

5.2 THE DEFINITE INTEGRAL

- Because $f(x) = e^x$ is positive, the integral in Example A represents the area shown in Figure 1.

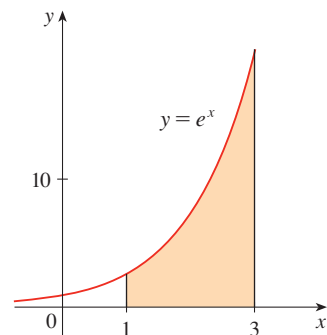


FIGURE 1

- A computer algebra system is able to find an explicit expression for this sum because it is a geometric series. The limit could be found using l'Hospital's Rule.

EXAMPLE A

- Set up an expression for $\int_1^3 e^x dx$ as a limit of sums.
- Use a computer algebra system to evaluate the expression.

SOLUTION

- Here we have $f(x) = e^x$, $a = 1$, $b = 3$, and

$$\Delta x = \frac{b - a}{n} = \frac{2}{n}$$

So $x_0 = 1$, $x_1 = 1 + 2/n$, $x_2 = 1 + 4/n$, $x_3 = 1 + 6/n$, and

$$x_i = 1 + \frac{2i}{n}$$

From Theorem 4, we get

$$\begin{aligned} \int_1^3 e^x dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(1 + \frac{2i}{n}\right) \frac{2}{n} \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n e^{1+2i/n} \end{aligned}$$

- If we ask a computer algebra system to evaluate the sum and simplify, we obtain

$$\sum_{i=1}^n e^{1+2i/n} = \frac{e^{(3n+2)/n} - e^{(n+2)/n}}{e^{2/n} - 1}$$

Now we ask the computer algebra system to evaluate the limit:

$$\int_1^3 e^x dx = \lim_{n \rightarrow \infty} \frac{2}{n} \cdot \frac{e^{(3n+2)/n} - e^{(n+2)/n}}{e^{2/n} - 1} = e^3 - e$$

We will learn a much easier method for the evaluation of integrals in the next section. ■