0.1	0.001088
0.05	0.000222
0.01	0.000101

x	$x^3 + \dfrac{\cos 5x}{10{,}000}$
0.005	0.00010009
0.001	0.00010000

EXAMPLE A Find $\lim_{x \to 0} \left(x^3 + \dfrac{\cos 5x}{10{,}000} \right)$.

SOLUTION As before, we construct a table of values. From the table in the margin it appears that

$$\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10{,}000} \right) = 0$$

But if we persevere with smaller values of x, the table at left suggests that

$$\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10{,}000} \right) = 0.000100 = \frac{1}{10{,}000}$$

Later we will see that $\lim_{x \to 0} \cos 5x = 1$; then it follows that the limit is 0.0001. ■

EXAMPLE B If $f(x) = x^2 - x + 2$, how close to 2 does x have to be to ensure that $f(x)$ is within a distance 0.1 of the number 4?

SOLUTION If the distance from $f(x)$ to 4 is less than 0.1, then $f(x)$ lies between 3.9 and 4.1, so the requirement is that

$$3.9 < x^2 - x + 2 < 4.1$$

Thus, we need to determine the values of x such that the curve $y = x^2 - x + 2$ lies between the horizontal lines $y = 3.9$ and $y = 4.1$. We graph the curve and lines near the point (2, 4) in Figure 1. With the cursor, we estimate that the x-coordinate of the point of intersection of the line $y = 3.9$ and the curve $y = x^2 - x + 2$ is about 1.966. Similarly, the curve intersects the line $y = 4.1$ when $x \approx 2.033$. So, rounding to be safe, we conclude that

$$3.9 < x^2 - x + 2 < 4.1 \quad \text{when} \quad 1.97 < x < 2.03$$

Therefore, $f(x)$ is within a distance 0.1 of 4 when x is within a distance 0.03 of 2. ■

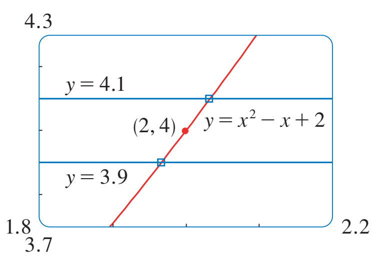

FIGURE 1