

4.7 ANTIDERIVATIVES

A Click here for answers.

1–19 ■ Find the most general antiderivative of the function. (Check your answer by differentiation.)

1. $f(x) = 12x^2 + 6x - 5$
2. $f(x) = x^3 - 4x^2 + 17$
3. $f(x) = 6x^9 - 4x^7 + 3x^2 + 1$
4. $f(x) = x^{99} - 2x^{49} - 1$
5. $f(x) = 6/x^5$
6. $f(x) = x^{2/3} + 2x^{-1/3}$
7. $f(x) = \sqrt{x} + 1/\sqrt{x}$
8. $g(t) = \sqrt[3]{t^4} + t^{-6}$
9. $f(x) = \sqrt{x} + \sqrt[3]{x}$
10. $f(x) = \sqrt[3]{x^2} - \sqrt{x^3}$
11. $f(x) = \frac{3}{x^2} - \frac{5}{x^4}$
12. $g(t) = \frac{t^3 + 2t^2}{\sqrt{t}}$
13. $f(t) = 3 \cos t - 4 \sin t$
14. $f(\theta) = e^\theta + \sec \theta \tan \theta$
15. $h(x) = \sin x - 2 \cos x$
16. $f(t) = \sin t - 2\sqrt{t}$
17. $f(t) = \sec^2 t + t^2$
18. $f'(x) = x^4 - 2x^2 + x - 1$
19. $f'(x) = \sin x - \sqrt[3]{x^2}$

20–42 ■ Find $f(x)$.

20. $f''(x) = x^2 + x^3$
21. $f''(x) = 60x^4 - 45x^2$
22. $f''(x) = 1$
23. $f''(x) = \sin x$
24. $f'''(x) = 24x$
25. $f'''(x) = \sqrt{x}$
26. $f'(x) = 4x + 3, f(0) = -9$
27. $f'(x) = 12x^2 - 24x + 1, f(1) = -2$
28. $f'(x) = 3\sqrt{x} - 1/\sqrt{x}, f(1) = 2$

S Click here for solutions.

29. $f'(x) = 1 + 1/x^2, x > 0, f(1) = 1$
30. $f'(x) = 3 \cos x + 5 \sin x, f(0) = 4$
31. $f'(x) = \sin x - 2\sqrt{x}, f(0) = 0$
32. $f'(x) = 2 + \sqrt[3]{x^3}, f(1) = 3$
33. $f'(x) = 4 - 3(1 + x^2)^{-1}, f(1) = 0$
34. $f''(x) = -8, f(0) = 6, f'(0) = 5$
35. $f''(x) = x, f(0) = -3, f'(0) = 2$
36. $f''(x) = 20x^3 - 10, f(1) = 1, f'(1) = -5$
37. $f''(x) = x^2 + 3 \cos x, f(0) = 2, f'(0) = 3$
38. $f''(x) = x + \sqrt{x}, f(1) = 1, f'(1) = 2$
39. $f''(x) = 6x + 6, f(0) = 4, f(1) = 3$
40. $f''(x) = 12x^2 - 6x + 2, f(0) = 1, f(2) = 11$
41. $f''(x) = 1/x^3, x > 0, f(1) = 0, f(2) = 0$
42. $f''(x) = 3e^x + 5 \sin x, f(0) = 1, f'(0) = 2$

43–46 ■ A particle is moving with the given data. Find the position of the particle.

43. $v(t) = 3 - 2t, s(0) = 4$
44. $v(t) = 3\sqrt{t}, s(1) = 5$
45. $a(t) = 3t + 8, s(0) = 1, v(0) = -2$
46. $a(t) = t^2 - t, s(0) = 0, s(6) = 12$

4.7 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $4x^3 + 3x^2 - 5x + C$
2. $\frac{1}{4}x^4 - \frac{4}{3}x^3 + 17x + C$
3. $\frac{3}{5}x^{10} - \frac{1}{2}x^8 + x^3 + x + C$
4. $\frac{1}{100}x^{100} - \frac{1}{25}x^{50} - x + C$
5. $-\frac{3}{2x^4} + C_1$ if $x < 0$, $-\frac{3}{2x^4} + C_2$ if $x > 0$
6. $\frac{3}{5}x^{5/3} + 3x^{2/3} + C_1$ if $x > 0$, $\frac{3}{5}x^{5/3} + 3x^{2/3} + C_2$ if $x < 0$
7. $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$
8. $\frac{5}{9}t^{9/5} - \frac{1}{5t^5} + C_1$ if $t > 0$, $\frac{5}{9}t^{9/5} - \frac{1}{5t^5} + C_2$ if $t < 0$
9. $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$
10. $\frac{3}{5}x^{5/3} - \frac{2}{5}x^{5/2} + C$
11. $-\frac{3}{x} + \frac{5}{3x^3} + C_1$ if $x < 0$, $-\frac{3}{x} + \frac{5}{3x^3} + C_2$ if $x > 0$
12. $\frac{2}{7}t^{7/2} + \frac{4}{5}t^{5/2} + C$
13. $3 \sin t + 4 \cos t + C$
14. $e^\theta + \sec \theta + C_n$ on the interval $(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2})$
15. $-\cos x - 2 \sin x + C$
16. $-\cos t - \frac{4}{3}t^{3/2} + C$
17. $\tan t + \frac{1}{3}t^3 + C_n$ on the interval $(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2})$
18. $\frac{1}{5}x^5 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$
19. $-\cos x - \frac{5}{7}x^{7/5} + C$
20. $\frac{1}{12}x^4 + \frac{1}{20}x^5 + Cx + D$
21. $2x^6 - \frac{15}{4}x^4 + Cx + D$
22. $\frac{1}{2}x^2 + Cx + D$
23. $-\sin x + Cx + D$
24. $x^4 + \frac{1}{2}Cx^2 + Dx + E$
25. $\frac{8}{105}x^{7/2} + \frac{1}{2}Cx^2 + Dx + E$
26. $2x^2 + 3x - 9$
27. $4x^3 - 12x^2 + x + 5$
28. $2x^{3/2} - 2x^{1/2} + 2$
29. $1 + x - 1/x$
30. $3 \sin x - 5 \cos x + 9$
31. $-\cos x - \frac{4}{3}x^{3/2} + 1$
32. $2x + \frac{5}{8}x^{8/5} + \frac{3}{8}$
33. $4x - 3 \arctan x + \frac{3\pi}{4} - 4$
34. $-4x^2 + 5x + 6$
35. $\frac{1}{6}x^3 + 2x - 3$
36. $x^5 - 5x^2 + 5$
37. $\frac{1}{12}x^4 - 3 \cos x + 3x + 5$
38. $\frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$
39. $x^3 + 3x^2 - 5x + 4$
40. $x^4 - x^3 + x^2 - x + 1$
41. $\frac{1}{2x} + \frac{1}{4}x - \frac{3}{4}$
42. $3e^x - 5 \sin x + 4x - 2$
43. $s(t) = 3t - t^2 + 4$
44. $s(t) = 2t^{3/2} + 3$
45. $s(t) = \frac{1}{2}t^3 + 4t^2 - 2t + 1$
46. $s(t) = \frac{1}{12}t^4 - \frac{1}{6}t^3 - 10t$

4.7 SOLUTIONS



1. $f(x) = 12x^2 + 6x - 5 \Rightarrow$
 $F(x) = 12\left(\frac{1}{3}x^3\right) + 6\left(\frac{1}{2}x^2\right) - 5x + C = 4x^3 + 3x^2 - 5x + C$
2. $f(x) = x^3 - 4x^2 + 17 \Rightarrow$
 $F(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 17x + C$
3. $f(x) = 6x^9 - 4x^7 + 3x^2 + 1 \Rightarrow$
 $F(x) = 6\left(\frac{1}{10}x^{10}\right) - 4\left(\frac{1}{8}x^8\right) + 3\left(\frac{1}{3}x^3\right) + x + C$
 $= \frac{3}{5}x^{10} - \frac{1}{2}x^8 + x^3 + x + C$
4. $f(x) = x^{99} - 2x^{49} - 1 \Rightarrow$
 $F(x) = \left(\frac{1}{100}x^{100}\right) - 2\left(\frac{1}{50}x^{50}\right) - x + C$
 $= \frac{1}{100}x^{100} - \frac{1}{25}x^{50} - x + C$
5. $f(x) = \frac{6}{x^5} = 6x^{-5} \Rightarrow$
 $F(x) = 6\frac{x^{-4}}{-4} + C_1 = -\frac{3}{2x^4} + C_1$ if $x < 0$ and
 $F(x) = -\frac{3}{2x^4} + C_2$ if $x > 0$.
6. $f(x) = x^{2/3} + 2x^{-1/3}$ has domain $(-\infty, 0) \cup (0, \infty)$, so
 $F(x) = \frac{x^{5/3}}{5/3} + \frac{2x^{2/3}}{2/3} + C_1 = \frac{3}{5}x^{5/3} + 3x^{2/3} + C_1$ if
 $x > 0$ and $F(x) = \frac{3}{5}x^{5/3} + 3x^{2/3} + C_2$ if $x < 0$.
7. $f(x) = \sqrt{x} + 1/\sqrt{x} = x^{1/2} + x^{-1/2} \Rightarrow$
 $F(x) = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$
8. $g(t) = t^{4/5} + t^{-6}$ has domain $(-\infty, 0) \cup (0, \infty)$, so
 $G(t) = \frac{t^{9/5}}{9/5} + \frac{t^{-5}}{-5} + C_1 = \frac{5}{9}t^{9/5} - \frac{1}{5t^5} + C_1$ if $t > 0$ and
 $G(t) = \frac{5}{9}t^{9/5} - \frac{1}{5t^5} + C_2$ if $t < 0$.
9. $f(x) = \sqrt{x} + \sqrt[3]{x} = x^{1/2} + x^{1/3} \Rightarrow$
 $F(x) = \frac{1}{3/2}x^{3/2} + \frac{1}{4/3}x^{4/3} + C = \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$
10. $f(x) = \sqrt[3]{x^2} - \sqrt{x^3} = x^{2/3} - x^{3/2} \Rightarrow$
 $F(x) = \frac{1}{5/3}x^{5/3} - \frac{1}{5/2}x^{5/2} + C = \frac{3}{5}x^{5/3} - \frac{2}{5}x^{5/2} + C$
11. $f(x) = 3x^{-2} - 5x^{-4}$ has domain $(-\infty, 0) \cup (0, \infty)$, so
 $F(x) = \frac{3x^{-1}}{-1} - \frac{5x^{-3}}{-3} + C_1 = -\frac{3}{x} + \frac{5}{3x^3} + C_1$ if $x < 0$
and $F(x) = -\frac{3}{x} + \frac{5}{3x^3} + C_2$ if $x > 0$.
12. $g(t) = \frac{t^3 + 2t^2}{\sqrt{t}} = t^{5/2} + 2t^{3/2} \Rightarrow$
 $G(t) = \frac{t^{7/2}}{7/2} + \frac{2t^{5/2}}{5/2} + C = \frac{2}{7}t^{7/2} + \frac{4}{5}t^{5/2} + C$. Note that
 g has domain $(0, \infty)$.
13. $f(t) = 3\cos t - 4\sin t \Rightarrow$
 $F(t) = 3(\sin t) - 4(-\cos t) + C = 3\sin t + 4\cos t + C$
14. $f(\theta) = e^\theta + \sec \theta \tan \theta \Rightarrow F(\theta) = e^\theta + \sec \theta + C_n$ on
the interval $(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2})$.
15. $h(x) = \sin x - 2\cos x \Rightarrow$
 $H(x) = -\cos x - 2\sin x + C$
16. $f(t) = \sin t - 2\sqrt{t} \Rightarrow$
 $F(t) = -\cos t - 2\left(\frac{1}{3/2}\right)t^{3/2} + C = -\cos t - \frac{4}{3}t^{3/2} + C$
17. $f(t) = \sec^2 t + t^2 \Rightarrow F(t) = \tan t + \frac{1}{3}t^3 + C_n$ on the
interval $(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2})$.
18. $f'(x) = x^4 - 2x^2 + x - 1 \Rightarrow$
 $f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$
19. $f'(x) = \sin x - x^{2/5} \Rightarrow f(x) = -\cos x - \frac{5}{7}x^{7/5} + C$
20. $f''(x) = x^2 + x^3 \Rightarrow f'(x) = \frac{1}{3}x^3 + \frac{1}{4}x^4 + C \Rightarrow$
 $f(x) = \frac{1}{12}x^4 + \frac{1}{20}x^5 + Cx + D$
21. $f''(x) = 60x^4 - 45x^2 \Rightarrow$
 $f'(x) = 60\left(\frac{1}{5}x^5\right) - 45\left(\frac{1}{3}x^3\right) + C = 12x^5 - 15x^3 + C$
 $\Rightarrow$
 $f(x) = 12\left(\frac{1}{6}x^6\right) - 15\left(\frac{1}{4}x^4\right) + Cx + D$
 $= 2x^6 - \frac{15}{4}x^4 + Cx + D$
22. $f''(x) = 1 \Rightarrow f'(x) = x + C \Rightarrow$
 $f(x) = \frac{1}{2}x^2 + Cx + D$
23. $f''(x) = \sin x \Rightarrow f'(x) = -\cos x + C \Rightarrow$
 $f(x) = -\sin x + Cx + D$
24. $f'''(x) = 24x \Rightarrow f''(x) = 12x^2 + C$
 $\Rightarrow f'(x) = 4x^3 + Cx + D \Rightarrow$
 $f(x) = x^4 + \frac{1}{2}Cx^2 + Dx + E$
25. $f'''(x) = x^{1/2} \Rightarrow f''(x) = \frac{2}{3}x^{3/2} + C \Rightarrow$
 $f'(x) = \frac{2}{3} \cdot \frac{2}{5}x^{5/2} + Cx + D = \frac{4}{15}x^{5/2} + Cx + D \Rightarrow$
 $f(x) = \frac{4}{15} \cdot \frac{2}{7}x^{7/2} + C\left(\frac{1}{2}x^2\right) + Dx + E$
 $= \frac{8}{105}x^{7/2} + \frac{1}{2}Cx^2 + Dx + E$
26. $f'(x) = 4x + 3 \Rightarrow f(x) = 2x^2 + 3x + C \Rightarrow$
 $-9 = f(0) = C \Rightarrow f(x) = 2x^2 + 3x - 9$
27. $f'(x) = 12x^2 - 24x + 1 \Rightarrow$
 $f(x) = 4x^3 - 12x^2 + x + C \Rightarrow$
 $f(1) = 4 - 12 + 1 + C = -2 \Rightarrow C = 5$, so
 $f(x) = 4x^3 - 12x^2 + x + 5$

28. $f'(x) = 3\sqrt{x} - 1/\sqrt{x} = 3x^{1/2} - x^{-1/2} \Rightarrow$
 $f(x) = 3\left(\frac{1}{3/2}\right)x^{3/2} - \frac{1}{1/2}x^{1/2} + C \Rightarrow$
 $2 = f(1) = 2 - 2 + C = C \Rightarrow$
 $f(x) = 2x^{3/2} - 2x^{1/2} + 2$
29. $f'(x) = 1 + x^{-2}, x > 0 \Rightarrow f(x) = x - 1/x + C$. Now
 $f(1) = 1 - 1 + C = 1 \Rightarrow C = 1$, so
 $f(x) = 1 + x - 1/x$.
30. $f'(x) = 3\cos x + 5\sin x \Rightarrow$
 $f(x) = 3\sin x - 5\cos x + C \Rightarrow 4 = f(0) = -5 + C$
 $\Rightarrow C = 9 \Rightarrow f(x) = 3\sin x - 5\cos x + 9$
31. $f'(x) = \sin x - 2x^{1/2} \Rightarrow f(x) = -\cos x - \frac{4}{3}x^{3/2} + C$
 $\Rightarrow f(0) = -1 - 0 + C = 0 \Rightarrow C = 1$, so
 $f(x) = -\cos x - \frac{4}{3}x^{3/2} + 1$.
32. $f'(x) = 2 + x^{3/5} \Rightarrow f(x) = 2x + \frac{5}{8}x^{8/5} + C \Rightarrow$
 $3 = f(1) = 2 + \frac{5}{8} + C \Rightarrow C = \frac{3}{8} \Rightarrow$
 $f(x) = 2x + \frac{5}{8}x^{8/5} + \frac{3}{8}$
33. $f'(x) = 4 - \frac{3}{1+x^2} \Rightarrow f(x) = 4x - 3\arctan x + C$.
Now $f(1) = 4 \cdot 1 - 3\arctan 1 + C = 0 \Rightarrow$
 $C = 3 \cdot \frac{\pi}{4} - 4$. Therefore $f(x) = 4x - 3\arctan x + \frac{3\pi}{4} - 4$.
34. $f''(x) = -8 \Rightarrow f'(x) = -8x + C \Rightarrow$
 $5 = f'(0) = C \Rightarrow f'(x) = -8x + 5 \Rightarrow$
 $f(x) = -4x^2 + 5x + D \Rightarrow 6 = f(0) = D \Rightarrow$
 $f(x) = -4x^2 + 5x + 6$
35. $f''(x) = x \Rightarrow f'(x) = \frac{1}{2}x^2 + C \Rightarrow$
 $2 = f'(0) = C \Rightarrow f'(x) = \frac{1}{2}x^2 + 2 \Rightarrow$
 $f(x) = \frac{1}{6}x^3 + 2x + D \Rightarrow -3 = f(0) = D \Rightarrow$
 $f(x) = \frac{1}{6}x^3 + 2x - 3$
36. $f''(x) = 20x^3 - 10 \Rightarrow f'(x) = 5x^4 - 10x + C \Rightarrow$
 $-5 = f'(1) = 5 - 10 + C \Rightarrow C = 0 \Rightarrow$
 $f'(x) = 5x^4 - 10x \Rightarrow f(x) = x^5 - 5x^2 + D \Rightarrow$
 $1 = f(1) = 1 - 5 + D \Rightarrow D = 5 \Rightarrow$
 $f(x) = x^5 - 5x^2 + 5$
37. $f''(x) = x^2 + 3\cos x \Rightarrow f'(x) = \frac{1}{3}x^3 + 3\sin x + C$
 $\Rightarrow 3 = f'(0) = C \Rightarrow f'(x) = \frac{1}{3}x^3 + 3\sin x + 3$
 $\Rightarrow f(x) = \frac{1}{12}x^4 - 3\cos x + 3x + D \Rightarrow$
 $2 = f(0) = -3 + D \Rightarrow D = 5 \Rightarrow$
 $f(x) = \frac{1}{12}x^4 - 3\cos x + 3x + 5$
38. $f''(x) = x + x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$
 $\Rightarrow 2 = f'(1) = \frac{1}{2} + \frac{2}{3} + C \Rightarrow$
 $C = \frac{5}{6} \Rightarrow f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6}$
 $\Rightarrow f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x + D \Rightarrow$
 $1 = f(1) = \frac{1}{6} + \frac{4}{15} + \frac{5}{6} + D \Rightarrow D = -\frac{4}{15} \Rightarrow$
 $f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$
39. $f''(x) = 6x + 6 \Rightarrow f'(x) = 3x^2 + 6x + C \Rightarrow$
 $f(x) = x^3 + 3x^2 + Cx + D$. $4 = f(0) = D$ and
 $3 = f(1) = 1 + 3 + C + D = 4 + C + 4 \Rightarrow C = -5$,
so $f(x) = x^3 + 3x^2 - 5x + 4$.
40. $f''(x) = 12x^2 - 6x + 2 \Rightarrow$
 $f'(x) = 4x^3 - 3x^2 + 2x + C \Rightarrow$
 $f(x) = x^4 - x^3 + x^2 + Cx + D$. $1 = f(0) = D$ and
 $11 = f(2) = 16 - 8 + 4 + 2C + D = 13 + 2C \Rightarrow$
 $C = -1$, so $f(x) = x^4 - x^3 + x^2 - x + 1$.
41. $f''(x) = x^{-3} \Rightarrow f'(x) = -\frac{1}{2}x^{-2} + C \Rightarrow$
 $f(x) = \frac{1}{2}x^{-1} + Cx + D \Rightarrow 0 = f(1) = \frac{1}{2} + C + D$
and $0 = f(2) = \frac{1}{4} + 2C + D$. Solving these equations, we
get $C = \frac{1}{4}$, $D = -\frac{3}{4}$, so $f(x) = 1/(2x) + \frac{1}{4}x - \frac{3}{4}$.
42. $f''(x) = 3e^x + 5\sin x \Rightarrow f'(x) = 3e^x - 5\cos x + C$
 $\Rightarrow 2 = f'(0) = 3 - 5 + C \Rightarrow C = 4$,
so $f'(x) = 3e^x - 5\cos x + 4 \Rightarrow$
 $f(x) = 3e^x - 5\sin x + 4x + D \Rightarrow 1 = f(0) = 3 + D$
 $\Rightarrow D = -2$, so $f(x) = 3e^x - 5\sin x + 4x - 2$.
43. $v(t) = s'(t) = 3 - 2t \Rightarrow s(t) = 3t - t^2 + C \Rightarrow$
 $4 = s(0) = C \Rightarrow s(t) = 3t - t^2 + 4$
44. $v(t) = s'(t) = 3\sqrt{t} \Rightarrow s(t) = 2t^{3/2} + C \Rightarrow$
 $5 = s(1) = 2 + C \Rightarrow C = 3$, so $s(t) = 2t^{3/2} + 3$
45. $a(t) = v'(t) = 3t + 8 \Rightarrow v(t) = \frac{3}{2}t^2 + 8t + C \Rightarrow$
 $-2 = v(0) = C \Rightarrow v(t) = \frac{3}{2}t^2 + 8t - 2 \Rightarrow$
 $s(t) = \frac{1}{2}t^3 + 4t^2 - 2t + D \Rightarrow 1 = s(0) = D \Rightarrow$
 $s(t) = \frac{1}{2}t^3 + 4t^2 - 2t + 1$
46. $a(t) = v'(t) = t^2 - t \Rightarrow v(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + C \Rightarrow$
 $s(t) = \frac{1}{12}t^4 - \frac{1}{6}t^3 + Ct + D \Rightarrow 0 = s(0) = D$ and
 $12 = s(6) = 108 - 36 + 6C + 0 \Rightarrow C = -10 \Rightarrow$
 $s(t) = \frac{1}{12}t^4 - \frac{1}{6}t^3 - 10t$