

3.4

 EXPONENTIAL GROWTH AND DECAY

EXAMPLE A A bacteria culture starts with 1000 bacteria, and after 2 h the population is 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 h.

SOLUTION Let $y(t)$ be the number of bacteria after t hours. Then $y(0) = 1000$ and $y(2) = 2500$. Since we are assuming $dy/dt = ky$, Theorem 2 gives

$$y(t) = y(0)e^{kt} = 1000e^{kt}$$

$$y(2) = 1000e^{2k} = 2500$$

Therefore

$$e^{2k} = 2.5 \quad \text{and} \quad 2k = \ln 2.5$$

Substituting the value of $k = \frac{1}{2} \ln 2.5$ back into the expression for $y(t)$, we have

$$\mathbf{I} \quad y(t) = 1000e^{\ln 2.5(t/2)}$$

Since $e^{\ln 2.5} = 2.5$, an alternative expression for Equation 1 is

$$y(t) = 1000(2.5)^{t/2}$$

and so

$$y(6) = 1000(2.5)^3 = 15,625 \quad \blacksquare$$

■ Figure 1 shows a graph of the size of the bacteria population in Example A.

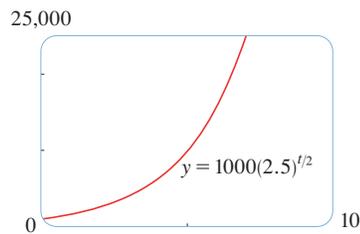


FIGURE 1