4.4 CURVE SKETCHING

A Click here for answers.

S Click here for solutions.

1–38 ▪ Use the guidelines of this section to sketch the curve.

1. $y = 1 - 3x + 5x^2 - x^3$

2. $y = 2x^3 - 6x^2 - 18x + 7$

3. $y = 4x^3 - x^4$

4. $y = 2 - x - x^9$

5. $y = \dfrac{1}{(x-1)(x+2)}$

6. $y = \dfrac{1}{x^2(x+3)}$

7. $y = \dfrac{1+x^2}{1-x^2}$

8. $y = \dfrac{4}{(x-5)^2}$

9. $y = \dfrac{x-3}{x+3}$

10. $y = \dfrac{1}{4x^3 - 9x}$

11. $y = \dfrac{x^3 - 1}{x}$

12. $y = \sqrt{x} - \sqrt{x-1}$

13. $y = \sqrt[4]{x^2 - 25}$

14. $y = x\sqrt{x^2 - 9}$

15. $y = \dfrac{x+1}{\sqrt{x^2+1}}$

16. $y = x + 3x^{2/3}$

17. $y = \sqrt{x - \sqrt{x}}$

18. $y = \dfrac{x^2}{\sqrt{1 - x^2}}$

19. $y = \cos x - \sin x$

20. $y = 2x + \cot x, \quad 0 < x < \pi$

21. $y = 2\cos x + \sin^2 x$

22. $y = \sin x + \cos x$

23. $y = \sin x + \sqrt{3}\cos x$

24. $y = 2\sin x + \sin^2 x$

25. $y = e^{-1/(x+1)}$

26. $y = xe^{x^2}$

27. $y = \ln(\cos x)$

28. $y = (\ln x)/x$

29. $y = \ln(x^2 - x)$

30. $y = \ln(1 + x^2)$

31. $y = \ln(\tan^2 x)$

32. $y = x^2 e^{-x}$

33. $y = x^2 \ln x$

34. $y = xe^{1/x}$

35. $y = x^2 e^{-x^2}$

36. $y = \dfrac{e^x}{x^2}$

37. $y = x^2 e^{-1/x}$

38. $y = x - \ln(1 + x)$

39–42 ▪ Produce graphs of f that reveal all the important aspects of the curve. In particular, you should use graphs of f' and f'' to estimate the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points.

39. $f(x) = 4x^4 - 7x^2 + 4x + 6$

40. $f(x) = 8x^5 + 45x^4 + 80x^3 + 90x^2 + 200x$

41. $f(x) = x^2 \sin x, \quad -7 \leqslant x \leqslant 7$

42. $f(x) = \sin x + \frac{1}{3}\sin 3x$

43–44 ▪ Produce graphs of f that show all the important aspects of the curve. Estimate the local maximum and minimum values and then use calculus to find these values exactly. Use a graph of f'' to estimate the inflection points.

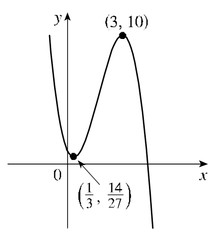
43. $f(x) = e^{x^3 - x}$

44. $f(x) = e^{\cos x}$

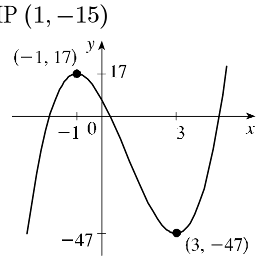
4.4 ANSWERS

E Click here for exercises.   S Click here for solutions.

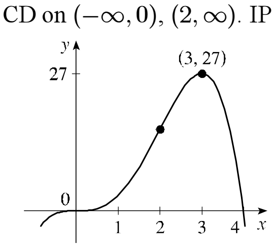
1. A. $\mathbb{R}$ B. y-int. 1 C. None D. None E. Inc. on $\left(\frac{1}{3}, 3\right)$; dec. on $\left(-\infty, \frac{1}{3}\right)$, $(3, \infty)$ F. Loc. min. $f\left(\frac{1}{3}\right) = \frac{14}{27}$, loc. max. $f(3) = 10$ G. CU on $\left(-\infty, \frac{5}{3}\right)$, CD on $\left(\frac{5}{3}, \infty\right)$, IP $\left(\frac{5}{3}, \frac{142}{27}\right)$
H.



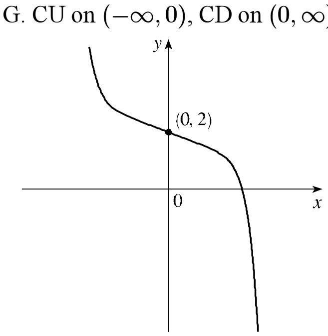
2. A. $\mathbb{R}$ B. y-int. 7 C. None D. None E. Inc. on $(-\infty, -1)$, $(3, \infty)$; dec. on $(-1, 3)$ F. Loc. max. $f(-1) = 17$, loc. min. $f(3) = -47$ G. CU on $(1, \infty)$, CD on $(-\infty, 1)$. IP $(1, -15)$
H.



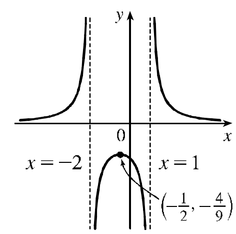
3. A. $\mathbb{R}$ B. y-int.0; x-int. 0, 4 C. None D. None E. Inc. on $(-\infty, 3)$, dec. on $(3, \infty)$ F. Loc. max. $f(3) = 27$ G. CU on $(0, 2)$; CD on $(-\infty, 0)$, $(2, \infty)$. IP $(0, 0)$, $(2, 16)$
H.



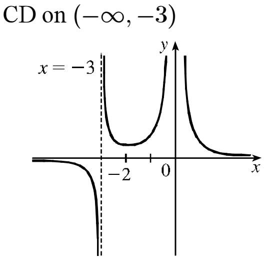
4. A. $\mathbb{R}$ B. y-int. 2, x-int. 1 C. None D. None E. Dec. on $\mathbb{R}$ F. None G. CU on $(-\infty, 0)$, CD on $(0, \infty)$. IP $(0, 2)$
H.



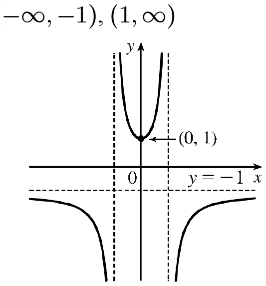
5. A. $\{x \mid x \neq -2, 1\}$ B. y-int. $f(0) = -\frac{1}{2}$ C. None D. HA $y = 0$; VA $x = -2$, $x = 1$ E. Inc. on $(-\infty, -2)$, $\left(-2, -\frac{1}{2}\right)$; dec. on $\left(-\frac{1}{2}, 1\right)$, $(1, \infty)$ F. Loc. max. $f\left(-\frac{1}{2}\right) = -\frac{4}{9}$ G. CD on $(-2, 1)$; CU on $(-\infty, -2)$, $(1, \infty)$
H.



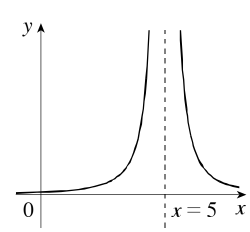
6. A. $\{x \mid x \neq 0, -3\}$ B. None C. None D. HA $y = 0$; VA $x = 0$, $x = -3$ E. Inc. on $(-2, 0)$; dec. on $(-\infty, -3)$, $(-3, -2)$, $(0, \infty)$ F. Loc. min. $f(-2) = \frac{1}{4}$ G. CU on $(-3, 0)$, $(0, \infty)$; CD on $(-\infty, -3)$
H.



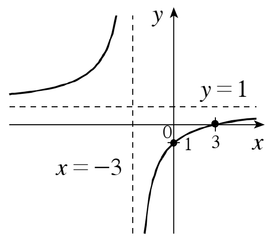
7. A. $\{x \mid x \neq \pm 1\}$ B. y-int.1 C. About y-axis D. HA $y = -1$, VA $x = \pm 1$ E. Inc. on $(0, 1)$, $(1, \infty)$; dec. on $(-\infty, -1)$, $(-1, 0)$ F. Loc. min. $f(0) = 1$ G. CU on $(-1, 1)$; CD on $(-\infty, -1)$, $(1, \infty)$
H.



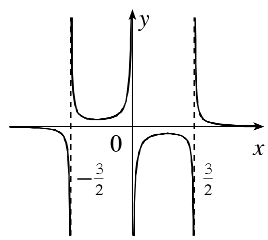
8. A. $\{x \mid x \neq 5\}$ B. y-int. $\frac{4}{25}$ C. None D. HA $y = 0$, VA $x = 5$ E. Inc. on $(-\infty, 5)$, dec. on $(5, \infty)$ F. None G. CU on $(-\infty, 5)$, $(5, \infty)$
H.



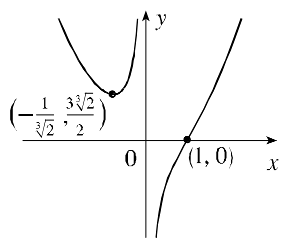
9. A. $\{x \mid x \neq -3\}$ B. x-int. 3, y-int. -1 C. None D. HA $y = 1$, VA $x = -3$ E. Inc. on $(-\infty, -3)$, $(3, \infty)$ F. None G. CU on $(-\infty, -3)$, CD on $(-3, \infty)$
H.



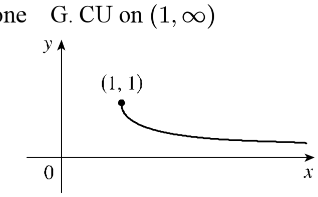
10. A. $\{x \mid x \neq 0, \pm\frac{3}{2}\}$ B. None C. About the origin D. HA $y = 0$, VA $x = 0$, $x = \pm\frac{3}{2}$ E. Inc. on $\left(-\frac{\sqrt{3}}{2}, 0\right)$, $\left(0, \frac{\sqrt{3}}{2}\right)$; dec. on $\left(-\infty, -\frac{3}{2}\right)$, $\left(-\frac{3}{2}, -\frac{\sqrt{3}}{2}\right)$, $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$, $\left(\frac{3}{2}, \infty\right)$ F. Loc. min. $f\left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{3\sqrt{3}}$, loc. max. $f\left(\frac{\sqrt{3}}{2}\right) = -\frac{1}{3\sqrt{3}}$ G. CU on $\left(-\frac{3}{2}, 0\right)$, $\left(\frac{3}{2}, \infty\right)$; CD on $\left(-\infty, -\frac{3}{2}\right)$, $\left(0, \frac{3}{2}\right)$
H.



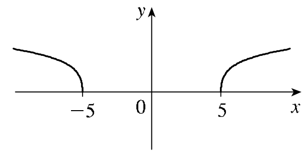
11. A. $\{x \mid x \neq 0\}$ B. x-int. 1 C. None D. VA $x = 0$ E. Inc. on $\left(-\frac{1}{\sqrt[3]{2}}, 0\right)$, $(0, \infty)$; dec. on $\left(-\infty, -\frac{1}{\sqrt[3]{2}}\right)$ F. Loc. min. $f\left(-\frac{1}{\sqrt[3]{2}}\right) = \frac{3\sqrt[3]{2}}{2}$ G. CU on $(-\infty, 0)$, $(1, \infty)$; CD on $(0, 1)$. IP $(1, 0)$
H.



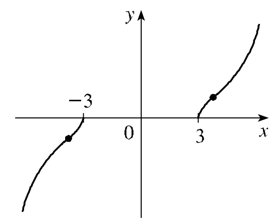
12. A. $[1, \infty)$ B. None C. None D. HA $y = 0$ E. Dec. on $(1, \infty)$ F. None G. CU on $(1, \infty)$
H.



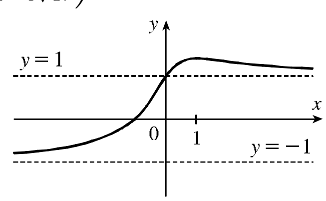
13. A. $(-\infty, -5) \cup (5, \infty)$ B. x-int. ± 5 C. About the y-axis D. None E. Inc. on $(5, \infty)$, dec. on $(-\infty, -5)$ F. None G. CD on $(-\infty, -5)$, $(5, \infty)$
H.



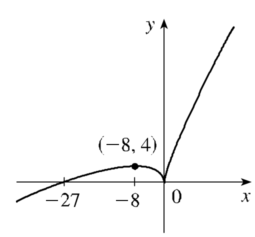
14. A. $(-\infty, -3] \cup [3, \infty)$ B. x-int. are ± 3 C. About the origin D. None E. Inc. on $(-\infty, -3)$, $(3, \infty)$ F. None G. CU on $\left(3\sqrt{\frac{3}{2}}, \infty\right)$, $\left(-3\sqrt{\frac{3}{2}}, -3\right)$; CD on $\left(-\infty, -3\sqrt{\frac{3}{2}}\right)$, $\left(3, 3\sqrt{\frac{3}{2}}\right)$. IP $\left(\pm 3\sqrt{\frac{3}{2}}, \pm\frac{9\sqrt{3}}{2}\right)$
H.



15. A. $\mathbb{R}$ B. x-int. -1, y-int. 1 C. None D. HA $y = \pm 1$ E. Inc. on $(-\infty, 1)$, dec. on $(1, \infty)$ F. Loc. max. $f(1) = \sqrt{2}$ G. CU on $\left(-\infty, \frac{3 - \sqrt{17}}{4}\right)$, $\left(\frac{3 + \sqrt{17}}{4}, \infty\right)$; CD on $\left(\frac{3 - \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{4}\right)$. IP $\left(\frac{3 + \sqrt{17}}{4}, \frac{7 + \sqrt{17}}{\sqrt{42 + 6\sqrt{17}}}\right)$, $\left(\frac{3 - \sqrt{17}}{4}, \frac{7 - \sqrt{17}}{\sqrt{42 - 6\sqrt{17}}}\right)$
H.

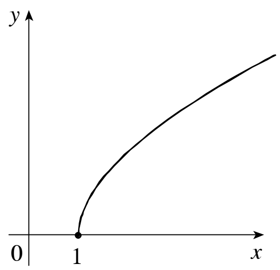


16. A. $\mathbb{R}$ B. x-int. 0, -27; y-int. 0 C. None D. None E. Inc. on $(-\infty, -8)$, $(0, \infty)$; dec. on $(-8, 0)$ F. Loc. max. $f(-8) = 4$, loc. min. $f(0) = 0$ G. CD on $(-\infty, 0)$, $(0, \infty)$
H.



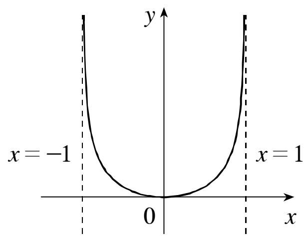
17. A. $[1, \infty)$ B. x-int. 1 C. None D. None E. Inc. on $(1, \infty)$ F. None G. CD on $(1, \infty)$

H.



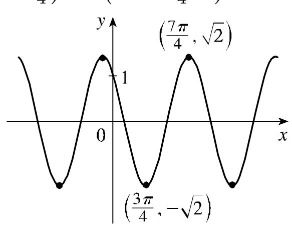
18. A. $(-1, 1)$ B. x-int.0, y-int 0 C. About y-axis D. VA $x = \pm 1$ E. Inc. on $(0, 1)$, dec. on $(-1, 0)$ F. Loc. min. $f(0) = 0$ G. CU on $(-1, 1)$

H.



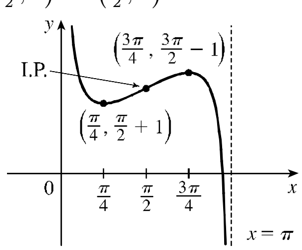
19. A. $\mathbb{R}$ B. x-int. $n\pi + \frac{\pi}{4}$, n an integer, y-int.1 C. Period 2π D. None E. Inc. on $\left(2n\pi + \frac{3\pi}{4}, 2n\pi + \frac{7\pi}{4}\right)$, dec. on $\left(2n\pi - \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right)$, n an integer F. Loc. max. $f\left(2n\pi - \frac{\pi}{4}\right) = \sqrt{2}$, loc. min. $f\left(2n\pi + \frac{3\pi}{4}\right) = -\sqrt{2}$, n an integer G. CU on $\left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{5\pi}{4}\right)$, CD on $\left(2n\pi - \frac{3\pi}{4}, 2n\pi + \frac{\pi}{4}\right)$, IP $\left(n\pi + \frac{\pi}{4}, 0\right)$, n an integer

H.



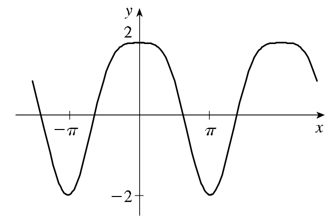
20. A. $(0, \pi)$ B. None C. None D. VA $x = 0$, $x = \pi$ E. Inc. on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$; dec. on $\left(0, \frac{\pi}{4}\right)$, $\left(\frac{3\pi}{4}, \pi\right)$ F. Loc. min. $f\left(\frac{\pi}{4}\right) = 1 + \frac{\pi}{2}$, loc. max. $f\left(\frac{3\pi}{4}\right) = \frac{3\pi}{2} - 1$ G. CU on $\left(0, \frac{\pi}{2}\right)$, CD on $\left(\frac{\pi}{2}, \pi\right)$. IP $\left(\frac{\pi}{2}, \pi\right)$

H.



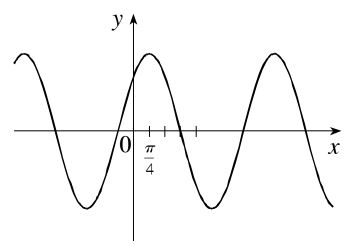
21. A. $\mathbb{R}$ B. y-int.2 C. About the y-axis, period 2π D. None E. Inc. on $((2n-1)\pi, 2n\pi)$, dec. on $(2n\pi, (2n+1)\pi)$, n an integer F. Loc. max. $f(2n\pi) = 2$, loc. min. $f((2n+1)\pi) = -2$, n an integer G. CU on $\left(2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3}\right)$, CD on $\left(2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right)$. IP $\left(2n\pi \pm \frac{2\pi}{3}, 0\right)$

H.



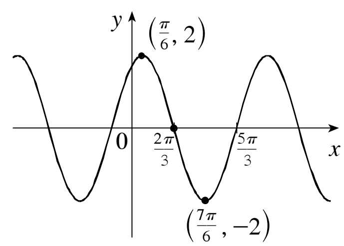
22. A. $\mathbb{R}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$ B. x-int. $\frac{3\pi}{4}, \frac{7\pi}{4}$; y-int.1 C. Period 2π D. None E. Inc. on $\left(0, \frac{\pi}{4}\right)$, $\left(\frac{5\pi}{4}, 2\pi\right)$; dec. on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ F. Loc. max. $f\left(\frac{\pi}{4}\right) = \sqrt{2}$, loc. min. $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$ G. CU on $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$; CD on $\left(0, \frac{3\pi}{4}\right)$, $\left(\frac{7\pi}{4}, 2\pi\right)$. IP $\left(\frac{3\pi}{4}, 0\right)$, $\left(\frac{7\pi}{4}, 0\right)$

H.

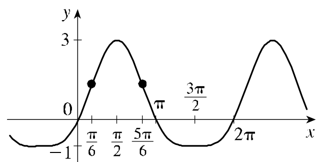


23. A. $\mathbb{R}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$ B. x-int. $\frac{2\pi}{3}, \frac{5\pi}{3}$; y-int.$\sqrt{3}$ C. Period 2π D. None E. Inc. on $\left(0, \frac{\pi}{6}\right)$, $\left(\frac{7\pi}{6}, 2\pi\right)$; dec. on $\left(\frac{\pi}{6}, \frac{7\pi}{6}\right)$ F. Loc. max. $f\left(\frac{\pi}{6}\right) = 2$, loc. min. $f\left(\frac{7\pi}{6}\right) = -2$ G. CU on $\left(\frac{2\pi}{3}, \frac{5\pi}{3}\right)$; CD on $\left(0, \frac{2\pi}{3}\right)$, $\left(\frac{5\pi}{3}, 2\pi\right)$. IP $\left(\frac{2\pi}{3}, 0\right)$, $\left(\frac{5\pi}{3}, 0\right)$

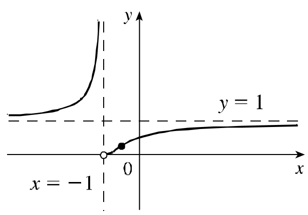
H.



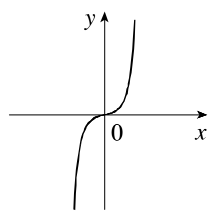
24. A. $\mathbb{R}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$ B. x-int. $0, \pi, 2\pi$; y-int.0 C. Period 2π D. None E. Inc. on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$; dec. on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ F. Loc. max. $f\left(\frac{\pi}{2}\right) = 3$, loc. min. $f\left(\frac{3\pi}{2}\right) = -1$ G. CU on $\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, 2\pi\right)$; CD on $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$. IP $\left(\frac{\pi}{6}, \frac{5}{4}\right), \left(\frac{5\pi}{6}, \frac{5}{4}\right)$
H.



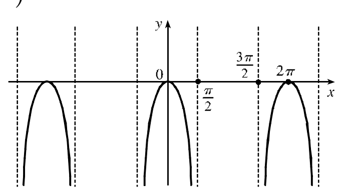
25. A. $\{x \mid x \neq -1\}$ B. y-int. e^{-1} C. None D. HA $y = 1$, VA $x = -1$ E. Inc. on $(-\infty, -1), (-1, \infty)$ F. None G. CU on $(-\infty, -1), \left(-1, -\frac{1}{2}\right)$; CD on $\left(-\frac{1}{2}, \infty\right)$. IP $\left(-\frac{1}{2}, e^{-2}\right)$
H.



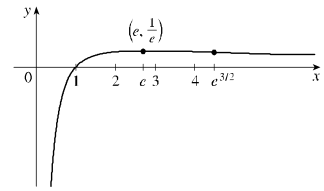
26. A. $\mathbb{R}$ B. x-int 0, y-int 0 C. About the origin D. None E. Inc. on $\mathbb{R}$ F. None G. CU on $(0, \infty)$, CD on $(-\infty, 0)$. IP $(0, 0)$
H.



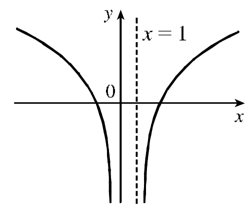
27. A. $\left\{x \mid 2n\pi - \frac{\pi}{2} < x < 2n\pi + \frac{\pi}{2}, n = 0, \pm1, \pm2, \ldots\right\}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$ B. x-int. $0, 2\pi$; y-int.0 C. About the y-axis, period 2π D. VA $x = \frac{\pi}{2}, \frac{3\pi}{2}$ E. Inc. on $\left(\frac{3\pi}{2}, 2\pi\right)$, dec. on $\left(0, \frac{\pi}{2}\right)$ F. Loc. max. $f(0) = f(2\pi) = 0$ G. CD on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$
H.



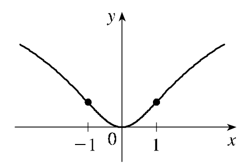
28. A. $(0, \infty)$ B. x-int.1 C. None D. HA $y = 0$, VA $x = 0$ E. Inc. on $(0, e)$, dec. on (e, ∞) F. Loc. max. $f(e) = 1/e$ G. CU on $\left(e^{3/2}, \infty\right)$, CD on $\left(0, e^{3/2}\right)$. IP $\left(e^{3/2}, \frac{3}{2}e^{-3/2}\right)$
H.



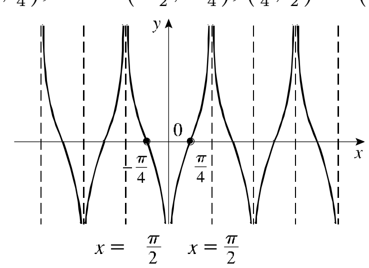
29. A. $(-\infty, 0) \cup (1, \infty)$ B. x-int. $\frac{1}{2}\left(1 \pm \sqrt{5}\right)$ C. None D. VA $x = 0, x = 1$ E. Inc. on $(1, \infty)$, dec. on $(-\infty, 0)$ F. None G. CD on $(-\infty, 0), (1, \infty)$
H.



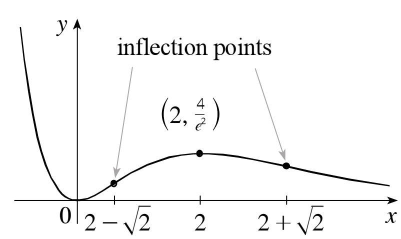
30. A. $\mathbb{R}$ B. x-int 0, y-int 0 C. About the y-axis D. None E. Inc. on $(0, \infty)$, dec. on $(-\infty, 0)$ F. Loc. min. $f(0) = 0$ G. CU on $(-1, 1)$; CD on $(-\infty, -1), (1, \infty)$. IP $(\pm1, \ln 2)$
H.



31. A. $\{x \mid x \neq n\pi/2\}$ *Note:* f is periodic with period π, so in B-G we consider only $-\frac{\pi}{2} < x < \frac{\pi}{2}$ B. x-int. $-\frac{\pi}{4}, \frac{\pi}{4}$ C. About the y-axis, period π D. VA $x = 0, x = \pm\frac{\pi}{2}$ E. Inc. on $\left(0, \frac{\pi}{2}\right)$, dec. on $\left(-\frac{\pi}{2}, 0\right)$ F. None G. CD on $\left(-\frac{\pi}{4}, 0\right), \left(0, \frac{\pi}{4}\right)$; CU on $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. IP $\left(\pm\frac{\pi}{4}, 0\right)$
H.

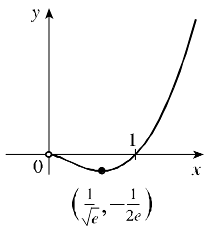


32. A. $\mathbb{R}$ B. x-int 0, y-int 0 C. None D. HA $y = 0$ E. Inc. on $(0, 2)$; dec. on $(-\infty, 0), (2, \infty)$ F. Loc. max. $f(2) = 4e^{-2}$, loc. min. $f(0) = 0$ G. CU on $(-\infty, 2 - \sqrt{2}), (2 + \sqrt{2}, \infty)$; CD on $(2 - \sqrt{2}, 2 + \sqrt{2})$. IP $\left(2 \pm \sqrt{2}, (6 \pm 4\sqrt{2})\, e^{\sqrt{2} \pm 2}\right)$
H.



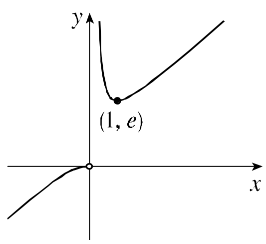
33. A. $(0,\infty)$ B. x-int. 1 C. None D. None E. Inc. on $(1/\sqrt{e}\,,\infty)$, dec. on $(0,/1\sqrt{e})$ F. Loc. min. $f(1/\sqrt{e}) = -1/(2e)$ G. CU on $\left(e^{-3/2},\infty\right)$, CD on $\left(0,e^{-3/2}\right)$. IP $\left(e^{-3/2},-3/(2e^3)\right)$

H.



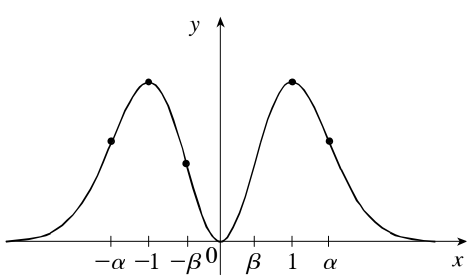
34. A. $\{x \mid x \neq 0\}$ B. None C. None D. VA $x=0$ E. Inc. on $(-\infty,0)$, $(1,\infty)$; dec. on $(0,1)$ F. Loc. min. $f(1)=e$ G. CU on $(0,\infty)$, CD on $(-\infty,0)$

H.



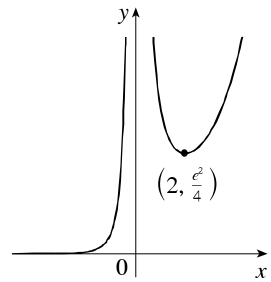
35. A. $\mathbb{R}$ B. x-int 0, y-int. 0 C. About the y-axis D. HA $y=0$ E. Inc. on $(0,1)$, $(-\infty,-1)$; dec. on $(-1,0)$, $(1,\infty)$ F. Loc. max. $f(\pm 1) = 1/e$, loc. min. $f(0)=0$ G. CU on $\left(-\infty,-\frac{1}{2}\sqrt{5+\sqrt{17}}\right)$, $\left(-\frac{1}{2}\sqrt{5-\sqrt{17}},\frac{1}{2}\sqrt{5-\sqrt{17}}\right)$, $\left(\frac{1}{2}\sqrt{5+\sqrt{17}},\infty\right)$; CD on $\left(-\frac{1}{2}\sqrt{5+\sqrt{17}},-\frac{1}{2}\sqrt{5-\sqrt{17}}\right)$, $\left(\frac{1}{2}\sqrt{5-\sqrt{17}},\frac{1}{2}\sqrt{5+\sqrt{17}}\right)$. IP at $x=\pm\frac{1}{2}\sqrt{5+\sqrt{17}}$, $\pm\frac{1}{2}\sqrt{5-\sqrt{17}}$.

H.



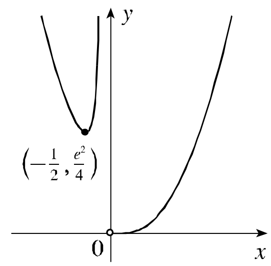
36. A. $\{x \mid x \neq 0\}$ B. None C. None D. HA $y=0$, VA $x=0$ E. Inc. on $(-\infty,0)$, $(2,\infty)$; dec. on $(0,2)$ F. Loc. min. $f(2) = \frac{1}{4}e^2$ G. CU on $(-\infty,0)$, $(0,\infty)$

H.



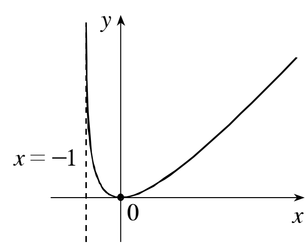
37. A. $\{x \mid x \neq 0\}$ B. None C. None D. VA $x=0$ E. Inc. on $\left(-\frac{1}{2},0\right)$, $(0,\infty)$; dec. on $\left(-\infty,-\frac{1}{2}\right)$ F. Loc. min. $f\left(-\frac{1}{2}\right) = \frac{1}{4}e^2$ G. CU on $(-\infty,0)$, $(0,\infty)$.

H.

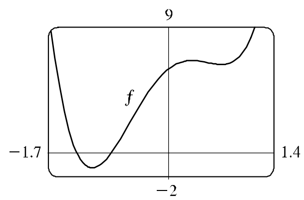


38. A. $(-1,\infty)$ B. x-int. 0, y-int. 0 C. None D. VA $x=-1$ E. Inc. on $(0,\infty)$, dec. on $(-1,0)$ F. Loc. min. $f(0)=0$ G. CU on $(-1,\infty)$

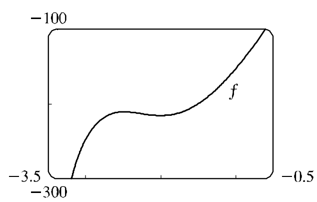
H.



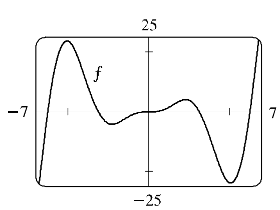
39. Inc. on $(-1.1, 0.3)$, $(0.7, \infty)$; dec. on $(-\infty, -1.1)$, $(0.3, 0.7)$; loc. max. $f(0.3) \approx 6.6$; loc. min. $f(-1.1) \approx -1.0$, $f(0.7) \approx 6.3$; CU on $(-\infty, -0.5)$, $(0.5, \infty)$; CD on $(-0.5, 0.5)$; IP $(-0.5, 2.1)$, $(0.5, 6.5)$



40. Inc. on $(-\infty, -2.5)$, $(-2.0, \infty)$; dec. on $(-2.5, -2.0)$; loc. max. $f(-2.5) \approx -211$, loc. min. $f(-2) \approx -216$; CU on $(-2.3, \infty)$, CD on $(-\infty, -2.3)$; IP $(-2.3, -213)$

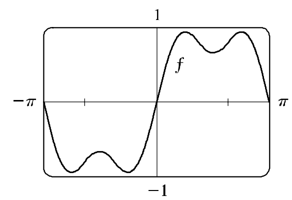


41. Inc. on $(-7, -5.1)$, $(-2.3, 2.3)$, $(5.1, 7)$; dec. on $(-5.1, -2.3)$, $(2.3, 5.1)$; loc. max. $f(-5.1) \approx 24.1$, $f(2.3) \approx 3.9$; loc. min. $f(-2.3) \approx -3.9$, $f(5.1) \approx -24.1$; CU on $(-7, -6.8)$, $(-4.0, -1.5)$, $(0, 1.5)$, $(4.0, 6.8)$; CD on $(-6.8, -4.0)$, $(-1.5, 0)$, $(1.5, 4.0)$, $(6.8, 7)$; IP $(-6.8, -24.4)$, $(-4.0, 12.0)$, $(-1.5, -2.3)$, $(0, 0)$, $(1.5, 2.3)$, $(4.0, -12.0)$, $(6.8, 24.4)$

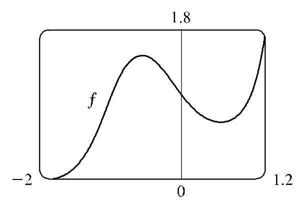


42. *Note:* Due to periodicity, we consider the function only on $[-\pi, \pi]$.

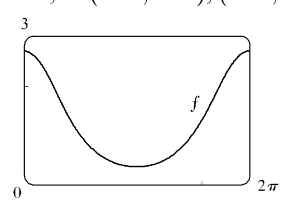
Inc. on $(-2.4, -1.6)$, $(-0.8, 0.8)$, $(1.6, 2.4)$; dec. on $(-\pi, -2.4)$, $(-1.6, -0.8)$, $(0.8, 1.6)$, $(2.4, \pi)$; loc. max. $f(-1.6) \approx -0.7$, $f(0.8) \approx 0.9$, $f(2.4) \approx 0.9$; loc. min. $f(-2.4) \approx -0.9$, $f(-0.8) \approx -0.9$, $f(1.6) \approx 0.7$; CU on $(-\pi, -2.0)$, $(-1.2, 0)$, $(1.2, 2)$; CD on $(-2.0, -1.2)$, $(0, 1.2)$, $(2.0, \pi)$; IP $(-\pi, 0)$, $(-2.0, -0.8)$, $(-1.2, -0.8)$, $(0, 0)$, $(1.2, 0.8)$, $(2.0, 0.8)$, $(\pi, 0)$



43. Loc. max. $f\left(-\frac{1}{\sqrt{3}}\right) = e^{2\sqrt{3}/9} \approx 1.5$, loc. min. $f\left(\frac{1}{\sqrt{3}}\right) = e^{-2\sqrt{3}/9} \approx 0.7$; IP $(-0.15, 1.15)$, $(-1.09, 0.82)$



44. Loc. max. $f(0) = f(2\pi) = e \approx 2.7$, loc. min. $f(\pi) = 1/e \approx 0.37$; IP $(0.90, 1.86)$, $(5.38, 1.86)$



4.4 SOLUTIONS

E Click here for exercises.

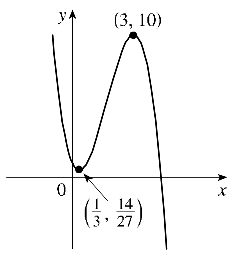
1. $y = f(x) = 1 - 3x + 5x^2 - x^3$

A. $D = \mathbb{R}$ **B.** y-intercept $= f(0) = 1$

C. No symmetry **D.** No asymptote

E. $f'(x) = -3 + 10x - 3x^2 = -(3x-1)(x-3) > 0$ $\Leftrightarrow$ $(3x-1)(x-3) < 0$ $\Leftrightarrow$ $\frac{1}{3} < x < 3$. $f'(x) < 0$ $\Leftrightarrow$ $x < \frac{1}{3}$ or $x > 3$. So f is increasing on $\left(\frac{1}{3}, 3\right)$ and decreasing on $\left(-\infty, \frac{1}{3}\right)$ and $(3, \infty)$. **F.** The critical numbers occur when $f'(x) = -(3x-1)(x-3) = 0$ $\Leftrightarrow$ $x = \frac{1}{3}, 3$. The local minimum is $f\left(\frac{1}{3}\right) = \frac{14}{27}$ and the local maximum is $f(3) = 10$. **G.** $f''(x) = 10 - 6x > 0$ $\Leftrightarrow$ $x < \frac{5}{3}$, so f is CU on $\left(-\infty, \frac{5}{3}\right)$ and CD on $\left(\frac{5}{3}, \infty\right)$. IP $\left(\frac{5}{3}, \frac{142}{27}\right)$

H.



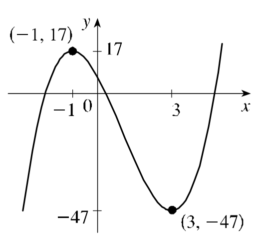
2. $y = f(x) = 2x^3 - 6x^2 - 18x + 7$

A. $D = \mathbb{R}$ **B.** y-intercept $= f(0) = 7$

C. No symmetry **D.** No asymptote

E. $f'(x) = 6x^2 - 12x - 18 = 6(x+1)(x-3) > 0$ $\Leftrightarrow$ $(x+1)(x-3) > 0$ $\Leftrightarrow$ $x < -1$ or $x > 3$. $f'(x) < 0$ $\Leftrightarrow$ $-1 < x < 3$. So f is increasing on $(-\infty, -1)$ and $(3, \infty)$ and decreasing on $(-1, 3)$. **F.** The critical numbers are $x = -1, 3$. The local maximum is $f(-1) = 17$ and the local minimum is $f(3) = -47$. **G.** $y'' = 12x - 12 > 0$ $\Leftrightarrow$ $x > 1$, so f is CU on $(1, \infty)$ and CD on $(-\infty, 1)$. IP $(1, -15)$

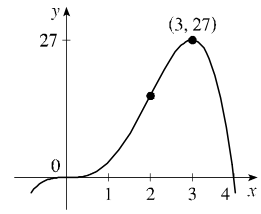
H.



3. $y = f(x) = 4x^3 - x^4$ **A.** $D = \mathbb{R}$

B. y-intercept $= f(0) = 0$, x-intercept $\Rightarrow$ $y = 0$ $\Leftrightarrow$ $x^3(4-x) = 0$ $\Leftrightarrow$ $x = 0, 4$ **C.** No symmetry **D.** No asymptote **E.** $y' = 12x^2 - 4x^3 = 4x^2(3-x) > 0$ $\Leftrightarrow$ $x < 3$, so f is increasing on $(-\infty, 3)$ and decreasing on $(3, \infty)$. **F.** Local maximum is $f(3) = 27$, no local minimum. **G.** $y'' = 12x(2-x) > 0$ $\Leftrightarrow$ $0 < x < 2$, so f is CU on $(0, 2)$ and CD on $(-\infty, 0)$ and $(2, \infty)$. IP $(0, 0)$ and $(2, 16)$

H.



4. $y = f(x) = 2 - x - x^9$

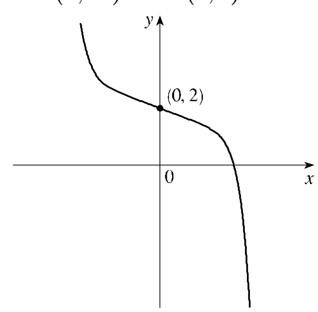
$$= -(x-1)\left(x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 2\right)$$

A. $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 2$; x-intercept: $f(x) = 0$ $\Leftrightarrow$ $x = 1$ (By part E below, f is decreasing on its domain, so it has only one x-intercept.) **C.** No symmetry **D.** No asymptote

E. $f'(x) = -1 - 9x^8 = -1\left(9x^8 + 1\right) < 0$ for all x, so f is decreasing on $\mathbb{R}$. **F.** No maximum or minimum

G. $f''(x) = -72x^7 > 0$ $\Leftrightarrow$ $x < 0$, so f is CU on $(-\infty, 0)$ and CD on $(0, \infty)$. IP at $(0, 2)$

H.



5. $y = f(x) = \dfrac{1}{(x-1)(x+2)} = \dfrac{1}{x^2 + x - 2}$

A. $D = \{x \mid x \neq -2, 1\} = (-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

B. y-intercept: $f(0) = -\frac{1}{2}$; no x-intercept **C.** No symmetry

D. $$\lim_{x \to \pm\infty} \frac{1}{x^2 + 2x - 2} = \lim_{x \to \pm\infty} \frac{1/x^2}{1 + 1/x - 2/x^2} = \frac{0}{1} = 0$$

so $y = 0$ is a HA. $x = -2$ and $x = 1$ are VA.

E. $$f'(x) = \frac{(x^2 + x - 2) \cdot 0 - 1(2x+1)}{(x-1)^2 (x+2)^2} = -\frac{2x+1}{(x-1)^2 (x+2)^2} > 0 \quad \Leftrightarrow$$

$x < -\frac{1}{2}$ $(x \neq -2)$; $f'(x) < 0$ $\Leftrightarrow$ $x > -\frac{1}{2}$ $(x \neq 1)$. So f is increasing on $(-\infty, -2)$ and $\left(-2, -\frac{1}{2}\right)$, and f is decreasing on $\left(-\frac{1}{2},1\right)$ and $(1,\infty)$. **F.** $f\left(-\frac{1}{2}\right)=-\frac{4}{9}$ is a local maximum.

G. $f''(x)$

$$=\frac{\left(x^2+x-2\right)^2(-2)-[-(2x+1)](2)\left(x^2+x-2\right)(2x+1)}{\left[(x-1)^2(x+2)^2\right]^2}$$

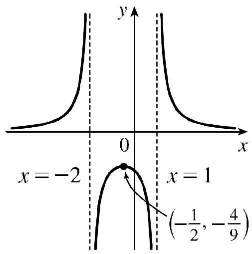
$$=\frac{2\left(x^2+x-2\right)\left[-1\left(x^2+x-2\right)+(2x+1)^2\right]}{(x-1)^4(x+2)^4}$$

$$=\frac{2\left(-x^2-x+2+4x^2+4x+1\right)}{(x-1)^3(x+2)^3}$$

$$=\frac{2\left(3x^2+3x+3\right)}{(x-1)^3(x+2)^3}=\frac{6\left(x^2+x+1\right)}{(x-1)^3(x+2)^3}$$

The numerator is always positive, so the sign of f'' is determined by the denominator, which is negative only for $-2<x<1$. Thus, f is CD on $(-2,1)$ and CU on $(-\infty,-2)$ and $(1,\infty)$. No IP.

H.



6. $y=f(x)=\dfrac{1}{x^2(x+3)}$

A. $D=\{x\mid x\neq 0,\ -3\}=(-\infty,-3)\cup(-3,0)\cup(0,\infty)$

B. No intercept **C.** No symmetry

D. $\displaystyle\lim_{x\to\pm\infty}\frac{1}{x^2(x+3)}=0$, so $y=0$ is a HA.

$\displaystyle\lim_{x\to 0}\frac{1}{x^2(x+3)}=\infty$ and $\displaystyle\lim_{x\to -3^+}\frac{1}{x^2(x+3)}=\infty$,

$\displaystyle\lim_{x\to -3^-}\frac{1}{x^2(x+3)}=-\infty$, so $x=0$ and $x=-3$ are VA.

E. $f'(x)=-\dfrac{3(x+2)}{x^3(x+3)^2}>0 \quad\Leftrightarrow\quad -2<x<0$;

$f'(x)<0 \quad\Leftrightarrow\quad x<-2$ or $x>0$. So f is increasing on $(-2,0)$ and decreasing on $(-\infty,-3)$, $(-3,-2)$, and $(0,\infty)$. **F.** $f(-2)=\frac{1}{4}$ is a local minimum.

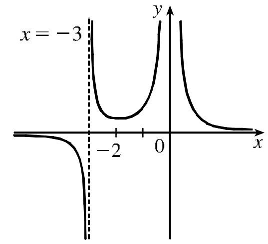
G. $f''(x)$

$$=-3\frac{x^3(x+3)^2-(x+2)\left[3x^2(x+3)^2+x^3 2(x+3)\right]}{x^6(x+3)^4}$$

$$=\frac{6\left(2x^2+8x+9\right)}{x^4(x+3)^3}$$

Since $2x^2+8x+9>0$ for all x, $f''(x)>0 \quad\Leftrightarrow\quad x>-3\ (x\neq 0)$, so f is CU on $(-3,0)$ and $(0,\infty)$, and CD on $(-\infty,-3)$. No IP

H.



7. $y=f(x)=\dfrac{1+x^2}{1-x^2}=-1+\dfrac{2}{1-x^2}$

A. $D=\{x\mid x\neq\pm 1\}$ **B.** No x-intercept, y-intercept $=f(0)=1$ **C.** $f(-x)=f(x)$, so f is even and the curve is symmetric about the y-axis.

D. $\displaystyle\lim_{x\to\pm\infty}\frac{1+x^2}{1-x^2}=\lim_{x\to\pm\infty}\frac{\left(1/x^2\right)+1}{\left(1/x^2\right)-1}=-1$, so $y=-1$ is a HA. $\displaystyle\lim_{x\to 1^-}\frac{1+x^2}{1-x^2}=\infty$, $\displaystyle\lim_{x\to 1^+}\frac{1+x^2}{1-x^2}=-\infty$,

$\displaystyle\lim_{x\to -1^-}\frac{1+x^2}{1-x^2}=-\infty$, $\displaystyle\lim_{x\to -1^+}\frac{1+x^2}{1-x^2}=\infty$. So $x=1$ and $x=-1$ are VA.

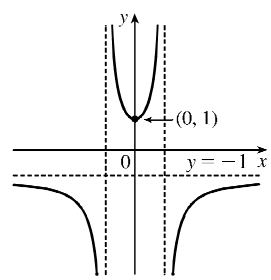
E. $f'(x)=\dfrac{4x}{\left(1-x^2\right)^2}>0 \quad\Leftrightarrow\quad x>0\ (x\neq 1)$, so f increases on $(0,1)$ and $(1,\infty)$, and decreases on $(-\infty,-1)$ and $(-1,0)$. **F.** $f(0)=1$ is a local minimum.

G. $y''=\dfrac{4\left(1-x^2\right)^2-4x\cdot 2\left(1-x^2\right)(-2x)}{\left(1-x^2\right)^4}$

$$=\frac{4\left(1+3x^2\right)}{\left(1-x^2\right)^3}>0 \quad\Leftrightarrow$$

$x^2<1 \quad\Leftrightarrow\quad -1<x<1$, so f is CU on $(-1,1)$ and CD on $(-\infty,-1)$ and $(1,\infty)$. No IP

H.



8. $y=f(x)=4/(x-5)^2$

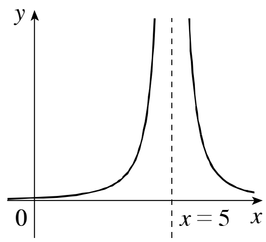
A. $D=\{x\mid x\neq 5\}=(-\infty,5)\cup(5,\infty)$

B. y-intercept $=f(0)=\frac{4}{25}$, no x-intercept **C.** No symmetry **D.** $\displaystyle\lim_{x\to\pm\infty}\frac{4}{(x-5)^2}=0$, so $y=0$ is a HA. $\displaystyle\lim_{x\to 5}\frac{4}{(x-5)^2}=\infty$, so $x=5$ is a VA.

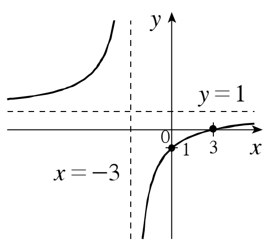
E. $f'(x)=-8/(x-5)^3>0 \quad\Leftrightarrow\quad x<5$ and $f'(x)<0 \quad\Leftrightarrow\quad x>5$. So f is increasing on $(-\infty,5)$ and decreasing on $(5,\infty)$. **F.** No maximum or minimum

G. $f''(x)=24/(x-5)^4>0$ for $x\neq 5$, so f is CU on $(-\infty,5)$ and $(5,\infty)$.

H.



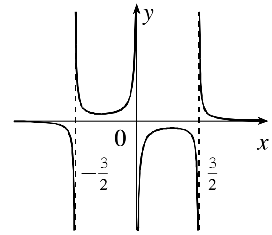
9. $y = f(x) = (x-3)/(x+3)$
A. $D = \{x \mid x \neq -3\} = (-\infty, -3) \cup (3, \infty)$
B. x-intercept is 3, y-intercept $= f(0) = -1$ **C.** No symmetry **D.** $\lim\limits_{x\to\pm\infty} \dfrac{x-3}{x+3} = \lim\limits_{x\to\pm\infty} \dfrac{1-3/x}{1+3/x} = 1$, so $y = 1$ is a HA. $\lim\limits_{x\to-3^-} \dfrac{x-3}{x+3} = \infty$ and $\lim\limits_{x\to-3^+} \dfrac{x-3}{x+3} = -\infty$, so $x = -3$ is a VA.
E. $f'(x) = \dfrac{(x+3)-(x-3)}{(x+3)^2} = \dfrac{6}{(x+3)^2} \Rightarrow$ $f'(x) > 0$ $(x \neq -3)$ so f is increasing on $(-\infty, -3)$ and $(3, \infty)$. **F.** No maximum or minimum
G. $f''(x) = -\dfrac{12}{(x+3)^3} > 0 \Leftrightarrow x < -3$, so f is CU on $(-\infty, -3)$ and CD on $(-3, \infty)$. No IP
H.



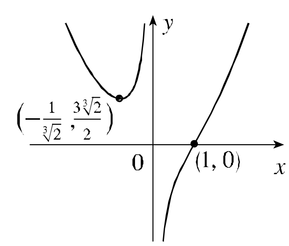
10. $y = f(x) = 1/\left[x\left(4x^2-9\right)\right]$ **A.** $D = \left\{x \mid x \neq 0, \pm\frac{3}{2}\right\}$
B. No intercept **C.** $f(-x) = -f(x)$, so the curve is symmetric about the origin. **D.** $\lim\limits_{x\to\pm\infty} \dfrac{1}{x\left(4x^2-9\right)} = 0$, so $y = 0$ is a HA. $\lim\limits_{x\to0^+} \dfrac{1}{x\left(4x^2-9\right)} = -\infty$,
$\lim\limits_{x\to0^-} \dfrac{1}{x\left(4x^2-9\right)} = \infty$, $\lim\limits_{x\to3/2^+} \dfrac{1}{x\left(4x^2-9\right)} = \infty$,
$\lim\limits_{x\to3/2^-} \dfrac{1}{x\left(4x^2-9\right)} = -\infty$, $\lim\limits_{x\to-3/2^+} \dfrac{1}{x\left(4x^2-9\right)} = \infty$,
and $\lim\limits_{x\to-3/2^-} \dfrac{1}{x\left(4x^2-9\right)} = -\infty$, so $x = 0$ and $x = \pm\frac{3}{2}$ are VA. **E.** $f'(x) = -\dfrac{12x^2-9}{\left(4x^3-9x\right)^2} > 0 \Leftrightarrow x^2 < \frac{3}{4}$ $\Leftrightarrow |x| < \frac{\sqrt{3}}{2} \Leftrightarrow -\frac{\sqrt{3}}{2} < x < \frac{\sqrt{3}}{2}$ and $f'(x) < 0 \Leftrightarrow$ $x > \frac{\sqrt{3}}{2}$ or $x < -\frac{\sqrt{3}}{2}$, so f is increasing on $\left(-\frac{\sqrt{3}}{2}, 0\right)$ and $\left(0, \frac{\sqrt{3}}{2}\right)$, and decreasing on $\left(-\infty, -\frac{3}{2}\right)$, $\left(-\frac{3}{2}, -\frac{\sqrt{3}}{2}\right)$, $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$, and $\left(\frac{3}{2}, \infty\right)$. **F.** $f\left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{3\sqrt{3}}$ is a local minimum, $f\left(\frac{\sqrt{3}}{2}\right) = -\frac{1}{3\sqrt{3}}$ is a local maximum.
G. $f''(x)$

$$= \frac{-24x\left(4x^3-9x\right)^2 + \left(12x^2-9\right)^2 2\left(4x^3-9x\right)}{\left(4x^3-9x\right)^4}$$
$$= \frac{6\left(32x^4-36x^2+27\right)}{x^3\left(4x^2-9\right)^3}$$

Since $32x^4 - 36x^2 + 27 > 0$ for all x, $f''(x) > 0 \Leftrightarrow$ $-\frac{3}{2} < x < 0$ or $x > \frac{3}{2}$, so f is CU on $\left(-\frac{3}{2}, 0\right)$ and $\left(\frac{3}{2}, \infty\right)$ and CD on $\left(-\infty, -\frac{3}{2}\right)$ and $\left(0, \frac{3}{2}\right)$.
H.



11. $y = f(x) = \dfrac{x^3-1}{x} = x^2 - \dfrac{1}{x}$ **A.** $D = \{x \mid x \neq 0\}$
B. x-intercept 1, no y-intercept **C.** No symmetry
D. $\lim\limits_{x\to\pm\infty} = \infty$, so no HA. $\lim\limits_{x\to0^+} \dfrac{x^3-1}{x} = -\infty$ and $\lim\limits_{x\to0^-} \dfrac{x^3-1}{x} = \infty$, so $x = 0$ is a VA.
E. $f'(x) = 2x + \dfrac{1}{x^2} = \dfrac{2x^3+1}{x^2} > 0 \Leftrightarrow 2x^3+1 > 0$ $\Leftrightarrow x > -\frac{1}{\sqrt[3]{2}}$ $(x \neq 0)$, so f is increasing on $\left(-\frac{1}{\sqrt[3]{2}}, 0\right)$ and $(0, \infty)$ and decreasing on $\left(-\infty, -\frac{1}{\sqrt[3]{2}}\right)$.
F. $f\left(-\frac{1}{\sqrt[3]{2}}\right) = \frac{3\sqrt[3]{2}}{2}$ is a local minimum.
G. $f''(x) = 2 - \dfrac{2}{x^3} = \dfrac{2\left(x^3-1\right)}{x^3} \Rightarrow f''(x) > 0 \Leftrightarrow$ $x > 1$ or $x < 0$, so f is CU on $(-\infty, 0)$ and $(1, \infty)$ and CD on $(0, 1)$. IP is $(1, 0)$.
H.

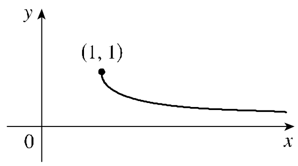


12. $y = f(x) = \sqrt{x} - \sqrt{x-1}$
A. $D = \{x \mid x \geq 0 \text{ and } x \geq 1\} = \{x \mid x \geq 1\} = [1, \infty)$
B. No intercept **C.** No symmetry
D. $\lim\limits_{x\to\infty} \left(\sqrt{x} - \sqrt{x-1}\right) =$
$\lim\limits_{x\to\infty} \left(\sqrt{x} - \sqrt{x-1}\right) \dfrac{\sqrt{x} + \sqrt{x-1}}{\sqrt{x} + \sqrt{x-1}} =$
$\lim\limits_{x\to\infty} \dfrac{1}{\sqrt{x} + \sqrt{x-1}} = 0$, so $y = 0$ is a HA.

E. $f'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{2\sqrt{x-1}} < 0$ for all $x > 1$, since $x - 1 < x \;\Rightarrow\; \sqrt{x-1} < \sqrt{x}$, so f is decreasing on $(1, \infty)$. **F.** No local maximum or minimum

G. $f''(x) = -\dfrac{1}{4}\left[\dfrac{1}{x^{3/2}} - \dfrac{1}{(x-1)^{3/2}}\right] \;\Rightarrow\; f''(x) > 0$ for $x > 1$, so f is CU on $(1, \infty)$.

H.



13. $y = f(x) = \sqrt[4]{x^2 - 25}$

A. $D = \{x \mid x^2 \geq 25\} = (-\infty, -5] \cup [5, \infty)$

B. x-intercepts are ± 5, no y-intercept **C.** $f(-x) = f(x)$, so the curve is symmetric about the y-axis.

D. $\lim\limits_{x \to \pm\infty} \sqrt[4]{x^2 - 25} = \infty$, no asymptote

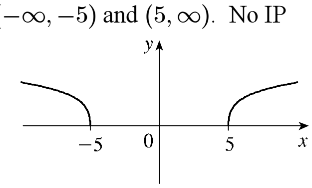
E. $f'(x) = \frac{1}{4}\left(x^2 - 25\right)^{-3/4}(2x) = \dfrac{x}{2\left(x^2 - 25\right)^{3/4}} > 0$ if $x > 5$, so f is increasing on $(5, \infty)$ and decreasing on $(-\infty, -5)$. **F.** No local maximum or minimum

G. $y'' = \dfrac{2\left(x^2 - 25\right)^{3/4} - 3x^2\left(x^2 - 25\right)^{-1/4}}{4\left(x^2 - 25\right)^{3/2}}$

$= -\dfrac{x^2 + 50}{4\left(x^2 - 25\right)^{7/4}} < 0$

so f is CD on $(-\infty, -5)$ and $(5, \infty)$. No IP

H.



14. $y = f(x) = x\sqrt{x^2 - 9}$

A. $D = \{x \mid x^2 \geq 9\} = (-\infty, -3] \cup [3, \infty)$

B. x-intercepts are ± 3, no y-intercept.

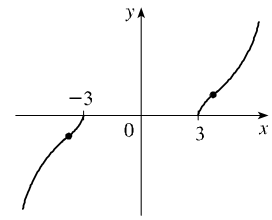
C. $f(-x) = -f(x)$, so the curve is symmetric about the origin. **D.** $\lim\limits_{x \to \infty} \sqrt{x^2 - 9} = \infty$, $\lim\limits_{x \to -\infty} \sqrt{x^2 - 9} = -\infty$, no asymptote **E.** $f'(x) = \sqrt{x^2 - 9} + \dfrac{x^2}{\sqrt{x^2 - 9}} > 0$ for $x \in D$, so f is increasing on $(-\infty, -3)$ and $(3, \infty)$. **F.** No maximum or minimum

G. $f''(x) = \dfrac{x}{\sqrt{x^2 - 9}} + \dfrac{2x\sqrt{x^2 - 9} - x^2\left(x/\sqrt{x^2 - 9}\right)}{x^2 - 9}$

$= \dfrac{x\left(2x^2 - 27\right)}{\left(x^2 - 9\right)^{3/2}} > 0 \;\Leftrightarrow$

$x > 3\sqrt{\frac{3}{2}}$ or $-3\sqrt{\frac{3}{2}} < x < 0$, so f is CU on $\left(3\sqrt{\frac{3}{2}}, \infty\right)$ and $\left(-3\sqrt{\frac{3}{2}}, -3\right)$ and CD on $\left(-\infty, -3\sqrt{\frac{3}{2}}\right)$ and $\left(3, 3\sqrt{\frac{3}{2}}\right)$. IP $\left(\pm 3\sqrt{\frac{3}{2}}, \pm\frac{9\sqrt{3}}{2}\right)$

H.



15. $y = f(x) = \dfrac{x + 1}{\sqrt{x^2 + 1}}$ **A.** $D = \mathbb{R}$ **B.** x-intercept -1, y-intercept 1 **C.** No symmetry **D.** $\lim\limits_{x \to \infty} \dfrac{x + 1}{\sqrt{x^2 + 1}} = 1$, and $\lim\limits_{x \to -\infty} \dfrac{x + 1}{\sqrt{x^2 + 1}} = -1$, so horizontal asymptotes are $y = \pm 1$.

E. $f'(x) = \dfrac{\sqrt{x^2 + 1} - \dfrac{1}{2\sqrt{x^2 + 1}}(2x)(x + 1)}{\left(x^2 + 1\right)}$

$= \dfrac{1 - x}{\left(x^2 + 1\right)^{3/2}} > 0 \;\Leftrightarrow\; x < 1$,

so f is increasing on $(-\infty, 1)$, and decreasing on $(1, \infty)$.

F. $f(1) = \sqrt{2}$ is a local maximum.

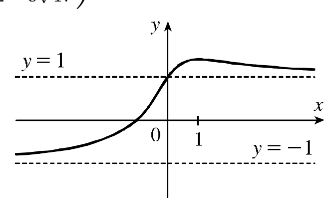
G. $f''(x) = \dfrac{-1\left(x^2 + 1\right)^{3/2} - \frac{3}{2}\left(x^2 + 1\right)^{1/2}(2x)(1 - x)}{\left(x^2 + 1\right)^3}$

$= \dfrac{2x^2 - 3x - 1}{\left(x^2 + 1\right)^{5/2}}$

$f''(x) = 0 \;\Leftrightarrow\; 2x^2 - 3x - 1 = 0 \;\Leftrightarrow$

$x = \dfrac{3 \pm \sqrt{9 - 4(2)(-1)}}{2(2)} = \dfrac{3 \pm \sqrt{17}}{4}$. $f(x)$ is CU on $\left(-\infty, \frac{3 - \sqrt{17}}{4}\right)$ and $\left(\frac{3 + \sqrt{17}}{4}, \infty\right)$ and CD on $\left(\frac{3 - \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{4}\right)$. IP $\left(\frac{3 + \sqrt{17}}{4}, \frac{7 + \sqrt{17}}{\sqrt{42 + 6\sqrt{17}}}\right)$, $\left(\frac{3 - \sqrt{17}}{4}, \frac{7 - \sqrt{17}}{\sqrt{42 - 6\sqrt{17}}}\right)$

H.



16. $y = f(x) = x + 3x^{2/3}$ **A.** $D = \mathbb{R}$

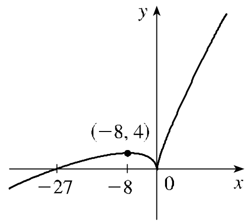
B. $y = x + 3x^{2/3} = x^{2/3}\left(x^{1/3} + 3\right) = 0$ if $x = 0$ or -27 (x-intercepts), y-intercept $= f(0) = 0$

C. No symmetry **D.** $\lim\limits_{x \to \infty}\left(x + 3x^{2/3}\right) = \infty$,

$\lim\limits_{x \to -\infty}\left(x + 3x^{2/3}\right) = \lim\limits_{x \to -\infty} x^{2/3}\left(x^{1/3} + 3\right) = -\infty$, no asymptote

E. $f'(x) = 1 + 2x^{-1/3} = \left(x^{1/3} + 2\right)/x^{1/3} > 0 \;\Leftrightarrow$ $x > 0$ or $x < -8$, so f increases on $(-\infty, -8)$, $(0, \infty)$ and decreases on $(-8, 0)$. **F.** Local maximum $f(-8) = 4$, local minimum $f(0) = 0$ **G.** $f''(x) = -\frac{2}{3}x^{-4/3} < 0$ $(x \neq 0)$ so f is CD on $(-\infty, 0)$ and $(0, \infty)$. No IP

H.



17. $y = f(x) = \sqrt{x - \sqrt{x}}$

A. $D = \{x \mid x \geq \sqrt{x}\} = \{x \mid x^2 \geq x\}$
$= \{x \mid x \geq 1\} = [1, \infty)$

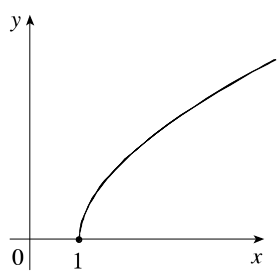
B. x-intercept is 1. **C.** No symmetry

D. $\lim\limits_{x \to \infty} \sqrt{x - \sqrt{x}} = \infty$, no asymptote

E. $f'(x) = \frac{1}{2}\left(x - \sqrt{x}\right)^{-1/2}\left(1 - \frac{1}{2}x^{-1/2}\right) > 0$ for all $x > 1$, so f is increasing on $(1, \infty)$. **F.** No local maximum or minimum.

G. $f''(x) = -\frac{1}{4}\left(x - \sqrt{x}\right)^{-3/2}\left(1 - \frac{1}{2}x^{-1/2}\right)^2$
$+\frac{1}{2}\left(x - \sqrt{x}\right)^{-1/2}\frac{1}{4}x^{-3/2}$
$$= \frac{-4x + 6\sqrt{x} - 3}{16x\left(x - \sqrt{x}\right)^{3/2}} < 0$$
since $-4x + 6\sqrt{x} - 3 < 0$ (negative discriminant as a quadratic in $\sqrt{x}$). So f is CD on $(1, \infty)$.

H.



18. $y = f(x) = x^2 / \sqrt{1 - x^2}$

A. $D = \left\{x \mid x^2 < 1\right\} = (-1, 1)$

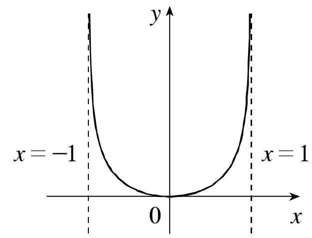
B. x-intercept $= 0 = y$-intercept **C.** $f(-x) = f(x)$, so f is even. The curve is symmetric about the y-axis.

D. $\lim\limits_{x \to 1^-} \dfrac{x^2}{\sqrt{1 - x^2}} = \infty = \lim\limits_{x \to -1^+} \dfrac{x^2}{\sqrt{1 - x^2}}$, so $x = \pm 1$ are VA.

E. $f'(x) = \dfrac{2x\sqrt{1 - x^2} - x^2\left(-x/\sqrt{1 - x^2}\right)}{1 - x^2}$
$$= \frac{x\left(2 - x^2\right)}{\left(1 - x^2\right)^{3/2}}$$
Since $2 - x^2 > 0$ and $\left(1 - x^2\right)^{3/2} > 0$, $f'(x) > 0$ if $0 < x < 1$ and $f'(x) < 0$ if $-1 < x < 0$, so f is increasing on $(0, 1)$ and decreasing on $(-1, 0)$. **F.** Local minimum $f(0) = 0$

G. $f''(x)$
$$= \frac{\left(1 - x^2\right)^{3/2}\left(2 - 3x^2\right) - \left(2x - x^3\right)\frac{3}{2}\left(1 - x^2\right)^{1/2}(-2x)}{\left(1 - x^2\right)^3}$$
$= \dfrac{x^2 + 2}{\left(1 - x^2\right)^{5/2}} > 0$ for all x, so f is CU on $(-1, 1)$.

H.



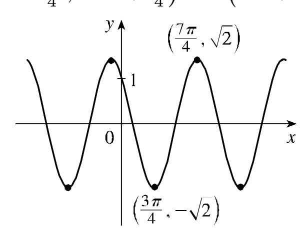
19. $y = f(x) = \cos x - \sin x$ **A.** $D = \mathbb{R}$ **B.** $y = 0$ $\Leftrightarrow$ $\cos x = \sin x$ $\Leftrightarrow$ $x = n\pi + \frac{\pi}{4}$, n an integer (x-intercepts), y-intercept $= f(0) = 1$.

C. Periodic with period 2π **D.** No asymptote

E. $f'(x) = -\sin x - \cos x = 0$ $\Leftrightarrow$ $\cos x = -\sin x$ $\Leftrightarrow$ $x = 2n\pi + \frac{3\pi}{4}$ or $2n\pi + \frac{7\pi}{4}$. $f'(x) > 0$ $\Leftrightarrow$ $\cos x < -\sin x$ $\Leftrightarrow$ $2n\pi + \frac{3\pi}{4} < x < 2n\pi + \frac{7\pi}{4}$, so f is increasing on $\left(2n\pi + \frac{3\pi}{4}, 2n\pi + \frac{7\pi}{4}\right)$ and decreasing on $\left(2n\pi - \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right)$. **F.** Local maxima $f\left(2n\pi - \frac{\pi}{4}\right) = \sqrt{2}$, local minima $f\left(2n\pi + \frac{3\pi}{4}\right) = -\sqrt{2}$.

G. $f''(x) = -\cos x + \sin x > 0$ $\Leftrightarrow$ $\sin x > \cos x$ $\Leftrightarrow$ $x \in \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{5\pi}{4}\right)$, so f is CU on these intervals and CD on $\left(2n\pi - \frac{3\pi}{4}, 2n\pi + \frac{\pi}{4}\right)$. IP $\left(n\pi + \frac{\pi}{4}, 0\right)$

H.



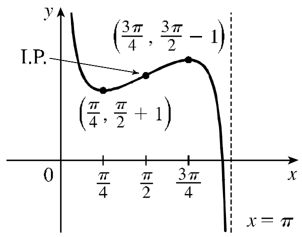
20. $y = f(x) = 2x + \cot x$, $0 < x < \pi$ **A.** $D = (0, \pi)$.

B. No y-intercept **C.** No symmetry

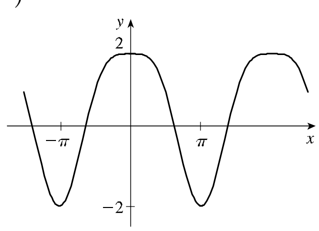
D. $\lim\limits_{x \to 0^+} (2x + \cot x) = \infty$, $\lim\limits_{x \to \pi^-} (2x + \cot x) = -\infty$, so $x = 0$ and $x = \pi$ are VA. **E.** $f'(x) = 2 - \csc^2 x > 0$ when $\csc^2 x < 2$ $\Leftrightarrow$ $\sin x > \frac{1}{\sqrt{2}}$ $\Leftrightarrow$ $\frac{\pi}{4} < x < \frac{3\pi}{4}$, so f is increasing on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and decreasing on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{3\pi}{4}, \pi\right)$. **F.** $f\left(\frac{\pi}{4}\right) = 1 + \frac{\pi}{2}$ is a local minimum, $f\left(\frac{3\pi}{4}\right) = \frac{3\pi}{2} - 1$ is a local maximum.

G. $f''(x) = -2\csc x\left(-\csc x \cot x\right) = 2\csc^2 x \cot x > 0$ $\Leftrightarrow \cot x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2}$, so f is CU on $\left(0, \frac{\pi}{2}\right)$, CD on $\left(\frac{\pi}{2}, \pi\right)$. IP $\left(\frac{\pi}{2}, \pi\right)$

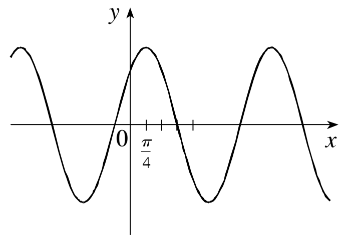
H.



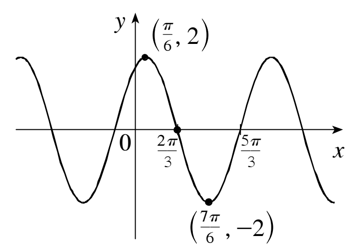
21. $y = f(x) = 2\cos x + \sin^2 x$ **A.** $D = \mathbb{R}$
B. y-intercept $= f(0) = 2$ **C.** $f(-x) = f(x)$, so the curve is symmetric about the y-axis. Periodic with period 2π
D. No asymptote
E. $f'(x) = -2\sin x + 2\sin x\cos x = 2\sin x(\cos x - 1)$
$> 0 \Leftrightarrow \sin x < 0 \Leftrightarrow (2n-1)\pi < x < 2n\pi$,
so f is increasing on $((2n-1)\pi, 2n\pi)$ and decreasing on $(2n\pi, (2n+1)\pi)$. **F.** $f(2n\pi) = 2$ is a local maximum. $f((2n+1)\pi) = -2$ is a local minimum.
G. $f''(x) = -2\cos x + 2\cos 2x = 2\left(2\cos^2 x - \cos x - 1\right)$
$= 2(2\cos x + 1)(\cos x - 1) > 0$
$\Leftrightarrow \cos x < -\frac{1}{2} \Leftrightarrow x \in \left(2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3}\right)$, so f is CU on these intervals and CD on $\left(2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right)$. IP $\left(2n\pi \pm \frac{2\pi}{3}, 0\right)$
H.



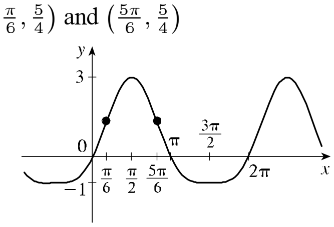
22. $y = f(x) = \sin x + \cos x$ **A.** $D = \mathbb{R}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$.
B. y-intercept $= f(0) = 1$, x-intercepts occur where $\sin x = -\cos x \Leftrightarrow \tan x = -1 \Leftrightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$.
C. $f(x + 2\pi) = f(x)$, so f is periodic with period 2π.
D. No asymptote **E.** $f'(x) = \cos x - \sin x > 0$ when $\cos x > \sin x \Leftrightarrow 0 < x < \frac{\pi}{4}$ or $\frac{5\pi}{4} < x < 2\pi$, $f'(x) < 0 \Leftrightarrow \frac{\pi}{4} < x < \frac{5\pi}{4}$, so f is increasing on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{5\pi}{4}, 2\pi\right)$ and decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$. **F.** $f\left(\frac{\pi}{4}\right) = \sqrt{2}$ is a local maximum, $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$ is a local minimum.
G. $f''(x) = -\sin x - \cos x > 0 \Leftrightarrow \frac{3\pi}{4} < x < \frac{7\pi}{4}$, so f is CU on $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ and CD on $\left(0, \frac{3\pi}{4}\right)$ and $\left(\frac{7\pi}{4}, 2\pi\right)$. IP $\left(\frac{3\pi}{4}, 0\right)$, $\left(\frac{7\pi}{4}, 0\right)$.
H.



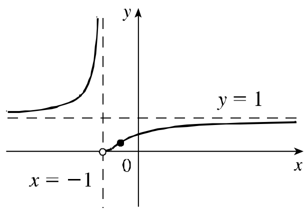
23. $y = f(x) = \sin x + \sqrt{3}\cos x$ **A.** $D = \mathbb{R}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$.
B. y-intercept $= \sqrt{3}$, x-intercepts occur where $\sin x = -\sqrt{3}\cos x \Leftrightarrow \tan x = -\sqrt{3} \Leftrightarrow x = \frac{2\pi}{3}, \frac{5\pi}{3}$. **C.** No symmetry other than periodicity. **D.** No asymptote **E.** $f'(x) = \cos x - \sqrt{3}\sin x = 0$ when $\cos x = \sqrt{3x} \Leftrightarrow \tan x = \frac{1}{\sqrt{3}} \Leftrightarrow x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$.
$f'(x) > 0 \Leftrightarrow 0 < x < \frac{\pi}{6}$ or $\frac{7\pi}{6} < x < 2\pi$, $f'(x) < 0 \Leftrightarrow \frac{\pi}{6} < x < \frac{7\pi}{6}$. So f is increasing on $\left(0, \frac{\pi}{6}\right)$ and $\left(\frac{7\pi}{6}, 2\pi\right)$ and decreasing on $\left(\frac{\pi}{6}, \frac{7\pi}{6}\right)$. **F.** $f\left(\frac{\pi}{6}\right) = 2$ is a local maximum, $f\left(\frac{7\pi}{6}\right) = -2$ is a local minimum.
G. $f''(x) = -\sin x - \sqrt{3}\cos x = 0$ when $\tan x = -\sqrt{3}$ $\Leftrightarrow x = \frac{2\pi}{3}$ or $\frac{5\pi}{3}$. $f''(x) > 0 \Leftrightarrow \frac{2\pi}{3} < x < \frac{5\pi}{3}$, so f is CU on $\left(\frac{2\pi}{3}, \frac{5\pi}{3}\right)$ and CD on $\left(0, \frac{2\pi}{3}\right)$ and $\left(\frac{5\pi}{3}, 2\pi\right)$. IP $\left(\frac{2\pi}{3}, 0\right)$, $\left(\frac{5\pi}{3}, 0\right)$.
H.



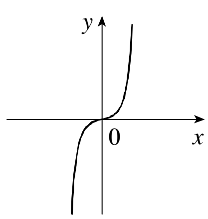
24. $y = f(x) = 2\sin x + \sin^2 x$ **A.** $D = \mathbb{R}$ *Note:* f is periodic with period 2π, so in B–G we consider only $[0, 2\pi]$. **B.** y-intercept $= 0$, x-intercepts occur where $2\sin x(2 + \sin x) = 0$ $\Leftrightarrow \sin x = 0 \Leftrightarrow x = 0, \pi, 2\pi$. **C.** No symmetry other than periodicity **D.** No asymptote
E. $f'(x) = 2\cos x + 2\sin x\cos x = 2\cos x(1 + \sin x) > 0$ $\Leftrightarrow \cos x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$, so f is increasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. **F.** $f\left(\frac{\pi}{2}\right) = 3$ is a local maximum, $f\left(\frac{3\pi}{2}\right) = -1$ is a local minimum.
G. $f''(x) = -2\sin x + 2\cos^2 x - 2\sin^2 x$
$= 2\left(-\sin x + 1 - 2\sin^2 x\right)$
$= 2(1 + \sin x)(1 - 2\sin x) > 0 \Leftrightarrow$
$1 - 2\sin x > 0 \Leftrightarrow \sin x < \frac{1}{2} \Leftrightarrow 0 \le x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x \le 2\pi$. So f is CU on $\left(0, \frac{\pi}{6}\right)$, $\left(\frac{5\pi}{6}, 2\pi\right)$, and CD on $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$. IP $\left(\frac{\pi}{6}, \frac{5}{4}\right)$ and $\left(\frac{5\pi}{6}, \frac{5}{4}\right)$
H.



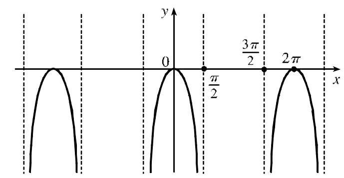
25. $y = f(x) = e^{-1/(x+1)}$
A. $D = \{x \mid x \neq -1\} = (-\infty, -1) \cup (-1, \infty)$ **B.** No x-intercept; y-intercept $= f(0) = e^{-1}$ **C.** No symmetry
D. $\lim\limits_{x \to \pm\infty} e^{-1/(x+1)} = 1$ since $-1/(x+1) \to 0$, so $y = 1$ is a HA. $\lim\limits_{x \to -1^+} e^{-1/(x+1)} = 0$ since $-1/(x+1) \to -\infty$, $\lim\limits_{x \to -1^-} e^{-1/(x+1)} = \infty$ since $-1/(x+1) \to \infty$, so $x = -1$ is a VA. **E.** $f'(x) = e^{-1/(x+1)}/(x+1)^2 \Rightarrow f'(x) > 0$ for all x except 1, so f is increasing on $(-\infty, -1)$ and $(-1, \infty)$. **F.** No maximum or minimum
G. $f''(x) = \dfrac{e^{-1/(x+1)}}{(x+1)^4} + \dfrac{e^{-1/(x+1)}(-2)}{(x+1)^3}$
$= -\dfrac{e^{-1/(x+1)}(2x+1)}{(x+1)^4} \Rightarrow$
$f''(x) > 0 \Leftrightarrow 2x + 1 < 0 \Leftrightarrow x < -\frac{1}{2}$, so f is CU on $(-\infty, -1)$ and $\left(-1, -\frac{1}{2}\right)$, and CD on $\left(-\frac{1}{2}, \infty\right)$. f has an IP at $\left(-\frac{1}{2}, e^{-2}\right)$.
H.



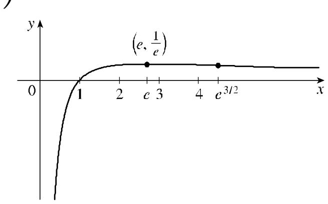
26. $y = f(x) = xe^{x^2}$ **A.** $D = \mathbb{R}$ **B.** Both intercepts are 0. **C.** $f(-x) = -f(x)$, so the curve is symmetric about the origin. **D.** $\lim\limits_{x \to \infty} xe^{x^2} = \infty$, $\lim\limits_{x \to -\infty} xe^{x^2} = -\infty$, no asymptote
E. $f'(x) = e^{x^2} + xe^{x^2}(2x) = e^{x^2}(1 + 2x^2) > 0$, so f is increasing on $\mathbb{R}$. **F.** No maximum or minimum
G. $f''(x) = e^{x^2}(2x)(1 + 2x^2) + e^{x^2}(4x)$
$= e^{x^2}(2x)(3 + 2x^2) > 0 \Leftrightarrow$
$x > 0$, so f is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. f has an inflection point at $(0, 0)$.
H.



27. $y = f(x) = \ln(\cos x)$
A. $D = \{x \mid \cos x > 0\} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \cup \cdots = \left\{x \mid 2n\pi - \frac{\pi}{2} < x < 2n\pi + \frac{\pi}{2}, n = 0, \pm 1, \pm 2, \ldots\right\}$
B. x-intercepts occur when $\ln(\cos x) = 0 \Leftrightarrow \cos x = 1 \Leftrightarrow x = 2n\pi$, y-intercept $= f(0) = 0$.
C. $f(-x) = f(x)$, so the curve is symmetric about the y-axis. $f(x + 2\pi) = f(x)$. f has period 2π, so in D–G we consider only $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **D.** $\lim\limits_{x \to \pi/2^-} \ln(\cos x) = -\infty$ and $\lim\limits_{x \to -\pi/2^+} \ln(\cos x) = -\infty$, so $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$ are VA. No HA. **E.** $f'(x) = (1/\cos x)(-\sin x) = -\tan x > 0 \Leftrightarrow -\frac{\pi}{2} < x < 0$, so f is increasing on $\left(-\frac{\pi}{2}, 0\right)$ and decreasing on $\left(0, \frac{\pi}{2}\right)$. **F.** $f(0) = 0$ is a local maximum.
G. $f''(x) = -\sec^2 x < 0 \Rightarrow f$ is CD on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. No IP.
H.



28. $f(x) = (\ln x)/x$ **A.** $D = (0, \infty)$ **B.** x-intercept $= 1$
C. No symmetry **D.** $\lim\limits_{x \to \infty} \dfrac{\ln x}{x} \overset{\text{H}}{=} \lim\limits_{x \to \infty} \dfrac{1/x}{1} = 0$, so $y = 0$ is a horizontal asymptote. Also $\lim\limits_{x \to 0^+} \dfrac{\ln x}{x} = -\infty$ since $\ln x \to -\infty$ and $x \to 0^+$, so $x = 0$ is a vertical asymptote. **E.** $f'(x) = \dfrac{1 - \ln x}{x^2} = 0$ when $\ln x = 1 \Leftrightarrow x = e$. $f'(x) > 0 \Leftrightarrow 1 - \ln x > 0 \Leftrightarrow \ln x < 1 \Leftrightarrow 0 < x < e$. $f'(x) < 0 \Leftrightarrow x > e$. So f is increasing on $(0, e)$ and decreasing on (e, ∞). **F.** Thus, $f(e) = 1/e$ is a local (and absolute) maximum.
G. $f''(x) = \dfrac{(-1/x)x^2 - (1 - \ln x)(2x)}{x^4} = \dfrac{2\ln x - 3}{x^3}$, so $f''(x) > 0 \Leftrightarrow 2\ln x - 3 > 0 \Leftrightarrow \ln x > \frac{3}{2} \Leftrightarrow x > e^{3/2}$. $f''(x) < 0 \Leftrightarrow 0 < x < e^{3/2}$. So f is CU on $\left(e^{3/2}, \infty\right)$ and CD on $\left(0, e^{3/2}\right)$. Inflection point: $\left(e^{3/2}, \frac{3}{2}e^{-3/2}\right)$
H.



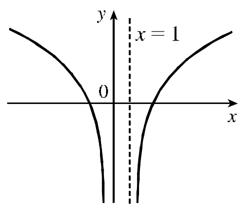
29. $y = f(x) = \ln(x^2 - x)$
A. $D = \{x \mid x^2 - x > 0\} = \{x \mid x < 0 \text{ or } x > 1\}$
$= (-\infty, 0) \cup (1, \infty)$
B. x-intercepts occur when $x^2 - x = 1 \Leftrightarrow x^2 - x - 1 = 0 \Leftrightarrow x = \frac{1}{2}\left(1 \pm \sqrt{5}\right)$. No y-intercept
C. No symmetry **D.** $\lim\limits_{x \to \infty} \ln(x^2 - x) = \infty$, no HA. $\lim\limits_{x \to 0^-} \ln(x^2 - x) = -\infty$, $\lim\limits_{x \to 1^+} \ln(x^2 - x) = -\infty$, so $x = 0$ and $x = 1$ are VA. **E.** $f'(x) = \dfrac{2x - 1}{x^2 - x} > 0$

when $x > 1$ and $f'(x) < 0$ when $x < 0$, so f is increasing on $(1, \infty)$ and decreasing on $(-\infty, 0)$. **F.** No maximum or minimum

G. $f''(x) = \dfrac{2(x^2 - x) - (2x - 1)^2}{(x^2 - x)^2} = \dfrac{-2x^2 + 2x - 1}{(x^2 - x)^2}$

$\Rightarrow$ $f''(x) < 0$ for all x since $-2x^2 + 2x - 1$ has a negative discriminant. So f is CD on $(-\infty, 0)$ and $(1, \infty)$. No IP.

H.



30. $y = f(x) = \ln(1 + x^2)$ **A.** $D = \mathbb{R}$ **B.** Both intercepts are 0. **C.** $f(-x) = f(x)$, so the curve is symmetric about the y-axis. **D.** $\lim\limits_{x \to \pm\infty} \ln(1 + x^2) = \infty$, no asymptote

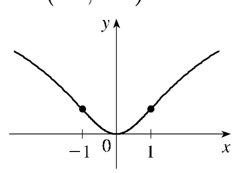
E. $f'(x) = \dfrac{2x}{1 + x^2} > 0 \quad \Leftrightarrow \quad x > 0$, so f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

F. $f(0) = 0$ is a local and absolute minimum.

G. $f''(x) = \dfrac{2(1 + x^2) - 2x(2x)}{(1 + x^2)^2} = \dfrac{2(1 - x^2)}{(1 + x^2)^2} > 0$

$\Leftrightarrow \quad |x| < 1$, so f is CU on $(-1, 1)$, CD on $(-\infty, -1)$ and $(1, \infty)$. IP $(1, \ln 2)$ and $(-1, \ln 2)$.

H.



31. $y = \ln(\tan^2 x)$ **A.** $D = \{x \mid x \neq n\pi/2\}$

B. x-intercepts $n\pi + \frac{\pi}{4}$, no y-intercept.

C. $f(-x) = f(x)$, so the curve is symmetric about the y-axis. Also $f(x + \pi) = f(x)$, so f is periodic with period π, and we consider D-G only for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

D. $\lim\limits_{x \to 0} \ln(\tan^2 x) = -\infty$ and $\lim\limits_{x \to \pi/2^-} \ln(\tan^2 x) = \infty$,

$\lim\limits_{x \to -\pi/2^+} \ln(\tan^2 x) = \infty$, so $x = 0$, $x = \pm\frac{\pi}{2}$ are VA.

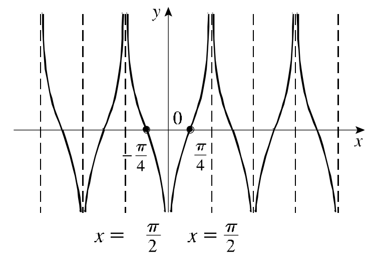
E. $f'(x) = \dfrac{2 \tan x \sec^2 x}{\tan^2 x} = 2\dfrac{\sec^2 x}{\tan x} > 0 \quad \Leftrightarrow$

$\tan x > 0 \quad \Leftrightarrow \quad 0 < x < \frac{\pi}{2}$, so f is increasing on $(0, \frac{\pi}{2})$ and decreasing on $(-\frac{\pi}{2}, 0)$. **F.** No maximum or minimum **G.** $f'(x) = \dfrac{2}{\sin x \cos x} = \dfrac{4}{\sin 2x} \quad \Rightarrow$

$f''(x) = \dfrac{-8 \cos 2x}{\sin^2 2x} < 0 \quad \Leftrightarrow \quad \cos 2x > 0 \quad \Leftrightarrow$

$-\frac{\pi}{4} < x < \frac{\pi}{4}$, so f is CD on $(-\frac{\pi}{4}, 0)$ and $(0, \frac{\pi}{4})$ and CU on $(-\frac{\pi}{2}, -\frac{\pi}{4})$ and $(\frac{\pi}{4}, \frac{\pi}{2})$. IP are $(\pm\frac{\pi}{4}, 0)$.

H.



32. $y = f(x) = x^2 e^{-x}$ **A.** $D = \mathbb{R}$

B. Intercepts are 0 **C.** No symmetry

D. $\lim\limits_{x \to \infty} x^2 e^{-x} = \lim\limits_{x \to \infty} \dfrac{x^2}{e^x} \overset{\text{H}}{=} \lim\limits_{x \to \infty} \dfrac{2x}{e^x} \overset{\text{H}}{=} \lim\limits_{x \to \infty} \dfrac{2}{e^x} = 0$,

so $y = 0$ is a HA. Also $\lim\limits_{x \to -\infty} x^2 e^{-x} = \infty$.

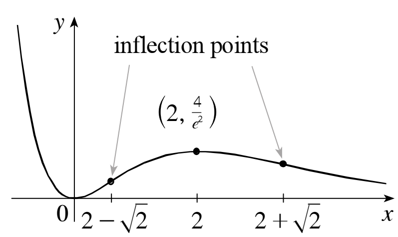
E. $f'(x) = 2xe^{-x} - x^2 e^{-x} = x(2 - x)e^{-x} > 0$ when $0 < x < 2$, so f is increasing on $(0, 2)$ and decreasing on $(-\infty, 0)$ and $(2, \infty)$. **F.** $f(0) = 0$ is a local minimum, $f(2) = 4e^{-2}$ is a local maximum.

G. $f''(x) = (2 - 2x)e^{-x} - (2x - x^2)e^{-x}$

$= (x^2 - 4x + 2)e^{-x} = 0$ when

$x^2 - 4x + 2 = 0 \quad \Leftrightarrow \quad x = 2 \pm \sqrt{2}$. $f''(x) > 0 \quad \Leftrightarrow$ $x < 2 - \sqrt{2}$ or $x > 2 + \sqrt{2}$, so f is CU on $(-\infty, 2 - \sqrt{2})$ and $(2 + \sqrt{2}, \infty)$ and CD on $(2 - \sqrt{2}, 2 + \sqrt{2})$. IP $\left(2 \pm \sqrt{2}, (6 \pm 4\sqrt{2})\, e^{\sqrt{2} \pm 2}\right)$

H.



33. $y = f(x) = x^2 \ln x$ **A.** $D = (0, \infty)$ **B.** x-intercept when $\ln x = 0 \quad \Leftrightarrow \quad x = 1$, no y-intercept **C.** No symmetry **D.** $\lim\limits_{x \to \infty} x^2 \ln x = \infty$,

$\lim\limits_{x \to 0^+} x^2 \ln x = \lim\limits_{x \to 0^+} \dfrac{\ln x}{1/x^2} \overset{\text{H}}{=} \lim\limits_{x \to 0^+} \dfrac{1/x}{-2/x^3}$

$= \lim\limits_{x \to 0^+} \left(-\dfrac{x^2}{2}\right) = 0$, no asymptote

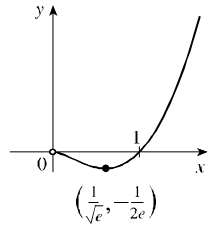
E. $f'(x) = 2x \ln x + x = x(2 \ln x + 1) > 0 \quad \Leftrightarrow$ $\ln x > -\frac{1}{2} \quad \Leftrightarrow \quad x > e^{-1/2}$, so f is increasing on $(1/\sqrt{e}, \infty)$ and decreasing on $(0, /1\sqrt{e})$.

F. $f(1/\sqrt{e}) = -1/(2e)$ is an absolute minimum.

G. $f''(x) = 2 \ln x + 3 > 0 \quad \Leftrightarrow \quad \ln x > -\frac{3}{2} \quad \Leftrightarrow$ $x > e^{-3/2}$, so f is CU on $\left(e^{-3/2}, \infty\right)$ and CD on

$\left(0, e^{-3/2}\right)$. IP is $\left(e^{-3/2}, -3/(2e^3)\right)$

H.



34. $y = f(x) = xe^{1/x}$ **A.** $D = \{x \mid x \neq 0\}$ **B.** No intercept **C.** No symmetry **D.** $\lim\limits_{x\to\infty} xe^{1/x} = \infty$, $\lim\limits_{x\to-\infty} xe^{1/x} = -\infty$, no HA.

$$\lim_{x\to0^+} xe^{1/x} = \lim_{x\to0^+} \frac{e^{1/x}}{1/x} \overset{\text{H}}{=} \lim_{x\to0^+} \frac{e^{1/x}\left(-1/x^2\right)}{-1/x^2} = \lim_{x\to0^+} e^{1/x} = \infty,$$

so $x = 0$ is a VA. Also $\lim\limits_{x\to0^-} xe^{1/x} = 0$ since $\frac{1}{x} \to -\infty \;\Rightarrow\; e^{1/x} \to 0$.

E. $f'(x) = e^{1/x} + xe^{1/x}\left(-\frac{1}{x^2}\right) = e^{1/x}\left(1 - \frac{1}{x}\right) > 0 \;\Leftrightarrow\; \frac{1}{x} < 1 \;\Leftrightarrow\; x < 0$ or $x > 1$, so f is increasing on $(-\infty, 0)$ and $(1, \infty)$ and decreasing on $(0, 1)$.
F. $f(1) = e$ is a local minimum.
G. $f''(x) = e^{1/x}\left(-1/x^2\right)(1 - 1/x) + e^{1/x}\left(1/x^2\right) = e^{1/x}/x^3 > 0 \;\Leftrightarrow\; x > 0$, so f is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. No IP

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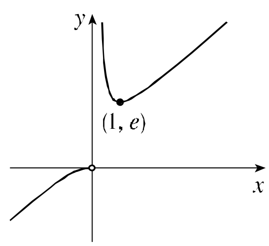
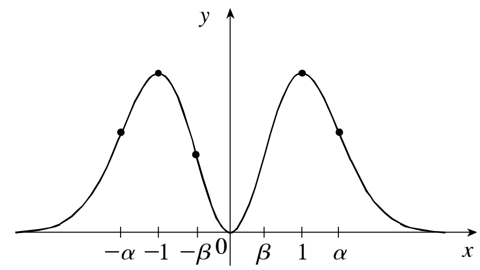


35. $y = f(x) = x^2e^{-x^2}$ **A.** $D = \mathbb{R}$ **B.** Intercepts are 0 **C.** $f(-x) = f(x)$, so the graph is symmetric about the y-axis.

D. $\lim\limits_{x\to\pm\infty} x^2e^{-x^2} = \lim\limits_{x\to\pm\infty} \frac{x^2}{e^{x^2}} \overset{\text{H}}{=} \lim\limits_{x\to\pm\infty} \frac{2x}{2xe^{x^2}} = \lim\limits_{x\to\pm\infty} e^{-x^2} = 0$, so $y = 0$ is a HA.

E. $f'(x) = 2xe^{-x^2} - 2x^3e^{-x^2} = 2x\left(1 - x^2\right)e^{-x^2} > 0 \;\Leftrightarrow\; 0 < x < 1$ or $x < -1$, so f is increasing on $(0, 1)$ and $(-\infty, -1)$ and decreasing on $(-1, 0)$ and $(1, \infty)$.
F. $f(0) = 0$ is a local minimum, $f(\pm1) = 1/e$ are maxima.
G. $f''(x) = 2e^{-x^2}\left(2x^4 - 5x^2 + 1\right) = 0$ where $x^2 = \frac{1}{4}\left(5 \pm \sqrt{17}\right) \;\Leftrightarrow\; x = \pm\frac{1}{2}\sqrt{5 \pm \sqrt{17}}$ (all four possibilities). Let $\alpha = \frac{1}{2}\sqrt{5 + \sqrt{17}}$ and $\beta = \frac{1}{2}\sqrt{5 - \sqrt{17}}$. Then $f''(x) > 0 \;\Leftrightarrow\; |x| > \alpha$ or $|x| < \beta$, so f is CU on $(-\infty, -\alpha)$, $(-\beta, \beta)$ and (α, ∞) and CD on $(-\alpha, -\beta)$ and (β, α). IP at $x = \pm\alpha, \pm\beta$.

H.



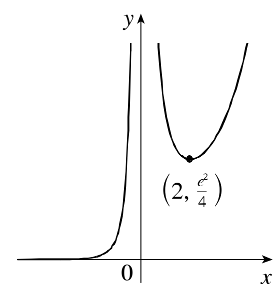
36. $y = f(x) = e^x/x^2$ **A.** $D = \{x \mid x \neq 0\}$ **B.** No intercept **C.** No symmetry

D. $\lim\limits_{x\to\infty} \frac{e^x}{x^2} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \frac{e^x}{2x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \frac{e^x}{2} = \infty$, $\lim\limits_{x\to-\infty} \frac{e^x}{x^2} = 0$, so $y = 0$ is a HA. $\lim\limits_{x\to0} \frac{e^x}{x^2} = \infty$, so $x = 0$ is a VA.

E. $f'(x) = \frac{x^2e^x - 2xe^x}{x^4} = \frac{(x-2)e^x}{x^3} > 0 \;\Leftrightarrow\; x < 0$ or $x > 0$, so f is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$. **F.** $f(2) = \frac{1}{4}e^2$ is a local minimum.

G. $f''(x) = \frac{x^3e^x(x-1) - 3x^2e^x(x-2)}{x^6} = \frac{e^x\left(x^2 - 4x + 6\right)}{x} > 0$ for all x since $x^2 - 4x + 6$ has positive discriminant, so f is CU on $(-\infty, 0)$ and $(0, \infty)$.

H.



37. $y = f(x) = x^2e^{-1/x}$ **A.** $D = \{x \mid x \neq 0\}$ **B.** No intercept **C.** No symmetry **D.** $\lim\limits_{x\to\pm\infty} x^2e^{-1/x} = \infty$, $x = \lim\limits_{x\to0^+} x^2e^{-1/x} = 0$,

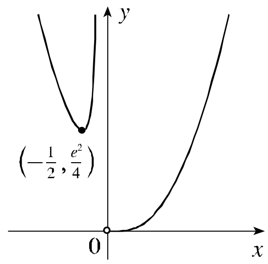
$$\lim_{x\to0^-} x^2e^{-1/x} = \lim_{x\to0^-} \frac{e^{-1/x}}{1/x^2} \overset{\text{H}}{=} \lim_{x\to0^-} \frac{e^{-1/x}\left(1/x^2\right)}{-2/x^3} = \lim_{x\to0^-} \frac{e^{-1/x}}{-2/x} \overset{\text{H}}{=} \lim_{x\to0^-} \frac{e^{-1/x}\left(1/x^2\right)}{2/x^2} = \lim_{x\to0^-} \tfrac{1}{2}e^{-1/x} = \infty,$$

so $x = 0$ is a VA.

E. $f'(x) = 2xe^{-1/x} + x^2e^{-1/x}\left(1/x^2\right) = e^{-1/x}(2x + 1) > 0 \;\Leftrightarrow\; x > -\frac{1}{2}$, so f is increasing on $\left(-\frac{1}{2}, 0\right)$ and $(0, \infty)$ and decreasing on $\left(-\infty,-\frac{1}{2}\right)$. **F.** $f\left(-\frac{1}{2}\right)=\frac{1}{4}e^2$ is a local minimum.

G. $f''(x)=e^{-1/x}\left(1/x^2\right)(2x+1)+2e^{-1/x}$
$$=e^{-1/x}\left(2x^2+2x+1\right)/x^2>0$$
for all x since $2x^2+2x+1$ has positive discriminant, so f is CU on $(-\infty,0)$ and $(0,\infty)$.

H.



38. $y=f(x)=x-\ln(1+x)$

A. $D=\{x\mid x>-1\}=(-1,\infty)$

B. Intercepts are 0. **C.** No symmetry

D. $\lim\limits_{x\to-1^+}[x-\ln(1+x)]=\infty$, so $x=-1$ is a VA.

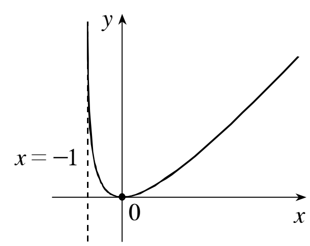
$$\lim_{x\to\infty}[x-\ln(1+x)]=\lim_{x\to\infty}x\left[1-\frac{\ln(1+x)}{x}\right]=\infty,$$

since $\lim\limits_{x\to\infty}\dfrac{\ln(1+x)}{x}\overset{\text{H}}{=}\lim\limits_{x\to\infty}\dfrac{1/(1+x)}{1}=0.$

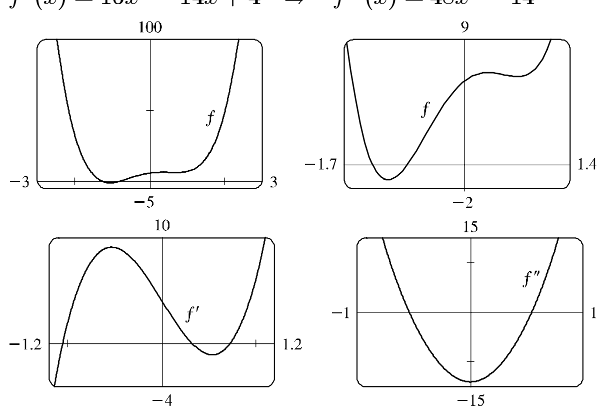
E. $f'(x)=1-\dfrac{1}{1+x}=\dfrac{x}{1+x}>0\quad\Leftrightarrow\quad x>0$ since $x+1>0$. So f is increasing on $(0,\infty)$ and decreasing on $(-1,0)$. **F.** $f(0)=0$ is a local and absolute minimum.

G. $f''(x)=1/(1+x)^2>0$, so f is CU on $(-1,\infty)$.

H.

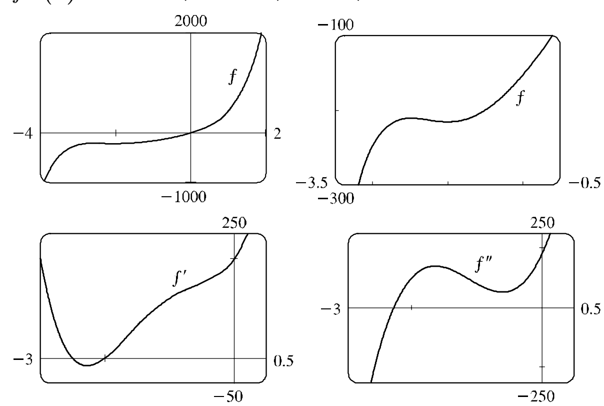


39. $f(x)=4x^4-7x^2+4x+6\quad\Rightarrow$
$f'(x)=16x^3-14x+4\quad\Rightarrow\quad f''(x)=48x^2-14$



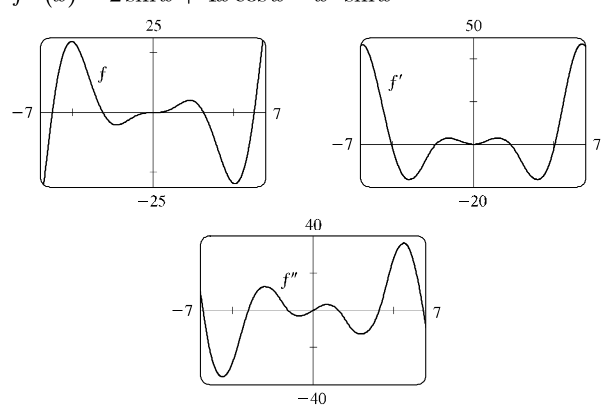
After finding suitable viewing rectangles (by ensuring that we have located all of the x-values where either $f'=0$ or $f''=0$) we estimate from the graph of f' that f is increasing on $(-1.1,0.3)$ and $(0.7,\infty)$ and decreasing on $(-\infty,-1.1)$ and $(0.3,0.7)$, with a local maximum of $f(0.3)\approx 6.6$ and minima of $f(-1.1)\approx -1.0$ and $f(0.7)\approx 6.3$. We estimate from the graph of f'' that f is CU on $(-\infty,-0.5)$ and $(0.5,\infty)$ and CD on $(-0.5,0.5)$, and that f has inflection points at about $(-0.5,2.1)$ and $(0.5,6.5)$.

40. $f(x)=8x^5+45x^4+80x^3+90x^2+200x\quad\Rightarrow$
$f'(x)=40x^4+180x^3+240x^2+180x+200\quad\Rightarrow$
$f''(x)=160x^3+540x^2+480x+180$



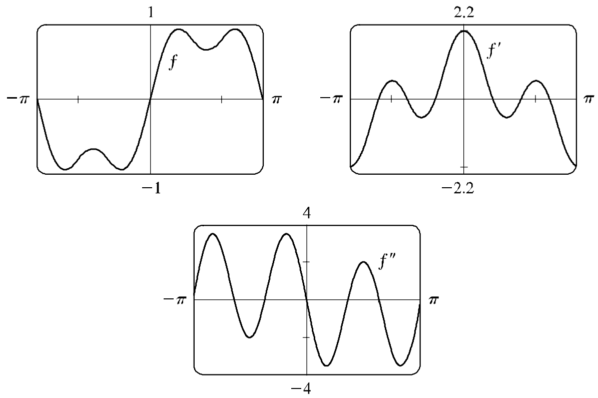
After finding suitable viewing rectangles, we estimate from the graph of f' that f is increasing on $(-\infty,-2.5)$ and $(-2.0,\infty)$ and decreasing on $(-2.5,-2.0)$. Maximum: $f(-2.5)\approx -211$. Minimum: $f(-2)\approx -216$. We estimate from the graph of f'' that f is CU on $(-2.3,\infty)$ and CD on $(-\infty,-2.3)$, and has an IP at $(-2.3,-213)$.

41. $f(x)=x^2\sin x\quad\Rightarrow\quad f'(x)=2x\sin x+x^2\cos x\quad\Rightarrow$
$f''(x)=2\sin x+4x\cos x-x^2\sin x$



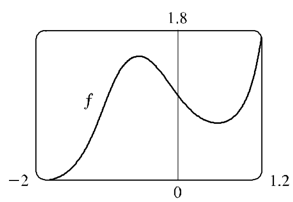
We estimate from the graph of f' that f is increasing on $(-7,-5.1)$, $(-2.3,2.3)$, and $(5.1,7)$ and decreasing on $(-5.1,-2.3)$, and $(2.3,5.1)$. Local maxima: $f(-5.1)\approx 24.1$, $f(2.3)\approx 3.9$. Local minima: $f(-2.3)\approx -3.9$, $f(5.1)\approx -24.1$. From the graph of f'', we estimate that f is CU on $(-7,-6.8)$, $(-4.0,-1.5)$, $(0,1.5)$, and $(4.0,6.8)$, and CD on $(-6.8,-4.0)$, $(-1.5,0)$, $(1.5,4.0)$, and $(6.8,7)$. f has IP at $(-6.8,-24.4)$, $(-4.0,12.0)$, $(-1.5,-2.3)$, $(0,0)$, $(1.5,2.3)$, $(4.0,-12.0)$ and $(6.8,24.4)$.

42. $f(x) = \sin x + \frac{1}{3}\sin 3x \quad \Rightarrow \quad f'(x) = \cos x + \cos 3x$
$\Rightarrow \quad f''(x) = -\sin x - 3\sin 3x$



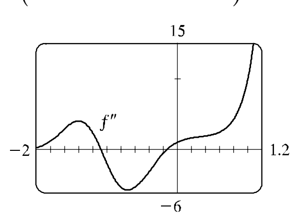
Note that f is periodic with period 2π, so we consider it on the interval $[-\pi, \pi]$. From the graph of f', we estimate that f is increasing on $(-2.4, -1.6)$, $(-0.8, 0.8)$, and $(1.6, 2.4)$ and decreasing on $(-\pi, -2.4)$, $(-1.6, -0.8)$, $(0.8, 1.6)$ and $(2.4, \pi)$. Maxima: $f(-1.6) \approx -0.7$, $f(0.8) \approx 0.9$, $f(2.4) \approx 0.9$. Minima: $f(-2.4) \approx -0.9$, $f(-0.8) \approx -0.9$, $f(1.6) \approx 0.7$. We estimate from the graph of f'' that f is CD on $(-2.0, -1.2)$, $(0, 1.2)$ and $(2.0, \pi)$ and CU on $(-\pi, -2.0)$, $(-1.2, 0)$ and $(1.2, 2)$. f has IP at $(-\pi, 0)$, $(-2.0, -0.8)$, $(-1.2, -0.8)$, $(0, 0)$, $(1.2, 0.8)$, $(2.0, 0.8)$, and $(\pi, 0)$.

43.



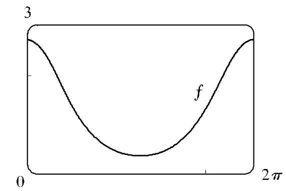
$f(x) = e^{x^3 - x} \to 0$ as $x \to -\infty$, and $f(x) \to \infty$ as $x \to \infty$. From the graph, it appears that f has a local minimum of about $f(0.58) = 0.68$, and a local maximum of about $f(-0.58) = 1.47$. To find the exact values, we calculate $f'(x) = (3x^2 - 1)\, e^{x^3 - x}$, which is 0 when $3x^2 - 1 = 0 \quad \Leftrightarrow \quad x = \pm\frac{1}{\sqrt{3}}$. The negative root corresponds to the local maximum $f\left(-\frac{1}{\sqrt{3}}\right) = e^{(-1/\sqrt{3})^3 - (-1/\sqrt{3})} = e^{2\sqrt{3}/9}$, and the positive root corresponds to the local minimum $f\left(\frac{1}{\sqrt{3}}\right) = e^{(1/\sqrt{3})^3 - (1/\sqrt{3})} = e^{-2\sqrt{3}/9}$. To estimate the inflection points, we calculate and graph

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left[(3x^2 - 1)\, e^{x^3 - x}\right] \\ &= (3x^2 - 1)\, e^{x^3 - x}\,(3x^2 - 1) + e^{x^3 - x}\,(6x) \\ &= e^{x^3 - x}\,(9x^4 - 6x^2 + 6x + 1) \end{aligned}$$



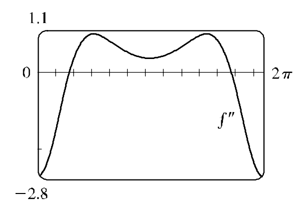
From the graph, it appears that $f''(x)$ changes sign (and thus f has inflection points) at $x \approx -0.15$ and $x \approx -1.09$. From the graph of f, we see that these x-values correspond to inflection points at about $(-0.15, 1.15)$ and $(-1.09, 0.82)$.

44.



The function $f(x) = e^{\cos x}$ is periodic with period 2π, so we consider it only on the interval $[0, 2\pi]$. We see that it has local maxima of about $f(0) \approx 2.72$ and $f(2\pi) \approx 2.72$, and a local minimum of about $f(3.14) \approx 0.37$. To find the exact values, we calculate $f'(x) = -\sin x\, e^{\cos x}$. This is 0 when $-\sin x = 0 \quad \Leftrightarrow \quad x = 0, \pi$ or 2π (since we are only considering $x \in [0, 2\pi]$). Also $f'(x) > 0 \quad \Leftrightarrow \quad \sin x < 0 \quad \Leftrightarrow \quad \pi < x < 2\pi$. So $f(0) = f(2\pi) = e$ (both maxima) and $f(\pi) = e^{\cos \pi} = 1/e$ (minimum). To find the inflection points, we calculate and graph

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left(-\sin x\, e^{\cos x}\right) \\ &= -\cos x\, e^{\cos x} - \sin x\left(e^{\cos x}\right)(-\sin x) \\ &= e^{\cos x}\left(\sin^2 x - \cos x\right) \end{aligned}$$



From the graph of $f''(x)$, we see that f has inflection points at $x \approx 0.90$ and at $x \approx 5.38$. These x-coordinates correspond to inflection points $(0.90, 1.86)$ and $(5.38, 1.86)$.