5.2 THE DEFINITE INTEGRAL

A Click here for answers.

1–7 ▪ Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

1. $\int_0^5 x^3\,dx,\quad n = 5$

2. $\int_1^3 \frac{1}{2x-7}\,dx,\quad n = 4$

3. $\int_1^2 \sqrt{1+x^2}\,dx,\quad n = 10$

4. $\int_0^{\pi/4} \tan x\,dx,\quad n = 4$

5. $\int_0^{10} \sin\sqrt{x}\,dx,\quad n = 5$

6. $\int_0^{\pi} \sec(x/3)\,dx,\quad n = 6$

7. $\int_2^4 x\ln x\,dx,\quad n = 4$

8–9 ▪ Express the limit as a definite integral on the given interval.

8. $\lim_{n\to\infty}\sum_{i=1}^{n}[2(x_i^*)^2 - 5x_i^*]\,\Delta x,\quad [0, 1]$

9. $\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{x_i^*}\,\Delta x,\quad [1, 4]$

10–19 ▪ Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

10. $\int_a^b c\,dx$

11. $\int_{-2}^7 (6 - 2x)\,dx$

12. $\int_1^4 (x^2 - 2)\,dx$

13. $\int_1^5 (2 + 3x - x^2)\,dx$

14. $\int_0^1 (ax + b)\,dx$

15. $\int_{-3}^0 (2x^2 - 3x - 4)\,dx$

16. $\int_{-1}^1 (t^3 - t^2 + 1)\,dt$

17. $\int_a^b (Px^2 + Qx + R)\,dx$

18. $\int_0^b (x^3 + 4x)\,dx$

19. $\int_2^5 (t^3 - 2t + 3)\,dt$

S Click here for solutions.

20–23 ▪ Evaluate the integral by interpreting it in terms of areas.

20. $\int_1^3 (1 + 2x)\,dx$

21. $\int_{-1}^3 (2 - x)\,dx$

22. $\int_{-2}^2 (1 - |x|)\,dx$

23. $\int_0^3 |3x - 5|\,dx$

24–27 ▪ Write the given sum or difference as a single integral in the form $\int_a^b f(x)\,dx$.

24. $\int_1^3 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^{12} f(x)\,dx$

25. $\int_5^8 f(x)\,dx + \int_0^5 f(x)\,dx$

26. $\int_2^{10} f(x)\,dx - \int_2^7 f(x)\,dx$

27. $\int_{-3}^5 f(x)\,dx - \int_{-3}^0 f(x)\,dx + \int_5^6 f(x)\,dx$

28. If $\int_2^8 f(x)\,dx = 1.7$ and $\int_5^8 f(x)\,dx = 2.5$, find $\int_2^5 f(x)\,dx$.

29. If $\int_0^1 f(t)\,dt = 2$, $\int_0^4 f(t)\,dt = -6$, and $\int_3^4 f(t)\,dt = 1$, find $\int_1^3 f(t)\,dt$.

30–32 ▪ Use Property 8 to estimate the value of the integral.

30. $\int_{-3}^0 (x^2 + 2x)\,dx$

31. $\int_{\pi/4}^{\pi/3} \cos x\,dx$

32. $\int_{-1}^1 \sqrt{1 + x^4}\,dx$

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5.2 ANSWERS

E Click here for exercises. S Click here for solutions.

1. 153.125
2. -0.7873
3. 1.8100
4. 0.3450
5. 6.4643
6. 3.9379
7. 6.6969
8. $\int_0^1 (2x^2 - 5x)\,dx$
9. $\int_1^4 \sqrt{x}\,dx$
10. $(b-a)\,c$
11. 9
12. 15
13. $\frac{8}{3}$
14. $\dfrac{a}{2} + b$
15. 19.5
16. $\frac{4}{3}$
17. $P\left(\dfrac{b^3}{3} - \dfrac{a^3}{3}\right) + Q\left(\dfrac{b^2}{2} - \dfrac{a^2}{2}\right) + R\,(b-a)$
18. $\dfrac{b^4}{4} + 2b^2$
19. 140.25
20. 10
21. 4
22. 0
23. $\frac{41}{6}$
24. $\int_1^{12} f(x)\,dx$
25. $\int_0^8 f(x)\,dx$
26. $\int_7^{10} f(x)\,dx$
27. $\int_0^6 f(x)\,dx$
28. -0.8
29. -9
30. $-3 \le \int_{-3}^0 (x^2 + 2x)\,dx \le 9.$
31. $\frac{\pi}{24} \le \int_{\pi/4}^{\pi/3} \cos x\,dx \le \frac{\sqrt{2}\pi}{24}$
32. $2 \le \int_{-1}^1 \sqrt{1+x^4}\,dx \le 2\sqrt{2}$

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5.2 SOLUTIONS

E Click here for exercises.

1. The width of the intervals is $\Delta x = (5-0)/5 = 1$ so the partition points are $0, 1, 2, 3, 4, 5$ and the midpoints are $0.5, 1.5, 2.5, 3.5, 4.5$. The Midpoint Rule gives

$$\begin{aligned}\int_0^5 x^3 dx &\approx \sum_{i=1}^{5} f(\overline{x}_i)\,\Delta x \\ &= (0.5)^3 + (1.5)^3 + (2.5)^3 + (3.5)^3 + (4.5)^3 \\ &= 153.125\end{aligned}$$

2. The width of the interval $\Delta x = (3-1)/4 = 0.5$ so the partition points are $1.0, 1.5, 2.0, 2.5, 3.0$ and the midpoints are $1.25, 1.75, 2.25, 2.75$.

$$\begin{aligned}\int_1^3 \frac{1}{2x-7}dx &\approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x \\ &= 0.5\left[\frac{1}{2(1.25)-7} + \frac{1}{2(1.75)-7}\right. \\ &\quad \left. + \frac{1}{2(2.25)-7} + \frac{1}{2(2.75)-7}\right] \\ &\approx -0.7873\end{aligned}$$

3. $\Delta x = (2-1)/10 = 0.1$ so the partition points are $1.0, 1.1, \ldots, 2.0$ and the midpoints are $1.05, 1.15, \ldots, 1.95$.

$$\begin{aligned}\int_1^2 \sqrt{1+x^2}dx &\approx \sum_{i=1}^{10} f(\overline{x}_i)\,\Delta x \\ &= 0.1\left[\sqrt{1+(1.05)^2} + \sqrt{1+(1.15)^2}\right. \\ &\quad \left. + \cdots + \sqrt{1+(1.95)^2}\right] \\ &\approx 1.8100\end{aligned}$$

4. $\Delta x = \frac{1}{4}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{16}$ so the partition points are $0, \frac{\pi}{16}, \frac{2\pi}{16}, \frac{3\pi}{16}, \frac{4\pi}{16}$ and the midpoints are $\frac{\pi}{32}, \frac{3\pi}{32}, \frac{5\pi}{32}, \frac{7\pi}{32}$. The Midpoint Rule gives

$$\begin{aligned}\int_0^{\pi/4} \tan x\,dx &\approx \left(\tfrac{\pi}{16}\right)\left(\tan\tfrac{\pi}{32} + \tan\tfrac{3\pi}{32}\right. \\ &\quad \left. + \tan\tfrac{5\pi}{32} + \tan\tfrac{7\pi}{32}\right) \\ &\approx 0.3450\end{aligned}$$

5. $\Delta x = (10-0)/5 = 2$, so the endpoints are $0, 2, 4, 6, 8$, and 10, and the midpoints are $1, 3, 5, 7$, and 9. The Midpoint Rule gives

$$\begin{aligned}\int_0^{10} \sin\sqrt{x}\,dx &\approx \sum_{i=1}^{5} f(\overline{x}_i)\,\Delta x \\ &= 2\left(\sin\sqrt{1} + \sin\sqrt{3}\right. \\ &\quad \left. + \sin\sqrt{5} + \sin\sqrt{7} + \sin\sqrt{9}\right) \\ &\approx 6.4643\end{aligned}$$

6. $\Delta x = (\pi - 0)/6 = \frac{\pi}{6}$, so the endpoints are $0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}$, and $\frac{6\pi}{6}$, and the midpoints are $\frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}$, and $\frac{11\pi}{12}$. The Midpoint Rule gives

$$\begin{aligned}\int_0^{\pi} \sec(x/3)\,dx &\approx \sum_{i=1}^{6} f(\overline{x}_i)\,\Delta x \\ &= \tfrac{\pi}{6}\left(\sec\tfrac{\pi}{36} + \sec\tfrac{3\pi}{36} + \sec\tfrac{5\pi}{36}\right. \\ &\quad \left. + \sec\tfrac{7\pi}{36} + \sec\tfrac{9\pi}{36} + \sec\tfrac{11\pi}{36}\right) \\ &\approx 3.9379\end{aligned}$$

7. $\Delta x = (4-2)/4 = 0.5$, so the endpoints are $2, 2.5, 3, 3.5$, and 4, and the midpoints are $2.25, 2.75, 3.25$, and 3.75. The Midpoint Rule gives

$$\begin{aligned}\int_2^4 x\ln x\,dx &\approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x \quad [f(x) = x\ln x] \\ &= 0.5\,[f(2.25) + f(2.75) + f(3.25) + f(3.75)] \\ &\approx 6.6969\end{aligned}$$

8. On $[0, 1]$,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[2(x_i^*)^2 - 5x_i^*\right]\Delta x = \int_0^1 \left(2x^2 - 5x\right)dx.$$

9. On $[1, 4]$, $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i^*}\,\Delta x = \int_1^4 \sqrt{x}\,dx.$

10. $$\begin{aligned}\int_a^b c\,dx &= \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} c = \lim_{n\to\infty} \frac{b-a}{n} nc \\ &= \lim_{n\to\infty} (b-a)\,c = (b-a)\,c\end{aligned}$$

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11. $\int_{-2}^{7}(6-2x)\,dx = \lim_{n\to\infty}\frac{9}{n}\sum_{i=1}^{n}\left[6-2\left(-2+\frac{9i}{n}\right)\right]$

$= \lim_{n\to\infty}\frac{9}{n}\sum_{i=1}^{n}\left[10-\frac{18i}{n}\right]$

$= \lim_{n\to\infty}\left[\frac{90}{n}\sum_{i=1}^{n}1-\frac{162}{n^2}\sum_{i=1}^{n}i\right]$

$= \lim_{n\to\infty}\left[\frac{90}{n}n-\frac{162}{n^2}\frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty}\left[90-81\cdot 1\left(1+\frac{1}{n}\right)\right]$

$= 90-81=9$

12. $\int_{1}^{4}\left(x^2-2\right)dx$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[\left(1+\frac{3i}{n}\right)^2-2\right]$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[\frac{9i^2}{n^2}+\frac{6i}{n}-1\right]$

$= \lim_{n\to\infty}\left[\frac{27}{n^3}\sum_{i=1}^{n}i^2+\frac{18}{n^2}\sum_{i=1}^{n}i-\frac{3}{n}\sum_{i=1}^{n}1\right]$

$= \lim_{n\to\infty}\left[\frac{27}{n^3}\frac{n(n+1)(2n+1)}{6}+\frac{18}{n^2}\frac{n(n+1)}{2}-\frac{3}{n}n\right]$

$= \lim_{n\to\infty}\left[\frac{9}{2}\cdot 1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+9\cdot 1\left(1+\frac{1}{n}\right)-3\right]$

$= \left(\frac{9}{2}\cdot 2\right)+9-3=15$

13. $\int_{1}^{5}\left(2+3x-x^2\right)dx$

$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left[2+3\left(1+\frac{4i}{n}\right)-\left(1+\frac{4i}{n}\right)^2\right]$

$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left[-\frac{16i^2}{n^2}+\frac{4i}{n}+4\right]$

$= \lim_{n\to\infty}\left[-\frac{64}{n^3}\sum_{i=1}^{n}i^2+\frac{16}{n^2}\sum_{i=1}^{n}i+\frac{16}{n}\sum_{i=1}^{n}1\right]$

$= \lim_{n\to\infty}\left[-\frac{64}{n^3}\frac{n(n+1)(2n+1)}{6}+\frac{16}{n^2}\frac{n(n+1)}{2}+\frac{16}{n}n\right]$

$= \lim_{n\to\infty}\left[-\frac{32}{3}\cdot 1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+8\cdot 1\left(1+\frac{1}{n}\right)+16\right]$

$= -\frac{64}{3}+8+16=\frac{8}{3}$

14. $\int_{0}^{1}(ax+b)\,dx = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left[a\left(\frac{i}{n}\right)+b\right]$

$= \lim_{n\to\infty}\left[\frac{a}{n^2}\sum_{i=1}^{n}i+\frac{b}{n}\sum_{i=1}^{n}1\right]$

$= \lim_{n\to\infty}\left[\frac{a}{n^2}\frac{n(n+1)}{2}+\frac{b}{n}n\right]$

$= \lim_{n\to\infty}\left[\frac{a}{2}\cdot 1\left(1+\frac{1}{n}\right)+b\right]=\frac{a}{2}+b$

15. $\int_{-3}^{0}\left(2x^2-3x-4\right)dx$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[2\left(-3+\frac{3i}{n}\right)^2-3\left(-3+\frac{3i}{n}\right)-4\right]$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[\frac{18i^2}{n^2}-\frac{45i}{n}+23\right]$

$= \lim_{n\to\infty}\left[\frac{54}{n^3}\sum_{i=1}^{n}i^2-\frac{135}{n^2}\sum_{i=1}^{n}i+\frac{69}{n}\sum_{i=1}^{n}1\right]$

$= \lim_{n\to\infty}\left[\frac{54}{n^3}\frac{n(n+1)(2n+1)}{6}-\frac{135}{n^2}\frac{n(n+1)}{2}+\frac{69}{n}n\right]$

$= \lim_{n\to\infty}\left[9\cdot 1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)-\frac{135}{2}\cdot 1\left(1+\frac{1}{n}\right)+69\right]$

$= 9\cdot 2-\frac{135}{2}+69=19.5$

16. $\int_{-1}^{1}\left(t^3-t^2+1\right)dt$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[\left(-1+\frac{2i}{n}\right)^3-\left(-1+\frac{2i}{n}\right)^2+1\right]$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{8i^3}{n^3}-\frac{12i^2}{n^2}+\frac{6i}{n}-1\right)\right.$
$\left.-\left(\frac{4i^2}{n^2}-\frac{4i}{n}+1\right)+1\right]$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[\frac{8i^3}{n^3}-\frac{16i^2}{n^2}+\frac{10i}{n}-1\right]$

$= \lim_{n\to\infty}\left[\frac{16}{n^4}\sum_{i=1}^{n}i^3-\frac{32}{n^3}\sum_{i=1}^{n}i^2+\frac{20}{n^2}\sum_{i=1}^{n}i-\frac{2}{n}\sum_{i=1}^{n}1\right]$

$= \lim_{n\to\infty}\left[\frac{16}{n^4}\frac{n^2(n+1)^2}{4}-\frac{32}{n^3}\frac{n(n+1)(2n+1)}{6}\right.$
$\left.+\frac{20}{n^2}\frac{n(n+1)}{2}-\frac{2}{n}n\right]$

$= \lim_{n\to\infty}\left[4\cdot 1^2\left(1+\frac{1}{n}\right)^2-\frac{16}{3}\cdot 1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right.$
$\left.+10\cdot 1\left(1+\frac{1}{n}\right)-2\right]$

$= 4-\frac{32}{3}+10-2=\frac{4}{3}$

17. $\int_{a}^{b}\left(Px^2+Qx+R\right)dx$

$= \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n}\left[P\left(a+\frac{b-a}{n}i\right)^2\right.$
$\left.+Q\left(a+\frac{b-a}{n}i\right)+R\right]$

$= \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n}\left[P\frac{(b-a)^2}{n^2}i^2\right.$
$\left.+(2Pa+Q)\frac{b-a}{n}i+\left(Pa^2+Qa+R\right)\right]$

$= \lim_{n\to\infty}\left[P\frac{(b-a)^3}{n^3}\sum_{i=1}^{n}i^2+(2Pa+Q)\frac{(b-a)^2}{n^2}\sum_{i=1}^{n}i\right.$
$\left.+\left(Pa^2+Qa+R\right)\frac{b-a}{n}\sum_{i=1}^{n}1\right]$

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$$= \lim_{n\to\infty}\left[P\frac{(b-a)^3}{n^3}\frac{n(n+1)(2n+1)}{6} + (2Pa+Q)\frac{(b-a)^2}{n^2}\frac{n(n+1)}{2} + \left(Pa^2+Qa+R\right)\frac{b-a}{n}n\right]$$

$$= \lim_{n\to\infty}\left[\frac{P(b-a)^3}{6}1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + \left(Pa+\frac{Q}{2}\right)(b-a)^2\,1\left(1+\frac{1}{n}\right) + \left(Pa^2+Qa+R\right)(b-a)\right]$$

$$= P\frac{(b-a)^3}{3} + \left(Pa+\frac{Q}{2}\right)(b-a)^2 + \left(Pa^2+Qa+R\right)(b-a)$$

$$= P\left(\frac{b^3}{3} - b^2a + ba^2 - \frac{a^3}{3} + ab^2 - 2a^2b + a^3 + a^2b - a^3\right) + Q\left(\frac{b^2}{2} - ab + \frac{a^2}{2} + ab - a^2\right) + R(b-a)$$

$$= P\left(\frac{b^3}{3} - \frac{a^3}{3}\right) + Q\left(\frac{b^2}{2} - \frac{a^2}{2}\right) + R(b-a)$$

18. $\int_0^b \left(x^3+4x\right)dx$

$$= \lim_{n\to\infty}\frac{b}{n}\sum_{i=1}^{n}\left[\left(\frac{bi}{n}\right)^3 + 4\left(\frac{bi}{n}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{b^4}{n^4}\sum_{i=1}^{n} i^3 + 4\frac{b^2}{n^2}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty}\left[\frac{b^4}{n^4}\frac{n^2(n+1)^2}{4} + \frac{4b^2}{n^2}\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{b^4}{4}\cdot 1^2\left(1+\frac{1}{n}\right)^2 + 2b^2\cdot 1\left(1+\frac{1}{n}\right)\right]$$

$$= \frac{b^4}{4} + 2b^2$$

19. $\int_2^5 \left(t^3-2t+3\right)dt$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[\left(2+\frac{3i}{n}\right)^3 - 2\left(2+\frac{3i}{n}\right) + 3\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[\frac{27i^3}{n^3} + \frac{54i^2}{n^2} + \frac{36i}{n} + 8 - 4 - \frac{6i}{n} + 3\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[\frac{27i^3}{n^3} + \frac{54i^2}{n^2} + \frac{30i}{n} + 7\right]$$

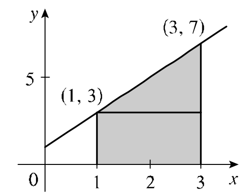
$$= \lim_{n\to\infty}\left[\frac{81}{n^4}\sum_{i=1}^{n} i^3 + \frac{162}{n^3}\sum_{i=1}^{n} i^2 + \frac{90}{n^2}\sum_{i=1}^{n} i + \frac{21}{n}\sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty}\left[\frac{81}{n^4}\frac{n^2(n+1)^2}{4} + \frac{162}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{90}{n^2}\frac{n(n+1)}{2} + \frac{21}{n}n\right]$$

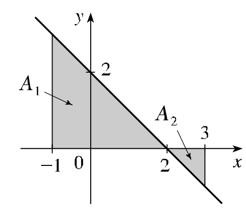
$$= \lim_{n\to\infty}\left[\frac{81}{4}\cdot 1^2\left(1+\frac{1}{n}\right)^2 + 27\cdot 1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 45\cdot 1\left(1+\frac{1}{n}\right) + 21\right]$$

$$= \tfrac{81}{4} + 54 + 45 + 21 = 140.25$$

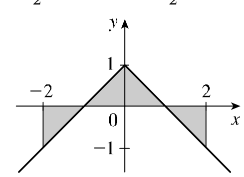
20. $\int_1^3 (1+2x)\,dx$ can be interpreted as the area under the graph of $f(x) = 1+2x$ between $x=1$ and $x=3$. This is equal to the area of the rectangle plus the area of the triangle, so $\int_1^3 (1+2x)\,dx = A = 2\cdot 3 + \frac{1}{2}\cdot 2\cdot 4 = 10$.
Or: Use the formula for the area of a trapezoid: $A = \frac{1}{2}(2)(3+7) = 10$.



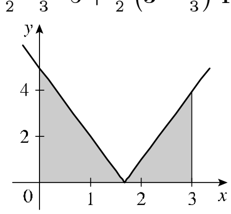
21. $\int_{-1}^3 (2-x)\,dx$ can be interpreted as $A_1 - A_2$, where A_1 and A_2 are the areas of the triangles shown. Thus, $\int_{-1}^3 (2-x)\,dx = \frac{1}{2}\cdot 3\cdot 3 - \frac{1}{2}\cdot 1\cdot 1 = 4$.



22. $\int_{-2}^2 (1-|x|)\,dx$ can be interpreted as the area of the middle triangle minus the areas of the outside ones, so $\int_{-2}^2 (1-|x|)\,dx = \frac{1}{2}\cdot 2\cdot 1 - 2\cdot\frac{1}{2}\cdot 1\cdot 1 = 0$.



23. $\int_0^3 |3x-5|\,dx$ can be interpreted as the area under the graph of the function $f(x) = |3x-5|$ between $x=0$ and $x=3$. This is equal to the sum of the areas of the two triangles, so $\int_0^3 |3x-5|\,dx = \frac{1}{2}\cdot\frac{5}{3}\cdot 5 + \frac{1}{2}\left(3-\frac{5}{3}\right)4 = \frac{41}{6}$.



24. $\int_1^3 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^{12} f(x)\,dx$
$= \int_1^6 f(x)\,dx + \int_6^{12} f(x)\,dx = \int_1^{12} f(x)\,dx$

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25. $\int_5^8 f(x)\,dx + \int_0^5 f(x)\,dx$

$= \int_0^5 f(x)\,dx + \int_5^8 f(x)\,dx = \int_0^8 f(x)\,dx$

26. $\int_2^{10} f(x)\,dx - \int_2^7 f(x)\,dx$

$= \int_2^7 f(x)\,dx + \int_7^{10} f(x)\,dx - \int_2^7 f(x)\,dx = \int_7^{10} f(x)\,dx$

27. $\int_{-3}^5 f(x)\,dx - \int_{-3}^0 f(x)\,dx + \int_5^6 f(x)\,dx$

$= \int_{-3}^0 f(x)\,dx + \int_0^5 f(x)\,dx - \int_{-3}^0 f(x)\,dx + \int_5^6 f(x)\,dx$

$= \int_0^6 f(x)\,dx$

28. $\int_2^5 f(x)\,dx + \int_5^8 f(x)\,dx = \int_2^8 f(x)\,dx \quad \Rightarrow$

$\int_2^5 f(x)\,dx + 2.5 = 1.7 \quad \Rightarrow \quad \int_2^5 f(x)\,dx = -0.8$

29. $\int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt + \int_3^4 f(t)\,dt = \int_0^4 f(t)\,dt$

$\Rightarrow \quad 2 + \int_1^3 f(t)\,dt + 1 = -6 \quad \Rightarrow$

$\int_1^3 f(t)\,dt = -6 - 2 - 1 = -9$

30. If $f(x) = x^2 + 2x$, $-3 \le x \le 0$, then $f'(x) = 2x + 2 = 0$ when $x = -1$, and $f(-1) = -1$. At the endpoints, $f(-3) = 3$, $f(0) = 0$. Thus the absolute minimum is $m = -1$ and the absolute maximum is $M = 3$. Thus $-1\,[0 - (-3)] \le \int_{-3}^0 \left(x^2 + 2x\right) dx \le 3\,[0 - (-3)]$ or $-3 \le \int_{-3}^0 \left(x^2 + 2x\right) dx \le 9$.

31. If $\frac{\pi}{4} \le x \le \frac{\pi}{3}$, then $\frac{1}{2} \le \cos x \le \frac{\sqrt{2}}{2}$, so $\frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \le \int_{\pi/4}^{\pi/3} \cos x\,dx \le \frac{\sqrt{2}}{2}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ or $\frac{\pi}{24} \le \int_{\pi/4}^{\pi/3} \cos x\,dx \le \frac{\sqrt{2}\pi}{24}$.

32. For $-1 \le x \le 1$, $0 \le x^4 \le 1$ and $1 \le \sqrt{1 + x^4} \le \sqrt{2}$, so $1\,[1 - (-1)] \le \int_{-1}^1 \sqrt{1 + x^4}\,dx \le \sqrt{2}\,[1 - (-1)]$ or $2 \le \int_{-1}^1 \sqrt{1 + x^4}\,dx \le 2\sqrt{2}$.

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