3.2 INVERSE FUNCTIONS AND LOGARITHMS

A Click here for answers.

S Click here for solutions.

1–10 ■ A function f is given by a graph or a formula. Determine whether f is one-to-one.

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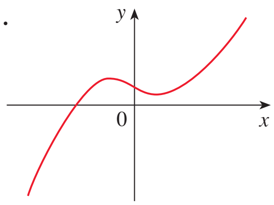

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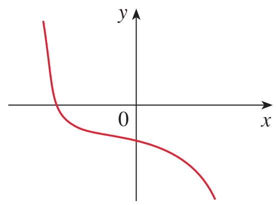

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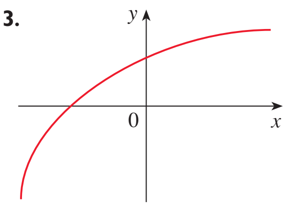

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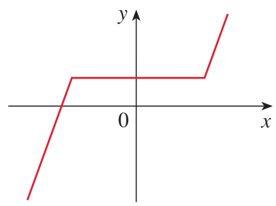

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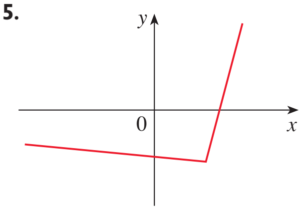

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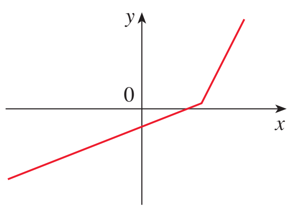


7. $f(x) = 7x - 3$

8. $f(x) = x^2 - 2x + 5$

9. $f(x) = 4 - 3x$

10. $f(x) = 3x^2 + 5x - 4$

11–16 ■ Find a formula for the inverse of the function.

11. $f(x) = 4x + 7$

12. $f(x) = \dfrac{x - 2}{x + 2}$

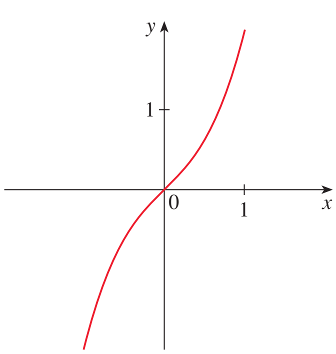
13. $f(x) = \dfrac{1 + 3x}{5 - 2x}$

14. $f(x) = 5 - 4x^3$

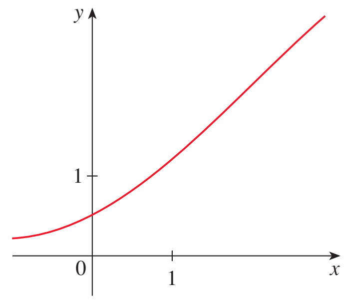
15. $f(x) = \sqrt{2 + 5x}$

16. $f(x) = x^2 + x, \quad x \geqslant -\frac{1}{2}$

17. Use the given graph of f to sketch the graph of f^{-1}.



18. Use the given graph of f to sketch the graphs of f^{-1} and $1/f$.



19. Find $g'(3)$, where g is the inverse function of $f(x) = 3 + x^2 + \sin \pi x$, $-0.4 < x < 0.4$.

20–32 ■ Find the exact value of the expression.

20. $\log_2 64$

21. $\log_6 \frac{1}{36}$

22. $\log_8 4$

23. $\log_3 \frac{1}{27}$

24. $e^{\ln 6}$

25. $\log_3 3^{\sqrt{5}}$

26. $\log_8 6 - \log_8 3 + \log_8 4$

27. $\log_8 2$

28. $\ln e^{\sqrt{2}}$

29. $\log_{10} 0.1$

30. $\log_3 108 - \log_3 4$

31. $\log_{10} 1.25 + \log_{10} 80$

32. $\log_5 10 + \log_5 20 - 3 \log_5 2$

33–35 ■ Express the quantity as a single logarithm.

33. $\log_2 x + 5 \log_2(x + 1) + \frac{1}{2} \log_2(x - 1)$

34. $\ln 10 + \frac{1}{2} \ln 9$

35. $\frac{1}{3} \ln x - 4 \ln(2x + 3)$

36. Use Formula 14 to evaluate each logarithm correct to six decimal places.

(a) $\log_{0.7} 14$ (b) $\log_2 5$

(c) $\log_5 26.05$ (d) $\log_3 e$

37–38 ■ Use Formula 14 to graph the given functions on a common screen. How are these graphs related?

37. $y = \log_3 x, \quad y = \log_{10} x$

38. $y = \log_2 x, \quad y = \ln x, \quad y = \log_{10} x$

39–47 ■ Make a rough sketch of the graph of each function. Do not use a calculator. Just use the graphs given in Figures 13, 14, and 15 and, if necessary, the transformations of Section 1.2.

39. $y = \log_{1.1} x$

40. $y = \log_{100} x$

41. $y = 1 + \log_5(x - 1)$

42. $y = \ln(-x)$

43. $y = -\ln(-x)$

44. $y = \ln(x + 3)$

45. $y = \ln|x + 3|$

46. $y = \ln(x^2)$

47. $y = \ln(1/x)$

48–63 ▪ Solve the equation for x.

48. $e^x = 16$

49. $\ln x = -1$

50. $\ln(2x - 1) = 3$

51. $e^{3x-4} = 2$

52. $\log_2 x = 3$

53. $2^{x-5} = 3$

54. $3^{x+2} = m$

55. $5^{\log_5(2x)} = 6$

56. $\ln x = \ln 5 + \ln 8$

57. $\ln x^2 = 2\ln 4 - 4\ln 2$

58. $\ln(e^{2x-1}) = 5$

59. $\ln(x + 6) + \ln(x - 3) = \ln 5 + \ln 2$

60. $\ln x + \ln(x - 1) = 1$

61. $2^{3^x} = 5$

62. $\log_2(\log_3(\log_4 x)) = C$

63. $\ln\left(\dfrac{x-2}{x-1}\right) = 1 + \ln\left(\dfrac{x-3}{x-1}\right)$

64–67 ▪ Find the solution of the equation correct to four decimal places.

64. $\ln(x - 5) = 3$

65. $e^{5x-1} = 12$

66. $e^{2-3x} = 20$

67. $2^{-x} = 5$

68–74 ▪ Find the limit.

68. $\lim_{x \to 5^+} \ln(x - 5)$

69. $\lim_{x \to 0^+} \log_{10}(4x)$

70. $\lim_{x \to \infty} \log_2(x^2 - x)$

71. $\lim_{x \to \infty} \ln(1 + x^2)$

72. $\lim_{x \to (\pi/2)^-} \log_{10}(\cos x)$

73. $\lim_{x \to \infty} \dfrac{\ln x}{1 + \ln x}$

74. $\lim_{x \to \infty} \ln\left(1 + e^{-x^2}\right)$

75–78 ▪ Find the domain and range of the function.

75. $g(x) = \ln(4 - x^2)$

76. $F(t) = \sqrt{t}\,\ln(t^2 - 1)$

77. $f(x) = \log_{10}(1 - x)$

78. $G(t) = \ln(t^3 - t)$

79. Find the inverse function of $y = e^{\sqrt{x}}$.

80. Without using a calculator, determine which of the numbers $\log_{10} 99$ or $\log_9 82$ is larger.

81. Solve the equation $4^x - 2^{x+3} + 12 = 0$.

3.2 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. No

2. Yes

3. Yes

4. No

5. No

6. Yes

7. Yes

8. No

9. Yes

10. No

11. $f^{-1}(x) = \frac{1}{4}(x-7)$

12. $f^{-1}(x) = \dfrac{2(1+x)}{1-x}$

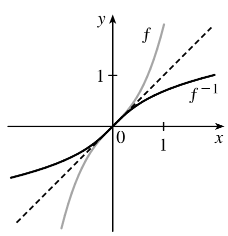
13. $f^{-1}(x) = \dfrac{5x-1}{2x+3}$

14. $f^{-1}(x) = \left(\dfrac{5-x}{4}\right)^{1/3}$

15. $f^{-1}(x) = \dfrac{x^2-2}{5}, x \geq 0$

16. $f^{-1}(x) = \frac{1}{2}\left(-1+\sqrt{1+4x}\right)$.

17.



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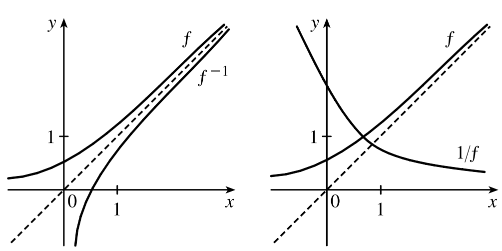


19. $\dfrac{1}{\pi}$

20. 6

21. -2

22. $\frac{2}{3}$

23. -3

24. 6

25. $\sqrt{5}$

26. 1

27. $\frac{1}{3}$

28. $\sqrt{2}$

29. -1

30. 3

31. 2

32. 2

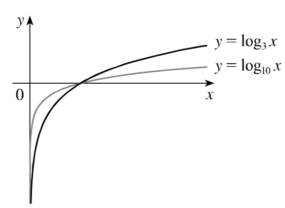
33. $\log_2\left(x(x+1)^5\sqrt{x-1}\right)$

34. $\ln 30$

35. $\ln\left(\dfrac{x^{1/3}}{(2x+3)^4}\right)$

36. (a) -7.399054 (b) 2.321928
(c) 2.025563 (d) 0.910239

37.



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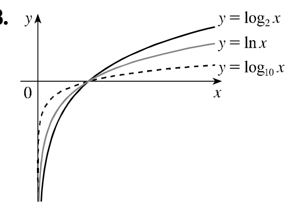

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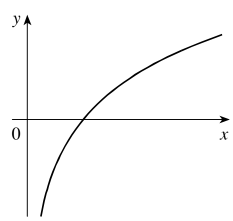

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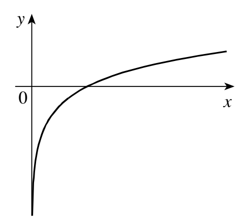

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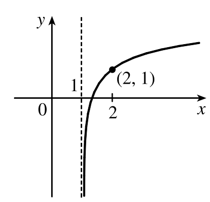

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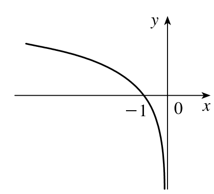

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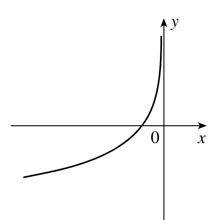

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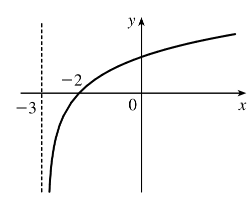

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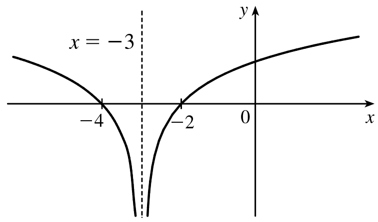

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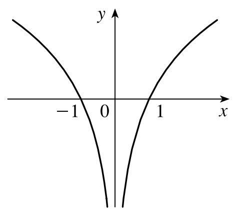

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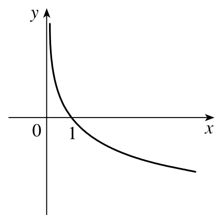


48. $x = 4\ln 2$

49. $x = 1/e$

50. $x = \frac{1}{2}\left(e^3 + 1\right)$

51. $x = \frac{1}{3}\left(\ln 2 + 4\right)$

52. $x = 8$

53. $x = 5 + \dfrac{\ln 3}{\ln 2}$

54. $x = \log_3 m - 2$

55. $x = 3$

56. $x = 40$

57. $x = \pm 1$

58. $x = 3$

59. $x = 4$

60. $x = \frac{1}{2}\left(1 + \sqrt{1 + 4e}\right)$

61. $x = \dfrac{\ln\left(\ln 5/\ln 2\right)}{\ln 3}$

62. $x = 4^{3^{2^C}}$

63. $x = \dfrac{2 - 3e}{1 - e}$

64. $x \approx 25.0855$

65. $x \approx 0.6970$

66. $x \approx -0.3319$

67. $x \approx -2.3219$

68. $-\infty$

69. $-\infty$

70. ∞

71. ∞

72. $-\infty$

73. 1

74. 0

75. $(-2, 2)$, $(-\infty, \ln\ 4]$

76. $(1, \infty)$, $\mathbb{R}$

77. $(-\infty, 1)$, $\mathbb{R}$

78. $(-1, 0) \cup (1, \infty)$, $\mathbb{R}$

79. $y = (\ln x)^2$, $x \geq 1$

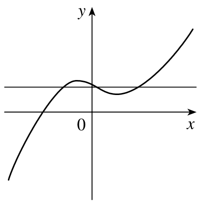
80. $\log_9 82$

81. $x = \log_2 6$, $x = 1$

3.2 SOLUTIONS

E Click here for exercises.

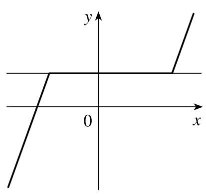
1. The diagram shows that there is a horizontal line which intersects the graph more than once, so the function is not one-to-one.



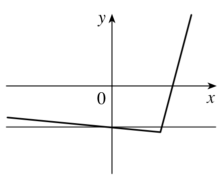
2. The function is one-to-one because no horizontal line intersects the graph more than once.

3. The function is one-to-one because no horizontal line intersects the graph more than once.

4. The diagram shows that there is a horizontal line which intersects the graph more than once, so the function is not one-to-one.



5. The diagram shows that there is a horizontal line which intersects the graph more than once, so the function is not one-to-one.



6. The function is one-to-one because no horizontal line intersects the graph more than once.

7. $x_1 \neq x_2 \quad \Rightarrow \quad 7x_1 \neq 7x_2 \quad \Rightarrow \quad 7x_1 - 3 \neq 7x_2 - 3 \quad \Rightarrow$ $f(x_1) \neq f(x_2)$, so f is 1-1.

8. $f(x) = x^2 - 2x + 5 \quad \Rightarrow \quad f(0) = 5 = f(2)$, so f is not one-to-one.

9. $f(x) = 4 - 3x \quad \Rightarrow \quad f'(x) = -3 < 0 \quad \Rightarrow \quad f$ is decreasing and hence one-to-one.

10. $f(x) = 3x^2 + 5x - 4 \quad \Rightarrow \quad f'(x) = 6x + 5 \quad \Rightarrow$ $f'(x) < 0$ and f is decreasing for $x < -\frac{5}{6}$ while $f'(x) > 0$ and f is increasing for $x > -\frac{5}{6}$. Thus, any horizontal line $y = k$ [where $k > f\left(-\frac{5}{6}\right)$] will intersect the graph of f more than once, so f fails the horizontal line test and is not 1-1.

11. $y = f(x) = 4x + 7 \quad \Rightarrow \quad 4x = y - 7 \quad \Rightarrow$ $x = (y - 7)/4$. Interchange x and y: $y = (x - 7)/4$. So $f^{-1}(x) = (x - 7)/4$.

12. $y = f(x) = \dfrac{x - 2}{x + 2} \quad \Rightarrow \quad xy + 2y = x - 2 \quad \Rightarrow$ $x(1 - y) = 2(y + 1) \quad \Rightarrow \quad x = \dfrac{2(1 + y)}{1 - y}$. Interchange x and y: $y = \dfrac{2(1 + x)}{1 - x}$. So $f^{-1}(x) = \dfrac{2(1 + x)}{1 - x}$.

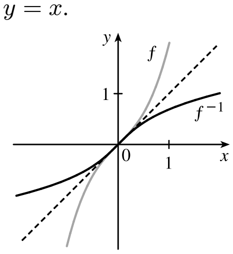
13. $y = f(x) = \dfrac{1 + 3x}{5 - 2x} \quad \Rightarrow \quad 5y - 2xy = 1 + 3x \quad \Rightarrow$ $5y - 1 = 3x + 2xy \quad \Rightarrow \quad x(3 + 2y) = 5y - 1 \quad \Rightarrow$ $x = \dfrac{5y - 1}{2y + 3}$. Interchange x and y: $y = \dfrac{5x - 1}{2x + 3}$. So $f^{-1}(x) = \dfrac{5x - 1}{2x + 3}$.

14. $y = f(x) = 5 - 4x^3 \quad \Rightarrow \quad 4x^3 = 5 - y \quad \Rightarrow$ $x^3 = (5 - y)/4 \quad \Rightarrow \quad x = \left(\dfrac{5 - y}{4}\right)^{1/3}$. Interchange x and y: $y = \left(\dfrac{5 - x}{4}\right)^{1/3}$. So $f^{-1}(x) = \left(\dfrac{5 - x}{4}\right)^{1/3}$.

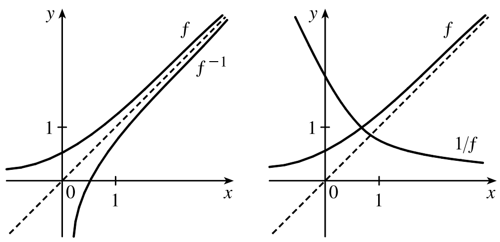
15. $y = f(x) = \sqrt{2 + 5x} \quad \Rightarrow \quad y^2 = 2 + 5x$ and $y \geq 0 \quad \Rightarrow$ $5x = y^2 - 2 \quad \Rightarrow \quad x = \dfrac{y^2 - 2}{5}$, $y \geq 0$. Interchange x and y: $y = \dfrac{x^2 - 2}{5}$, $x \geq 0$. So $f^{-1}(x) = \dfrac{x^2 - 2}{5}$, $x \geq 0$.

16. $y = f(x) = x^2 + x \quad \Rightarrow \quad x^2 + x - y = 0 \quad \Rightarrow$ $x = \frac{1}{2}\left(-1 \pm \sqrt{1 + 4y}\right)$ by the quadratic formula. But $x \geq -\frac{1}{2} \quad \Rightarrow \quad x = \frac{1}{2}\left(-1 + \sqrt{1 + 4y}\right)$. Interchange x and y: $y = \frac{1}{2}\left(-1 + \sqrt{1 + 4x}\right)$. So $f^{-1}(x) = \frac{1}{2}\left(-1 + \sqrt{1 + 4x}\right)$.

17. The function f is one-to-one, so its inverse exists and the graph of its inverse can be obtained by reflecting the graph of f through the line $y = x$.



18. For the graph of $1/f$, the y-coordinates are simply the reciprocals of f. For example, if $f(0) = \frac{1}{2}$, then $1/f(0) = 2$. If we draw the horizontal line $y = 1$, we see that the only place where the graphs intersect is on that line.



19. $f(0) = 3 \;\Rightarrow\; g(3) = 0$ and $f'(x) = 2x + \pi\cos\pi x \;\Rightarrow\; f'(0) = \pi$. Therefore, $g'(3) = \dfrac{1}{f'(g(3))} = \dfrac{1}{f'(0)} = \dfrac{1}{\pi}$.

20. $\log_2 64 = 6$ since $2^6 = 64$.

21. $\log_6 \frac{1}{36} = -2$ since $6^{-2} = \frac{1}{36}$.

22. $\log_8 4 = \frac{2}{3}$ since $8^{2/3} = 4$.

23. $\log_3 \frac{1}{27} = -3$ since $3^{-3} = \frac{1}{27}$.

24. $e^{\ln 6} = 6$

25. $\log_3 3^{\sqrt{5}} = \sqrt{5}$

26. $\log_8 6 - \log_8 3 + \log_8 4 = \log_8 \frac{6\cdot 4}{3} = \log_8 8 = 1$

27. $\log_8 2 = \frac{1}{3}$ since $8^{1/3} = 2$.

28. $\ln e^{\sqrt{2}} = \sqrt{2}$

29. $\log_{10} 0.1 = \log_{10} 10^{-1} = -1$

30. $\log_3 108 - \log_3 4 = \log_3 \frac{108}{4} = \log_3 27 = 3$

31. $\log_{10} 1.25 + \log_{10} 80 = \log_{10}(1.25 \cdot 80)$
$= \log_{10} 100 = \log_{10} 10^2 = 2$

32. $\log_5 10 + \log_5 20 - 3\log_5 2 = \log_5(10\cdot 20) - \log_5 2^3$
$= \log_5 \frac{200}{8} = \log_5 25$
$= \log_5 5^2 = 2$

33. $\log_2 x + 5\log_2(x+1) + \frac{1}{2}\log_2(x-1)$
$= \log_2 x + \log_2(x+1)^5 + \log_2\sqrt{x-1}$
$= \log_2\left(x(x+1)^5\sqrt{x-1}\right)$

34. $\ln 10 + \frac{1}{2}\ln 9 = \ln 10 + \ln 9^{1/2} = \ln 10 + \ln 3 = \ln 30$

35. $\frac{1}{3}\ln x - 4\ln(2x+3) = \ln\left(x^{1/3}\right) - \ln(2x+3)^4$
$= \ln\left(\dfrac{x^{1/3}}{(2x+3)^4}\right)$

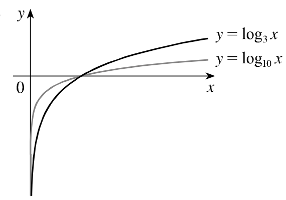
36. (a) $\log_{0.7} 14 = \dfrac{\ln 14}{\ln 0.7} \approx -7.399054$

(b) $\log_2 5 = \dfrac{\ln 5}{\ln 2} \approx 2.321928$

(c) $\log_5 26.05 = \dfrac{\ln 26.05}{\ln 5} \approx 2.025563$

(d) $\log_3 e = \dfrac{1}{\ln 3} \approx 0.910239$

37.



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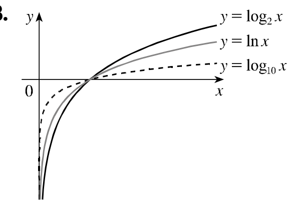

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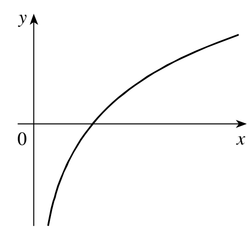
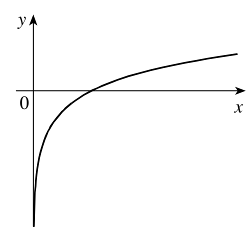


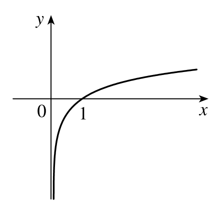
$y = \log_{1.1} x$

40.

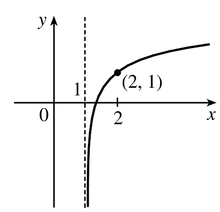


$y = \log_{100} x$

41.

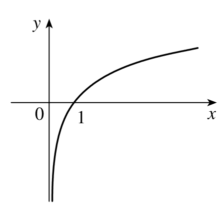


$y = \log_5 x$

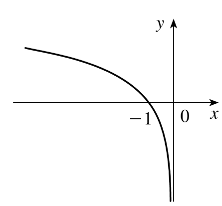


$y = 1 + \log_5(x-1)$

42.

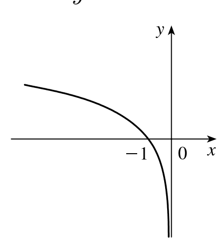


$y = \ln x$

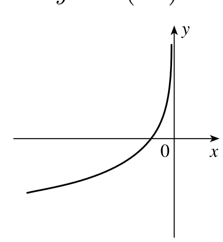


$y = \ln(-x)$

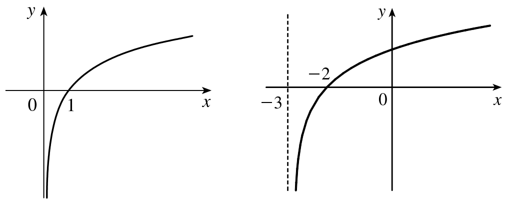
43.



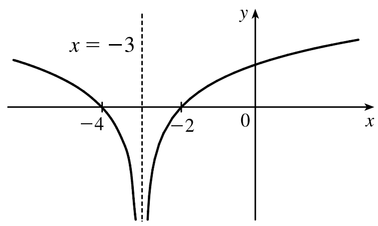
$y = \ln(-x)$



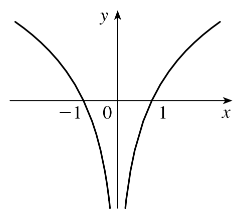
$y = -\ln(-x)$

44. 

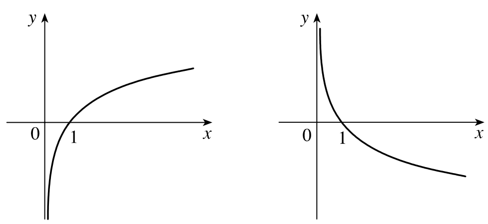
$y = \ln x$ $\qquad$ $y = \ln(x+3)$

45. 

$y = \ln|x+3|$

46. 

$y = \ln(x^2) = 2\ln|x|$

47. 

$y = \ln x$ $\qquad$ $y = \ln 1/x = -\ln x$

48. $e^x = 16 \;\Leftrightarrow\; \ln e^x = \ln 16 \;\Leftrightarrow\; x = \ln 16 = \ln 2^4 = 4\ln 2$

49. $\ln x = -1 \;\Leftrightarrow\; e^{\ln x} = e^{-1} \;\Leftrightarrow\; x = 1/e$

50. $\ln(2x-1) = 3 \;\Leftrightarrow\; e^{\ln(2x-1)} = e^3 \;\Leftrightarrow\; 2x - 1 = e^3 \;\Leftrightarrow\; x = \frac{1}{2}(e^3+1)$

51. $e^{3x-4} = 2 \;\Leftrightarrow\; \ln(e^{3x-4}) = \ln 2 \;\Leftrightarrow\; 3x - 4 = \ln 2 \;\Leftrightarrow\; x = \frac{1}{3}(\ln 2 + 4)$

52. $\log_2 x = 3 \;\Leftrightarrow\; x = 2^3 = 8$

53. $2^{x-5} = 3 \;\Leftrightarrow\; \log_2 3 = x - 5 \;\Leftrightarrow\; x = 5 + \log_2 3$.
Or: $2^{x-5} = 3 \;\Leftrightarrow\; \ln(2^{x-5}) = \ln 3 \;\Leftrightarrow\; (x-5)\ln 2 = \ln 3 \;\Leftrightarrow\; x - 5 = \dfrac{\ln 3}{\ln 2} \;\Leftrightarrow\; x = 5 + \dfrac{\ln 3}{\ln 2}$

54. $3^{x+2} = m \;\Leftrightarrow\; \log_3 m = x + 2 \;\Leftrightarrow\; x = \log_3 m - 2$

55. $6 = 5^{\log_5(2x)} = 2x \;\Leftrightarrow\; x = 3$

56. $\ln x = \ln 5 + \ln 8 = \ln 40 \;\Leftrightarrow\; x = 40$

57. $\ln x^2 = 2\ln 4 - 4\ln 2 = 4\ln 2 - 4\ln 2 = 0 \;\Leftrightarrow\; x^2 = 1 \;\Leftrightarrow\; x = \pm 1$

58. $5 = \ln(e^{2x-1}) = 2x - 1 \;\Leftrightarrow\; x = 3$

59. $\ln(x+6) + \ln(x-3) = \ln 5 + \ln 2 \;\Leftrightarrow\; \ln((x+6)(x-3)) = \ln 10 \;\Leftrightarrow\; (x+6)(x-3) = 10 \;\Leftrightarrow\; x^2 + 3x - 18 = 10 \;\Leftrightarrow\; x^2 + 3x - 28 = 0 \;\Leftrightarrow\; (x+7)(x-4) = 0 \;\Leftrightarrow\; x = -7$ or 4. However, $x = -7$ is not a solution since $\ln(-7+6)$ is not defined. So $x = 4$ is the only solution.

60. $\ln x + \ln(x-1) = \ln(x(x-1)) = 1 \;\Leftrightarrow\; x(x-1) = e^1 \;\Leftrightarrow\; x^2 - x - e = 0$. The quadratic formula gives $x = \frac{1}{2}\left(1 \pm \sqrt{1+4e}\right)$, but we reject the negative root since the natural logarithm is not defined for $x < 0$. So $x = \frac{1}{2}\left(1 + \sqrt{1+4e}\right)$.

61. $2^{3^x} = 5 \;\Leftrightarrow\; 3^x = \log_2 5 \;\Leftrightarrow\; \log_3(\log_2 5) = x$.
Or: $2^{3^x} = 5 \;\Leftrightarrow\; \ln 2^{3^x} = \ln 5 \;\Leftrightarrow\; 3^x \ln 2 = \ln 5 \;\Leftrightarrow\; 3^x = \dfrac{\ln 5}{\ln 2}$. Hence $\ln 3^x = x\ln 3 = \ln\left(\dfrac{\ln 5}{\ln 2}\right) \;\Leftrightarrow\; x = \dfrac{\ln(\ln 5/\ln 2)}{\ln 3}$.

62. $\log_2[\log_3(\log_4 x)] = C \;\Leftrightarrow\; 2^C = \log_3(\log_4 x) \;\Leftrightarrow\; 3^{2^C} = \log_4 x \;\Leftrightarrow\; x = 4^{3^{2^C}}$

63. $\ln\left(\dfrac{x-2}{x-1}\right) = 1 + \ln\left(\dfrac{x-3}{x-1}\right) \;\Leftrightarrow\; \ln\left(\dfrac{x-2}{x-1}\right) - \ln\left(\dfrac{x-3}{x-1}\right) = 1 \;\Leftrightarrow\; \ln\left(\dfrac{x-2}{x-3}\right) = 1 \;\Leftrightarrow\; \dfrac{x-2}{x-3} = e^1 = e$. Thus $x - 2 = (x-3)e \;\Leftrightarrow\; x(1-e) = 2 - 3e \;\Leftrightarrow\; x = \dfrac{2-3e}{1-e}$.

64. $\ln(x-5) = 3 \;\Rightarrow\; x - 5 = e^3 \;\Rightarrow\; x = e^3 + 5 \approx 25.0855$

65. $e^{5x-1} = 12 \;\Rightarrow\; 5x - 1 = \ln 12 \;\Rightarrow\; x = \frac{1}{5}(\ln 12 + 1) \approx 0.6970$

66. $e^{2-3x} = 20 \;\Rightarrow\; 2 - 3x = \ln 20 \;\Rightarrow\; x = \frac{1}{3}(2 - \ln 20) \approx -0.3319$

67. $2^{-x} = 5 \;\Rightarrow\; -x\ln 2 = \ln 5 \;\Rightarrow\; x = -\ln 5/\ln 2 \approx -2.3219$

68. $\lim\limits_{x\to 5^+} \ln(x-5) = -\infty$ since $x - 5 \to 0^+$ as $x \to 5^+$.

69. $\lim\limits_{x\to 0^+} \log_{10}(4x) = -\infty$ since $4x \to 0^+$ as $x \to 0^+$.

70. $\lim\limits_{x\to\infty} \log_2(x^2 - x) = \infty$ since $x^2 - x \to \infty$ as $x \to \infty$.

71. Let $t = 1 + x^2$. As $x \to \infty$, $t \to \infty$. $\lim\limits_{x\to\infty} \ln(1+x^2) = \lim\limits_{t\to\infty} \ln t = \infty$ by (8).

72. $\lim\limits_{x\to(\pi/2)^-} \log_{10}(\cos x) = -\infty$ since $\cos x \to 0^+$ as $x \to (\pi/2)^-$.

73. $\lim\limits_{x\to\infty} \dfrac{\ln x}{1+\ln x} = \lim\limits_{x\to\infty} \dfrac{1}{(1/\ln x) + 1} = \dfrac{1}{0+1} = 1$

74. $\lim_{x\to\infty} \ln\left(1+e^{-x^2}\right) = \ln\left(1+\lim_{x\to\infty} e^{-x^2}\right)$
$= \ln(1+0) = 0$

75. $g(x) = \ln(4-x^2)$
Domain $(g) = \{x \mid 4-x^2 > 0\}$
$= \{x \mid |x| < 2\} = (-2, 2)$
Since $4 - x^2 \le 4$, we have $\ln(4-x^2) \le \ln 4$. Also $\lim_{x\to 2^-} g(x) = -\infty$, so range $(g) = (-\infty, \ln 4]$.

76. $F(t) = \sqrt{t}\ln(t^2-1)$
Domain $(F) = \{t \mid t \ge 0 \text{ and } t^2 - 1 > 0\}$
$= \{t \mid t > 1\} = (1, \infty)$
Range $(F) = \mathbb{R}$

77. $f(x) = \log_{10}(1-x)$
Domain $(f) = \{x \mid 1 - x > 0\} = \{x \mid x < 1\}$
$= (-\infty, 1)$
Range $(f) = \mathbb{R}$.

78. $G(t) = \ln(t^3 - t)$
Domain $(G) = \{t \mid t^3 - t > 0\} = \{t \mid t(t^2-1) > 0\}$
$= \{t \mid t > 1 \text{ or } -1 < t < 0\}$
$= (-1, 0) \cup (1, \infty)$
Range $(G) = \mathbb{R}$.

79. $y = e^{\sqrt{x}} \Rightarrow \ln y = \ln e^{\sqrt{x}} = \sqrt{x} \Rightarrow x = (\ln y)^2$.
Also note that $\sqrt{x} \ge 0 \Rightarrow y = e^{\sqrt{x}} \ge 1$. Interchange x and y: the inverse function is $y = (\ln x)^2$, $x \ge 1$.

80. Let $x = \log_{10} 99$, $y = \log_9 82$. Then $10^x = 99 < 10^2 \Rightarrow x < 2$, and $9^y = 82 > 9^2 \Rightarrow y > 2$. Therefore, $y = \log_9 82$ is larger.

81. Notice that $4^x = (2^2)^x = 2^{2x} = (2^x)^2$, and $2^{x+3} = 2^3 2^x = 8 \cdot 2^x$, so that $4^x - 2^{x+3} + 12 = 0 \Leftrightarrow (2^x)^2 - 8 \cdot 2x + 12 = (2^x - 6)(2^x - 2) = 0 \Leftrightarrow 2^x = 6$ or $2^x = 2$. Hence, $x = \log_2 6$ or $x = 1$.