

## 4.7 ANTIDERIVATIVES

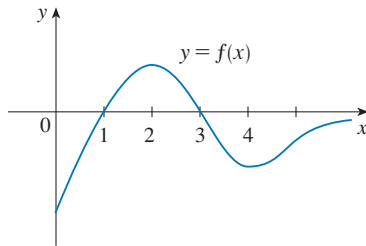


FIGURE 1

**EXAMPLE A** The graph of a function  $f$  is given in Figure 1. Make a rough sketch of an antiderivative  $F$ , given that  $F(0) = 2$ .

**SOLUTION** We are guided by the fact that the slope of  $y = F(x)$  is  $f(x)$ . We start at the point  $(0, 2)$  and draw  $F$  as an initially decreasing function since  $f(x)$  is negative when  $0 < x < 1$ . Notice that  $f(1) = f(3) = 0$ , so  $F$  has horizontal tangents when  $x = 1$  and  $x = 3$ . For  $1 < x < 3$ ,  $f(x)$  is positive and so  $F$  is increasing. We see that  $F$  has a local minimum when  $x = 1$  and a local maximum when  $x = 3$ . For  $x > 3$ ,  $f(x)$  is negative and so  $F$  is decreasing on  $(3, \infty)$ . Since  $f(x) \rightarrow 0$  as  $x \rightarrow \infty$ , the graph of  $F$  becomes flatter as  $x \rightarrow \infty$ . Also notice that  $F''(x) = f'(x)$  changes from positive to negative at  $x = 2$  and from negative to positive at  $x = 4$ , so  $F$  has inflection points when  $x = 2$  and  $x = 4$ . We use this information to sketch the graph of the antiderivative in Figure 2.

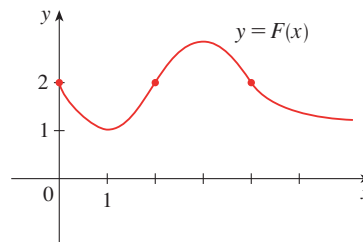


FIGURE 2