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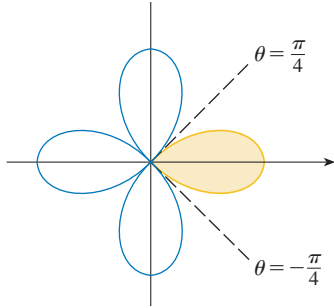
DOUBLE INTEGRALS IN POLAR COORDINATES


FIGURE 1

EXAMPLE A Use a double integral to find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$.

SOLUTION From the sketch of the curve in Figure 1 we see that a loop is given by the region

$$D = \{(r, \theta) \mid -\pi/4 \leq \theta \leq \pi/4, 0 \leq r \leq \cos 2\theta\}$$

So the area is

$$\begin{aligned} A(D) &= \iint_D dA = \int_{-\pi/4}^{\pi/4} \int_0^{\cos 2\theta} r \, dr \, d\theta \\ &= \int_{-\pi/4}^{\pi/4} \left[\frac{1}{2} r^2 \right]_0^{\cos 2\theta} d\theta = \frac{1}{2} \int_{-\pi/4}^{\pi/4} \cos^2 2\theta \, d\theta \\ &= \frac{1}{4} \int_{-\pi/4}^{\pi/4} (1 + \cos 4\theta) \, d\theta = \frac{1}{4} \left[\theta + \frac{1}{4} \sin 4\theta \right]_{-\pi/4}^{\pi/4} = \frac{\pi}{8} \end{aligned}$$

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