3.3 DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

A Click here for answers.

S Click here for solutions.

1–9 ▪ Differentiate f and find the domain of f.

1. $f(x) = \ln(x + 1)$

2. $f(x) = \cos(\ln x)$

3. $f(x) = \log_3(x^2 - 4)$

4. $f(x) = \sqrt{3 - 2^x}$

5. $f(x) = \ln(2 - x - x^2)$

6. $f(x) = \ln\left(\sqrt{x} - \sqrt{x - 1}\right)$

7. $f(x) = \dfrac{1}{1 + \ln x}$

8. $f(x) = x^2 \ln(1 - x^2)$

9. $f(x) = \ln \ln \ln x$

10–37 ▪ Differentiate the function.

10. $f(x) = \ln(2 - x)$

11. $F(x) = \ln \sqrt{x}$

12. $G(x) = \sqrt[3]{\ln x}$

13. $F(x) = e^x \ln x$

14. $h(y) = \ln(y^3 \sin y)$

15. $y = \dfrac{\ln x}{1 + x}$

16. $y = (\ln \tan x)^2$

17. $y = \ln |x^3 - x^2|$

18. $G(x) = \sqrt{\ln x}$

19. $f(t) = \log_2(t^4 - t^2 + 1)$

20. $g(u) = \dfrac{1 - \ln u}{1 + \ln u}$

21. $y = (\ln \sin x)^3$

22. $y = \dfrac{\ln x}{1 + x^2}$

23. $y = \ln\left(x\sqrt{1 - x^2} \sin x\right)$

24. $y = \ln |\tan 2x|$

25. $G(x) = 5^{\tan x}$

26. $f(t) = \pi^{-t}$

27. $g(x) = 1.6^x + x^{1.6}$

28. $g(t) = \sin(\ln t)$

29. $k(r) = r \sin r \ln r$

30. $y = \ln\left(\dfrac{x + 1}{x - 1}\right)^{3/5}$

31. $y = \tan[\ln(ax + b)]$

32. $h(\theta) = 10^{\sec\theta}$

33. $y = \ln\left(e^{2x} + \sqrt{e^{4x} + 1}\right)$

34. $y = x^{1/\ln x}$

35. $y = (\sin x)^{\cos x}$

36. $y = \cos\left(x^{\sqrt{x}}\right)$

37. $y = x^{x^x}$

38. If $f(x) = \dfrac{x}{\ln x}$, find $f'(e)$.

39. If $f(x) = x^2 \ln x$, find $f'(1)$.

40–44 ▪ Find an equation of the tangent line to the curve at the given point.

40. $y = \ln \ln x$, $(e, 0)$

41. $y = 10^x$, $(1, 10)$

42. $y = \ln(x^2 + 1)$, $(1, \ln 2)$

43. $y = \ln(1 + e^x)$, $(0, \ln 2)$

44. $y = (x + 1)^x$, $(1, 2)$

45–60 ▪ Differentiate the function.

45. $y = e^{-mx}$

46. $g(x) = e^{-5x} \cos 3x$

47. $f(x) = e^{\sqrt{x}}$

48. $h(t) = \sqrt{1 - e^t}$

49. $h(\theta) = e^{\sin 5\theta}$

50. $y = e^{x \cos x}$

51. $y = \dfrac{e^{3x}}{1 + e^x}$

52. $f(x) = xe^{-x^2}$

53. $y = xe^{2x}$

54. $y = \dfrac{e^{-x^2}}{x}$

55. $y = e^{-1/x}$

56. $y = e^{x + e^x}$

57. $y = \tan(e^{3x-2})$

58. $y = \sqrt[3]{2x + e^{3x}}$

59. $y = x^e$

60. $y = \sec\left(e^{\tan x^2}\right)$

61–62 ▪ Find an equation of the tangent line to the curve at the given point.

61. $y = x^2 e^{-x}$, $(1, 1/e)$

62. $y = e^{-x} \sin x$, $(\pi, 0)$

63. Find y' if $\cos(x - y) = xe^x$.

64. Find an equation of the tangent line to the curve $2e^{xy} = x + y$ at the point $(0, 2)$.

65. Show that the function $y = e^{2x} + e^{-3x}$ satisfies the differential equation $y'' + y' - 6y = 0$.

66. For what values of r does the function $y = e^{rx}$ satisfy the equation $y'' + 5y' - 6y = 0$?

3.3 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $f'(x) = 1/(x+1)$, $(-1, \infty)$

2. $f'(x) = -\dfrac{\sin(\ln x)}{x}$, $(0, \infty)$

3. $f'(x) = \dfrac{2x}{(x^2-4)\ln 3}$, $(-\infty, -2) \cup (2, \infty)$

4. $f'(x) = -\ln 2 \dfrac{2^{x-1}}{\sqrt{3-2^x}}$, $(-\infty, \log_2 3]$

5. $f'(x) = \dfrac{-1-2x}{2-x-x^2}$, $(-2, 1)$

6. $f'(x) = -\dfrac{1}{2\sqrt{x}\sqrt{x-1}}$, $[1, \infty)$

7. $f'(x) = -\dfrac{1}{x(1+\ln x)^2}$, $(0, 1/e) \cup (1/e, \infty)$

8. $f'(x) = 2x\ln\left(1-x^2\right) - \dfrac{2x^3}{1-x^2}$, $(-1, 1)$

9. $f'(x) = \dfrac{1}{\ln\ln x} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}$, (e, ∞)

10. $f'(x) = \dfrac{1}{x-2}$

11. $F'(x) = \dfrac{1}{2x}$

12. $G'(x) = \dfrac{1}{3x(\ln x)^{2/3}}$

13. $F'(x) = e^x\left(\ln x + \dfrac{1}{x}\right)$

14. $h'(y) = \dfrac{3}{y} + \cot y$

15. $y' = \dfrac{1+x-x\ln x}{x(1+x)^2}$

16. $y' = \dfrac{2(\ln\tan x)\sec^2 x}{\tan x}$

17. $y' = \dfrac{3x-2}{x(x\quad 1)}$

18. $G'(x) = \dfrac{1}{2x\sqrt{\ln x}}$

19. $f'(t) = \dfrac{4t^3-2t}{(t^4-t^2+1)\ln 2}$

20. $g'(u) = -\dfrac{2}{u(1+\ln u)^2}$

21. $y' = 3(\ln\sin x)^2\cot x$

22. $y' = \dfrac{1+x^2-2x^2\ln x}{x(1+x^2)^2}$

23. $y' = \dfrac{1}{x} - \dfrac{x}{1-x^2} + \cot x$

24. $y' = \dfrac{2\sec^2 2x}{\tan 2x}$

25. $G'(x) = 5^{\tan x}(\ln 5)\sec^2 x$

26. $f'(t) = -\pi^{-t}\ln\pi$

27. $g'(x) = 1.6^x\ln(1.6) + 1.6x^{0.6}$

28. $g'(t) = \dfrac{\cos(\ln t)}{t}$

29. $k'(r) = \sin r\ln r + r\cos r\ln r + \sin r$

30. $y' = -\dfrac{6}{5(x^2-1)}$

31. $y' = \sec^2(\ln(ax+b))\dfrac{a}{ax+b}$

32. $h'(\theta) = 10^{\sec\theta}(\ln 10)\sec\theta\tan\theta$

33. $y' = \dfrac{2e^{2x}}{\sqrt{e^{4x}+1}}$

34. $y' = 0$

35. $y' = (\sin x)^{\cos x}(-\sin x\ln\sin x + \cos x\cot x)$

36. $y' = -\sin\left(x^{\sqrt{x}}\right)x^{\sqrt{x}}\left(\dfrac{\ln x+2}{2\sqrt{x}}\right)$

37. $y' = x^{x^x}\left[x^x(\ln x+1)\ln x + x^{x-1}\right]$

38. 0

39. 1

40. $y = \dfrac{1}{e}(x-e)$

41. $y = 10[(x-1)\ln 10 + 1]$

42. $y = x + \ln 2 - 1$

43. $y = \frac{1}{2}x + \ln 2$

44. $y = (1+2\ln 2)x + 1 - 2\ln 2$

45. $y' = -me^{-mx}$

46. $g'(x) = -5e^{-5x}\cos 3x - 3e^{-5x}\sin 3x$

47. $f'(x) = e^{\sqrt{x}}/(2\sqrt{x})$

48. $h'(t) = -e^t/\left(2\sqrt{1-e^t}\right)$

49. $h'(\theta) = 5\cos(5\theta)e^{\sin 5\theta}$

50. $y' = e^{x\cos x}(\cos x - x\sin x)$

51. $y' = \dfrac{3e^{3x}+2e^{4x}}{(1+e^x)^2}$

52. $f'(x) = e^{-x^2}\left(1 - 2x^2\right)$ **53.** $y' = e^{2x}(1 + 2x)$

54. $y' = \dfrac{e^{-x^2}\left(-2x^2 - 1\right)}{x^2}$ **55.** $y' = e^{-1/x}/x^2$

56. $y' = e^{x+e^x}(1 + e^x)$

57. $y' = 3e^{3x-2}\sec^2\left(e^{3x-2}\right)$

58. $y' = \frac{1}{3}\left(2 + 3e^{3x}\right)\left(2x + e^{3x}\right)^{-2/3}$

59. $y' = ex^{e-1}$

60. $y' = 2xe^{\tan x^2}\sec^2\left(x^2\right)\sec\left(e^{\tan x^2}\right)\tan\left(e^{\tan x^2}\right)$

61. $y = x/e$

62. $y = -e^{-\pi}x + \pi e^{-\pi}$

63. $1 + \dfrac{e^x(1 + x)}{\sin(x - y)}$

64. $y = 3x + 2$

66. $1, -6$

3.3 SOLUTIONS

E Click here for exercises.

1. $f(x) = \ln(x+1) \quad \Rightarrow \quad f'(x) = 1/(x+1)$,
$\text{Dom}(f) = \{x \mid x+1 > 0\} = \{x \mid x > -1\} = (-1, \infty)$

2. $f(x) = \cos(\ln x) \quad \Rightarrow \quad f'(x) = -\dfrac{\sin(\ln x)}{x}$,
$\text{Dom}(f) = (0, \infty)$

3. $f(x) = \log_3(x^2 - 4) \quad \Rightarrow \quad f'(x) = \dfrac{2x}{(x^2-4)\ln 3}$

$$\begin{aligned}\text{Dom}(f) &= \{x \mid x^2 - 4 > 0\} \\ &= \{x \mid |x| > 2\} = (-\infty, -2) \cup (2, \infty)\end{aligned}$$

4. $f(x) = \sqrt{3 - 2^x} \quad \Rightarrow$

$$f'(x) = \frac{1}{2\sqrt{3-2^x}}\left(-2^x \ln 2\right) = -\ln 2 \frac{2^{x-1}}{\sqrt{3-2^x}}$$

$\text{Dom}(f) = \{x \mid 2^x \le 3\} = \{x \mid x \le \log_2 3\} = (-\infty, \log_2 3]$

5. $f(x) = \ln(2 - x - x^2) \quad \Rightarrow \quad f'(x) = \dfrac{-1-2x}{2-x-x^2}$

$$\begin{aligned}\text{Dom}(f) &= \{x \mid 2 - x - x^2 > 0\} \\ &= \{x \mid (2+x)(1-x) > 0\} \\ &= \{x \mid -2 < x < 1\} = (-2, 1)\end{aligned}$$

6. $f(x) = \ln\left(\sqrt{x} - \sqrt{x-1}\right) \quad \Rightarrow$

$$\begin{aligned}f'(x) &= \frac{1}{\sqrt{x} - \sqrt{x-1}}\left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x-1}}\right) \\ &= \frac{1}{\sqrt{x} - \sqrt{x-1}} \frac{\sqrt{x-1} - \sqrt{x}}{2\sqrt{x}\sqrt{x-1}} \\ &= -\frac{1}{2\sqrt{x}\sqrt{x-1}}\end{aligned}$$

$\text{Dom}(f) = \{x \mid x \ge 1\} = [1, \infty)$

7. $f(x) = \dfrac{1}{1 + \ln x} \quad \Rightarrow$

$$\begin{aligned}f'(x) &= -\frac{1/x}{(1+\ln x)^2} \quad \text{(Reciprocal Rule)} \\ &= -\frac{1}{x(1+\ln x)^2}\end{aligned}$$

$$\begin{aligned}\text{Dom}(f) &= \{x \mid x > 0 \text{ and } \ln x \ne -1\} \\ &= \{x \mid x > 0 \text{ and } x \ne 1/e\} \\ &= (0, 1/e) \cup (1/e, \infty)\end{aligned}$$

8. $f(x) = x^2 \ln(1 - x^2) \quad \Rightarrow$

$$\begin{aligned}f'(x) &= 2x \ln(1-x^2) + \frac{x^2(-2x)}{1-x^2} \\ &= 2x\ln(1-x^2) - \frac{2x^3}{1-x^2}\end{aligned}$$

$\text{Dom}(f) = \{x \mid 1 - x^2 > 0\} = \{x \mid |x| < 1\} = (-1, 1)$

9. $f(x) = \ln\ln\ln x \quad \Rightarrow \quad f'(x) = \dfrac{1}{\ln\ln x} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}$.

$$\begin{aligned}\text{Dom}(f) &= \{x \mid \ln\ln x > 0\} = \{x \mid \ln x > 1\} \\ &= \{x \mid x > e\} = (e, \infty)\end{aligned}$$

10. $f(x) = \ln(2 - x) \quad \Rightarrow$

$$f'(x) = \frac{1}{2-x}\frac{d}{dx}(2-x) = \frac{-1}{2-x} = \frac{1}{x-2}$$

11. $F(x) = \ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2}\ln x \quad \Rightarrow$

$$F'(x) = \frac{1}{2}\left(\frac{1}{x}\right) = \frac{1}{2x}$$

12. $G(x) = \sqrt[3]{\ln x} = (\ln x)^{1/3} \quad \Rightarrow$

$$G'(x) = \tfrac{1}{3}(\ln x)^{-2/3} \cdot \frac{1}{x} = \frac{1}{3x(\ln x)^{2/3}}$$

13. $F(x) = e^x \ln x \quad \Rightarrow$

$$F'(x) = e^x \ln x + e^x\left(\frac{1}{x}\right) = e^x\left(\ln x + \frac{1}{x}\right)$$

14. $h(y) = \ln(y^3 \sin y) = 3\ln y + \ln(\sin y) \quad \Rightarrow$

$$h'(y) = \frac{3}{y} + \frac{1}{\sin y}(\cos y) = \frac{3}{y} + \cot y$$

15. $y = \dfrac{\ln x}{1+x} \quad \Rightarrow$

$$\begin{aligned}y' &= \frac{(1+x)(1/x) - (\ln x)(1)}{(1+x)^2} = \frac{\dfrac{1+x}{x} - \dfrac{x\ln x}{x}}{(1+x)^2} \\ &= \frac{1 + x - x\ln x}{x(1+x)^2}\end{aligned}$$

16. $y = (\ln\tan x)^2 \quad \Rightarrow$

$$y' = 2(\ln\tan x) \cdot \frac{1}{\tan x} \cdot \sec^2 x = \frac{2(\ln\tan x)\sec^2 x}{\tan x}$$

17. $y = \ln\left|x^3 - x^2\right| \quad\Rightarrow$

$y' = \dfrac{1}{x^3 - x^2}\left(3x^2 - 2x\right) = \dfrac{x\,(3x-2)}{x^2\,(x-1)} = \dfrac{3x-2}{x\,(x-1)}$

18. $G(x) = \sqrt{\ln x} \quad\Rightarrow\quad G'(x) = \dfrac{1}{2\sqrt{\ln x}}\left(\dfrac{1}{x}\right) = \dfrac{1}{2x\sqrt{\ln x}}$

19. $f(t) = \log_2\left(t^4 - t^2 + 1\right) \quad\Rightarrow$

$f'(t) = \dfrac{4t^3 - 2t}{(t^4 - t^2 + 1)\ln 2}$

20. $g(u) = \dfrac{1 - \ln u}{1 + \ln u} \quad\Rightarrow$

$g'(u) = \dfrac{(1+\ln u)(-1/u) - (1-\ln u)(1/u)}{(1+\ln u)^2}$

$= -\dfrac{2}{u\,(1+\ln u)^2}$

21. $y = (\ln\sin x)^3 \quad\Rightarrow$

$y' = 3\,(\ln\sin x)^2\,\dfrac{\cos x}{\sin x} = 3\,(\ln\sin x)^2\cot x$

22. $y = \dfrac{\ln x}{1+x^2} \quad\Rightarrow$

$y' = \dfrac{\left(1+x^2\right)(1/x) - 2x\ln x}{(1+x^2)^2} = \dfrac{1 + x^2 - 2x^2\ln x}{x\,(1+x^2)^2}$

23. $y = \ln\left(x\sqrt{1-x^2}\,\sin x\right) = \ln x + \frac{1}{2}\ln\left(1-x^2\right) + \ln\sin x$

$\Rightarrow\quad y' = \dfrac{1}{x} + \dfrac{1}{2}\left(\dfrac{-2x}{1-x^2}\right) + \dfrac{\cos x}{\sin x} = \dfrac{1}{x} - \dfrac{x}{1-x^2} + \cot x$

24. $y = \ln|\tan 2x| \quad\Rightarrow\quad y' = \dfrac{2\sec^2 2x}{\tan 2x}$

25. $G(x) = 5^{\tan x} \quad\Rightarrow\quad G'(x) = 5^{\tan x}(\ln 5)\sec^2 x$

26. $f(t) = \pi^{-t} \quad\Rightarrow\quad f'(t) = \pi^{-t}(\ln\pi)(-1) = -\pi^{-t}\ln\pi$

27. $g(x) = 1.6^x + x^{1.6} \quad\Rightarrow\quad g'(x) = 1.6^x\ln(1.6) + 1.6x^{0.6}$

28. $g(t) = \sin(\ln t) \quad\Rightarrow\quad g'(t) = \dfrac{\cos(\ln t)}{t}$

29. $k(r) = r\sin r\ln r \quad\Rightarrow$

$k'(r) = \sin r\ln r + r\cos r\ln r + \sin r$

30. $y = \ln\left(\dfrac{x+1}{x-1}\right)^{3/5} = \frac{3}{5}\left[\ln(x+1) - \ln(x-1)\right] \quad\Rightarrow$

$y' = \dfrac{3}{5}\left(\dfrac{1}{x+1} - \dfrac{1}{x-1}\right) = -\dfrac{6}{5\,(x^2-1)}$

31. $y = \tan(\ln(ax+b)) \quad\Rightarrow$

$y' = \sec^2(\ln(ax+b))\,\dfrac{a}{ax+b}$

32. $h(\theta) = 10^{\sec\theta} \quad\Rightarrow\quad h'(\theta) = 10^{\sec\theta}(\ln 10)\sec\theta\tan\theta$

33. $y = \ln\left(e^{2x} + \sqrt{e^{4x}+1}\right) \quad\Rightarrow$

$y' = \dfrac{2e^{2x} + 2e^{4x}/\sqrt{e^{4x}+1}}{e^{2x} + \sqrt{e^{4x}+1}} = \dfrac{2e^{2x}}{\sqrt{e^{4x}+1}}$

34. $y = x^{1/\ln x} \quad\Rightarrow\quad \ln y = \left(\dfrac{1}{\ln x}\right)\ln x = 1 \quad\Rightarrow\quad y = e$

$\Rightarrow\quad y' = 0$

35. $y = (\sin x)^{\cos x} \quad\Rightarrow\quad \ln y = \cos x\ln(\sin x) \quad\Rightarrow$

$\dfrac{y'}{y} = -\sin x\ln\sin x + \cos x\left(\dfrac{\cos x}{\sin x}\right) \quad\Rightarrow$

$y' = (\sin x)^{\cos x}\left(-\sin x\ln\sin x + \cos x\cot x\right)$

36. $y = \cos\left(x^{\sqrt{x}}\right) \quad\Rightarrow\quad y' = -\sin\left(x^{\sqrt{x}}\right)x^{\sqrt{x}}\left(\dfrac{\ln x + 2}{2\sqrt{x}}\right)$

by Example 11

37. $y = x^{x^x} \quad\Rightarrow\quad \ln y = x^x\ln x \quad\Rightarrow$

$\dfrac{y'}{y} = x^x(\ln x + 1)\ln x + x^x\left(\dfrac{1}{x}\right)$ [since

$z = x^x \quad\Rightarrow\quad \ln z = x\ln x \quad\Rightarrow$

$\dfrac{z'}{z} = \ln x + x\left(\dfrac{1}{x}\right) \quad\Rightarrow\quad z' = x^x(\ln x + 1)$] $\quad\Rightarrow$

$y' = x^{x^x}\left[x^x(\ln x + 1)\ln x + x^{x-1}\right]$

38. $f(x) = \dfrac{x}{\ln x} \quad\Rightarrow\quad f'(x) = \dfrac{\ln x - x\,(1/x)}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$

$\Rightarrow\quad f'(e) = \dfrac{1-1}{1^2} = 0$

39. $f(x) = x^2\ln x \quad\Rightarrow$

$f'(x) = 2x\ln x + x^2\left(\frac{1}{x}\right) = 2x\ln x + x \quad\Rightarrow$

$f'(1) = 2\ln 1 + 1 = 1$

40. $y = f(x) = \ln\ln x \quad\Rightarrow\quad f'(x) = \dfrac{1}{\ln x}\left(\dfrac{1}{x}\right) \quad\Rightarrow$

$f'(e) = \dfrac{1}{e}$, so an equation of the tangent at $(e, 0)$ is

$y = \dfrac{1}{e}(x - e)$.

41. $f(x) = 10^x \quad\Rightarrow\quad f'(x) = 10^x\ln 10$, so the slope of the tangent at $(1, 10)$ is $f'(1) = 10\ln 10$ and an equation is $y - 10 = 10\ln 10\,(x - 1)$ or $y = 10\left[(x-1)\ln 10 + 1\right]$.

42. $y = f(x) = \ln\left(x^2+1\right) \quad\Rightarrow$

$f'(x) = \dfrac{1}{x^2+1}\cdot 2x = \dfrac{2x}{x^2+1} \quad\Rightarrow\quad f'(1) = 1$, so an equation of the tangent line at $(1, \ln 2)$ is $y - \ln 2 = 1\,(x-1)$, or $y = x + \ln 2 - 1$.

43. $y = f(x) = \ln(1 + e^x) \quad\Rightarrow\quad f'(x) = \dfrac{e^x}{1+e^x} \quad\Rightarrow$

$f'(0) = \dfrac{1}{1+1} = \dfrac{1}{2}$ and so an equation of the tangent line at $(0, \ln 2)$ is $y - \ln 2 = \frac{1}{2}x$ or $y = \frac{1}{2}x + \ln 2$.

44. $y = (x+1)^x \quad\Rightarrow\quad \ln y = x\ln(x+1)$

$\Rightarrow\quad \dfrac{y'}{y} = \ln(x+1) + \dfrac{x}{x+1} \quad\Rightarrow$

$y' = (x+1)^x\left[\ln(x+1) + \dfrac{x}{x+1}\right]$, so the slope of the tangent at $(1, 2)$ is $f'(1) = 2\left(\ln 2 + \frac{1}{2}\right) = 1 + 2\ln 2$ and an equation is $y - 2 = (1 + 2\ln 2)(x - 1)$ or $y = (1 + 2\ln 2)\,x + 1 - 2\ln 2$.

45. $y = e^{-mx} \Rightarrow$
$y' = e^{-mx} \frac{d}{dx}(-mx) = e^{-mx}(-m) = -me^{-mx}$

46. $g(x) = e^{-5x} \cos 3x \Rightarrow$
$g'(x) = -5e^{-5x} \cos 3x - 3e^{-5x} \sin 3x$

47. $f(x) = e^{\sqrt{x}} \Rightarrow f'(x) = e^{\sqrt{x}} / (2\sqrt{x})$

48. $h(t) = \sqrt{1 - e^t} \Rightarrow h'(t) = -e^t / \left(2\sqrt{1 - e^t}\right)$

49. $h(\theta) = e^{\sin 5\theta} \Rightarrow h'(\theta) = 5 \cos(5\theta) e^{\sin 5\theta}$

50. $y = e^{x \cos x} \Rightarrow y' = e^{x \cos x}(\cos x - x \sin x)$

51. $y = \dfrac{e^{3x}}{1 + e^x} \Rightarrow$

$$y' = \frac{3e^{3x}(1 + e^x) - e^{3x}(e^x)}{(1 + e^x)^2} = \frac{3e^{3x} + 3e^{4x} - e^{4x}}{(1 + e^x)^2} = \frac{3e^{3x} + 2e^{4x}}{(1 + e^x)^2}$$

52. $f(x) = xe^{-x^2} \Rightarrow$
$f'(x) = e^{-x^2} + xe^{-x^2}(-2x) = e^{-x^2}\left(1 - 2x^2\right)$

53. $y = xe^{2x} \Rightarrow y' = e^{2x} + xe^{2x}(2) = e^{2x}(1 + 2x)$

54. $y = \dfrac{e^{-x^2}}{x} \Rightarrow$

$$y' = \frac{xe^{-x^2}(-2x) - e^{-x^2}}{x^2} = \frac{e^{-x^2}\left(-2x^2 - 1\right)}{x^2}$$

55. $y = e^{-1/x} \Rightarrow y' = e^{-1/x} / x^2$

56. $y = e^{x + e^x} \Rightarrow y' = e^{x + e^x}(1 + e^x)$

57. $y = \tan\left(e^{3x-2}\right) \Rightarrow y' = 3e^{3x-2} \sec^2\left(e^{3x-2}\right)$

58. $y = \left(2x + e^{3x}\right)^{1/3} \Rightarrow$
$y' = \frac{1}{3}\left(2 + 3e^{3x}\right)\left(2x + e^{3x}\right)^{-2/3}$

59. $y = x^e \Rightarrow y' = ex^{e-1}$

60. $y = \sec\left(e^{\tan x^2}\right) \Rightarrow$

$$\begin{aligned} y' &= \sec\left(e^{\tan x^2}\right) \tan\left(e^{\tan x^2}\right)\left(e^{\tan x^2}\right)\left[\sec^2\left(x^2\right)\right](2x) \\ &= 2xe^{\tan x^2} \sec^2\left(x^2\right) \sec\left(e^{\tan x^2}\right) \tan\left(e^{\tan x^2}\right) \end{aligned}$$

61. $y' = 2xe^{-x} - x^2e^{-x}$. At $(1, 1/e)$,
$y' = 2e^{-1} - e^{-1} = 1/e$. So an equation of the tangent line
is $y - \dfrac{1}{e} = \dfrac{1}{e}(x - 1) \Rightarrow y = \dfrac{x}{e}$.

62. $y = f(x) = e^{-x} \sin x \Rightarrow$
$f'(x) = -e^{-x} \sin x + e^{-x} \cos x \Rightarrow$
$f'(\pi) = e^{-\pi}(\cos \pi - \sin \pi) = -e^{-\pi}$, so an equation of the tangent line at $(\pi, 0)$ is $y - 0 = -e^{-\pi}(x - \pi)$, or
$y = -e^{-\pi}x + \pi e^{-\pi}$, or $x + e^{\pi}y = \pi$.

63. $\cos(x - y) = xe^x \Rightarrow$
$-\sin(x - y)(1 - y') = e^x + xe^x \Rightarrow$
$y' = 1 + \dfrac{e^x(1 + x)}{\sin(x - y)}$

64. Using implicit differentiation, $2e^{xy} = x + y$
$\Rightarrow (y + xy')\, 2e^{xy} = 1 + y' \Rightarrow$
$y'\left(2xe^{xy} - 1\right) = 1 - 2ye^{xy} \Rightarrow$
$y' = \left(1 - 2ye^{xy}\right) / \left(2xe^{xy} - 1\right)$. So at $(0, 2)$, $m = y' = 3$, and an equation of the tangent line is $y - 2 = 3(x - 0) \Rightarrow$
$y = 3x + 2$.

65. $y = e^{2x} + e^{-3x} \Rightarrow y' = 2e^{2x} - 3e^{-3x} \Rightarrow$
$y'' = 4e^{2x} + 9e^{-3x}$, so

$$\begin{aligned} y'' + y' - 6y &= \left(4e^{2x} + 9e^{-3x}\right) + \left(2e^{2x} - 3e^{-3x}\right) \\ &\qquad - 6\left(e^{2x} + e^{-3x}\right) \\ &= 0 \end{aligned}$$

66. $y = e^{rx} \Rightarrow y' = re^{rx} \Rightarrow y'' = r^2e^{rx}$, so
$y'' + 5y' - 6y = r^2e^{rx} + 5re^{rx} - 6e^{rx} =$
$e^{rx}\left(r^2 + 5r - 6\right) = e^{rx}(r + 6)(r - 1) = 0 \Rightarrow$
$(r + 6)(r - 1) = 0 \Rightarrow r = 1$ or -6.