

## 10.5 EQUATIONS OF LINES AND PLANES

**EXAMPLE A** Find the point at which the line with parametric equations  $x = 2 + 3t$ ,  $y = -4t$ ,  $z = 5 + t$  intersects the plane  $4x + 5y - 2z = 18$ .

**SOLUTION** We substitute the expressions for  $x$ ,  $y$ , and  $z$  from the parametric equations into the equation of the plane:

$$4(2 + 3t) + 5(-4t) - 2(5 + t) = 18$$

This simplifies to  $-10t = 20$ , so  $t = -2$ . Therefore, the point of intersection occurs when the parameter value is  $t = -2$ . Then  $x = 2 + 3(-2) = -4$ ,  $y = -4(-2) = 8$ ,  $z = 5 - 2 = 3$  and so the point of intersection is  $(-4, 8, 3)$ . ■

**EXAMPLE B** In Example 3 we showed that the lines

$$\begin{aligned} L_1: \quad x &= 1 + t & y &= -2 + 3t & z &= 4 - t \\ L_2: \quad x &= 2s & y &= 3 + s & z &= -3 + 4s \end{aligned}$$

are skew. Find the distance between them.

**SOLUTION** Since the two lines  $L_1$  and  $L_2$  are skew, they can be viewed as lying on two parallel planes  $P_1$  and  $P_2$ . The distance between  $L_1$  and  $L_2$  is the same as the distance between  $P_1$  and  $P_2$ , which can be computed as in Example 8. The common normal vector to both planes must be orthogonal to both  $\mathbf{v}_1 = \langle 1, 3, -1 \rangle$  (the direction of  $L_1$ ) and  $\mathbf{v}_2 = \langle 2, 1, 4 \rangle$  (the direction of  $L_2$ ). So a normal vector is

$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -1 \\ 2 & 1 & 4 \end{vmatrix} = 13\mathbf{i} - 6\mathbf{j} - 5\mathbf{k}$$

If we put  $s = 0$  in the equations of  $L_2$ , we get the point  $(0, 3, -3)$  on  $L_2$  and so an equation for  $P_2$  is

$$13(x - 0) - 6(y - 3) - 5(z + 3) = 0 \quad \text{or} \quad 13x - 6y - 5z + 3 = 0$$

If we now set  $t = 0$  in the equations for  $L_1$ , we get the point  $(1, -2, 4)$  on  $P_1$ . So the distance between  $L_1$  and  $L_2$  is the same as the distance from  $(1, -2, 4)$  to  $13x - 6y - 5z + 3 = 0$ . By Formula 9, this distance is

$$D = \frac{|13(1) - 6(-2) - 5(4) + 3|}{\sqrt{13^2 + (-6)^2 + (-5)^2}} = \frac{8}{\sqrt{230}} \approx 0.53 \quad \blacksquare$$