3.1 EXPONENTIAL FUNCTIONS

A Click here for answers.

S Click here for solutions.

1–8 ▪ Make a rough sketch of the graph of each function. Do not use a calculator. Just use the graphs given in Figure 3 and, if necessary, the transformations of Section 1.2.

1. $y = 2^x + 1$

2. $y = 2^{x+1}$

3. $y = 3^{-x}$

4. $y = -3^x$

5. $y = -3^{-x}$

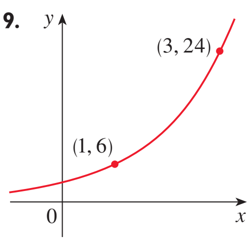
6. $y = 10^x - 1$

7. $y = 3 + 2^{-x}$

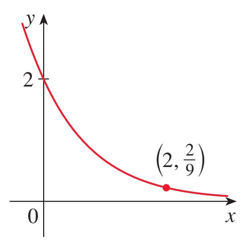
8. $y = 2^{|x|}$

9–10 ▪ Find the exponential function $f(x) = Ca^x$ whose graph is given.

9.



10.



11–19 ▪ Find the limit.

11. $\lim_{x \to \infty} (1.1)^x$

12. $\lim_{x \to -\infty} (1.1)^x$

13. $\lim_{x \to -\infty} (\pi/4)^x$

14. $\lim_{x \to \infty} (2\pi/7)^x$

15. $\lim_{x \to 1^-} e^{2/(x-1)}$

16. $\lim_{x \to 1^+} e^{2/(x-1)}$

17. $\lim_{x \to -\infty} \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

18. $\lim_{x \to (\pi/2)^-} \dfrac{2}{1 + e^{\tan x}}$

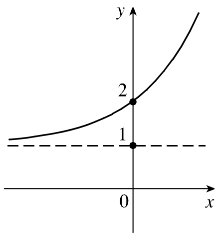
19. $\lim_{x \to 0^-} \dfrac{2}{1 + e^{\cot x}}$

3.1 ANSWERS

E Click here for exercises.

S Click here for solutions.

1.



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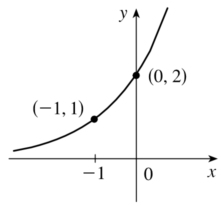

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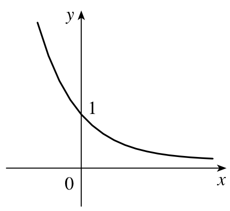

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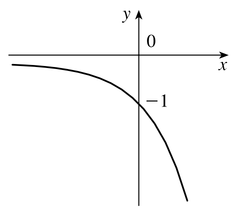

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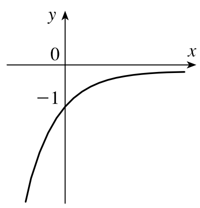

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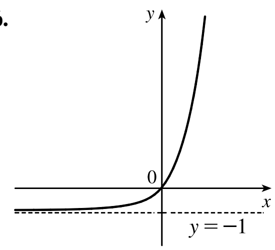

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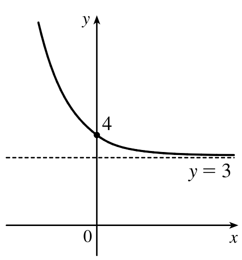

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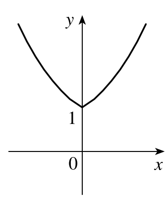


9. $f(x) = 3 \cdot 2^x$

10. $f(x) = 2(3)^{-x}$

11. ∞

12. 0

13. ∞

14. 0

15. 0

16. ∞

17. -1

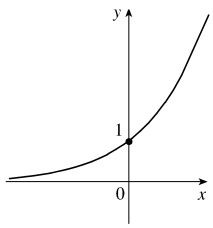
18. 0

19. 2

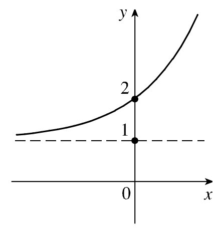
3.1 SOLUTIONS

E Click here for exercises.

1.

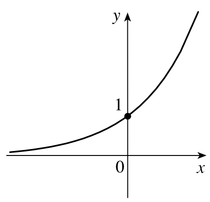


$y = 2^x$

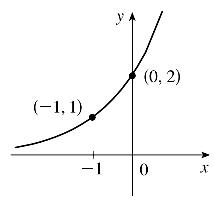


$y = 2^x + 1$

2.

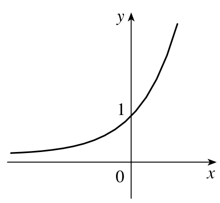


$y = 2^x$

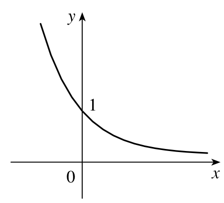


$y = 2^{x+1}$

3.

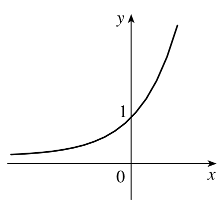


$y = 3^x$

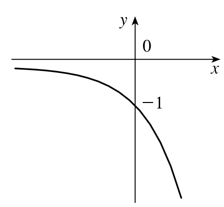


$y = 3^{-x}$

4.

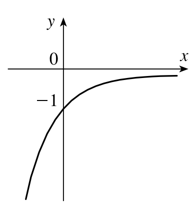


$y = 3^x$



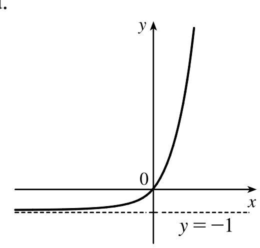
$y = -3^x$

5.

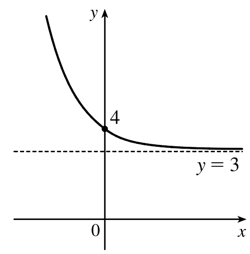


$y = -3^{-x}$

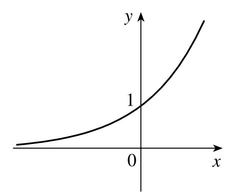
6. We start with the graph of $y = 10^x$ (Figure 3) and shift it 1 unit downward.



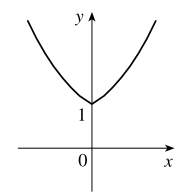
7. We start with the graph of $y = 2^x$ (Figure 3), reflect it about the y-axis ($y = 2^{-x}$), and then shift 3 units upward.



8. We reflect the part of $y = 2^x$ for $x > 0$ through the y-axis to obtain the part of $y = 2^{|x|}$ for $x < 0$.



$y = 2^x$



$y = 2^{|x|}$

9. Use $y = Ca^x$ with the points $(1, 6)$ and $(3, 24)$. $6 = Ca^1$ and $24 = Ca^3$ $\Rightarrow$ $24 = \left(\dfrac{6}{a}\right) a^3$ $\Rightarrow$ $4 = a^2$ $\Rightarrow$ $a = 2$ (since $a > 0$) and $C = 3$. The function is $f(x) = 3 \cdot 2^x$.

10. Given the y-intercept $(0, 2)$, we have $y = Ca^x = 2a^x$. Using the point $\left(2, \frac{2}{9}\right)$ gives us $\frac{2}{9} = 2a^2$ $\Rightarrow$ $\frac{1}{9} = a^2$ $\Rightarrow$ $a = \frac{1}{3}$ (since $a > 0$). The function is $f(x) = 2\left(\frac{1}{3}\right)^x$ or $f(x) = 2(3)^{-x}$.

11. $\lim\limits_{x \to \infty} (1.1)^x = \infty$ by Equation 3 since $1.1 > 1$.

12. $\lim\limits_{x \to -\infty} (1.1)^x = 0$ by Equation 3 since $1.1 > 1$.

13. $\lim\limits_{x \to -\infty} \left(\frac{\pi}{4}\right)^x = \infty$ since $0 < \frac{\pi}{4} < 1$.

14. $\lim\limits_{x \to \infty} \left(\frac{2\pi}{7}\right)^x = 0$ since $0 < \frac{2\pi}{7} < 1$.

15. $\lim\limits_{x \to 1^-} e^{2/(x-1)} = 0$ since $\dfrac{2}{x-1} \to -\infty$ as $x \to 1^-$.

16. $\lim\limits_{x \to 1^+} e^{2/(x-1)} = \infty$ since $\dfrac{2}{x-1} \to \infty$ as $x \to 1^+$.

17. Divide numerator and denominator by e^{-3x}:

$$\lim_{x \to -\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x \to -\infty} \frac{e^{6x} - 1}{e^{6x} + 1} = \frac{0 - 1}{0 + 1} = -1$$

18. $\lim\limits_{x \to (\pi/2)^-} \dfrac{2}{1 + e^{\tan x}} = 0$ since $\tan x \to \infty$ $\Rightarrow$ $e^{\tan x} \to \infty$.

19. As $x \to 0^-$, $\cot x = \dfrac{\cos x}{\sin x} \to -\infty$, so $e^{\cot x} \to 0$ and $\lim\limits_{x \to 0} \dfrac{2}{1 + e^{\cot x}} = \dfrac{2}{1 + 0} = 2$.