3.6 HYPERBOLIC FUNCTIONS

A Click here for answers.

S Click here for solutions.

1–12 ■ Find the derivative of the function.

1. $f(x) = e^x \sinh x$

2. $f(x) = \tanh 3x$

3. $g(x) = \cosh^4 x$

4. $h(x) = \cosh(x^4)$

5. $F(x) = e^{\cosh 2x}$

6. $G(x) = x^2 \operatorname{sech} x$

7. $y = \cos(\sinh x)$

8. $y = x^{\cosh x}$

9. $y = e^{\tanh x} \cosh(\cosh x)$

10. $y = \cosh^{-1}(x^2)$

11. $y = \sqrt{x} \sinh^{-1}\sqrt{x}$

12. $y = x \ln(\operatorname{sech} 4x)$

3.6 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $f'(x) = e^x \sinh x + e^x \cosh x$ **2.** $f'(x) = 3 \operatorname{sech}^2 3x$

3. $g'(x) = 4 \cosh^3 x \sinh x$ **4.** $h'(x) = 4x^3 \sinh(x^4)$

5. $F'(x) = -2e^{\coth 2x} \operatorname{csch}^2 2x$

6. $G'(x) = 2x \operatorname{sech} x - x^2 \operatorname{sech} x \tanh x$

7. $y' = -\sin(\sinh x) \cosh x$

8. $y' = x^{\cosh x}\left(\sinh x \ln x + \dfrac{\cosh x}{x}\right)$

9. $y' = e^{\tanh x} \operatorname{sech}^2 x \cosh(\cosh x) + e^{\tanh x} \sinh(\cosh x) \sinh x$

10. $y' = \dfrac{2x}{\sqrt{x^4 - 1}}$ **11.** $y' = \dfrac{1}{2\sqrt{x}} \sinh^{-1}\sqrt{x} + \dfrac{1}{2\sqrt{1 + x}}$

12. $y' = \ln(\operatorname{sech} 4x) - 4x \tanh 4x$

3.6 SOLUTIONS

E Click here for exercises.

1. $f(x) = e^x \sinh x \quad \Rightarrow \quad f'(x) = e^x \sinh x + e^x \cosh x$

2. $f(x) = \tanh 3x \quad \Rightarrow \quad f'(x) = 3\operatorname{sech}^2 3x$

3. $g(x) = \cosh^4 x \quad \Rightarrow \quad g'(x) = 4\cosh^3 x \sinh x$

4. $h(x) = \cosh\left(x^4\right) \quad \Rightarrow$
$h'(x) = \sinh\left(x^4\right) 4x^3 = 4x^3 \sinh\left(x^4\right)$

5. $F(x) = e^{\coth 2x} \quad \Rightarrow$
$F'(x) = e^{\coth 2x}\left(-\operatorname{csch}^2 2x\right)(2) = -2e^{\coth 2x} \operatorname{csch}^2 2x$

6. $G(x) = x^2 \operatorname{sech} x \quad \Rightarrow$
$G'(x) = 2x \operatorname{sech} x - x^2 \operatorname{sech} x \tanh x$

7. $y = \cos(\sinh x) \quad \Rightarrow \quad y' = -\sin(\sinh x)\cosh x$

8. $y = x^{\cosh x} \quad \Rightarrow \quad \ln y = \cosh x \ln x$

$$\Rightarrow \quad \frac{y'}{y} = \sinh x \ln x + \frac{\cosh x}{x} \quad \Rightarrow$$

$$y' = x^{\cosh x}\left(\sinh x \ln x + \frac{\cosh x}{x}\right)$$

9. $y = e^{\tanh x} \cosh(\cosh x) \quad \Rightarrow$

$$y' = e^{\tanh x} \operatorname{sech}^2 x \cosh(\cosh x) + e^{\tanh x} \sinh(\cosh x) \sinh x$$

10. $y = \cosh^{-1}\left(x^2\right) \quad \Rightarrow$

$$y' = \frac{1}{\sqrt{(x^2)^2 - 1}}(2x) = \frac{2x}{\sqrt{x^4 - 1}}$$

11. $y = \sqrt{x} \sinh^{-1}\sqrt{x} \quad \Rightarrow$

$$y' = \frac{1}{2\sqrt{x}} \sinh^{-1}\sqrt{x} + \sqrt{x}\frac{1}{\sqrt{1 + (\sqrt{x})^2}}\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \sinh^{-1}\sqrt{x} + \frac{1}{2\sqrt{1+x}}$$

12. $y = x \ln(\operatorname{sech} 4x) \quad \Rightarrow$

$$y' = \ln(\operatorname{sech} 4x) + x\frac{-\operatorname{sech} 4x \tanh 4x}{\operatorname{sech} 4x}(4)$$

$$= \ln(\operatorname{sech} 4x) - 4x \tanh 4x$$