5.4 THE FUNDAMENTAL THEOREM OF CALCULUS

A Click here for answers.

S Click here for solutions.

1. Sketch the area represented by

$$g(x) = \int_{\pi}^{x} (2 + \cos t)\, dt$$

Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

2–12 ■ Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

2. $g(x) = \int_{1}^{x} (t^2 - 1)^{20}\, dt$

3. $g(x) = \int_{-1}^{x} \sqrt{t^3 + 1}\, dt$

4. $g(u) = \int_{\pi}^{u} \frac{1}{1 + t^4}\, dt$

5. $g(t) = \int_{0}^{t} \sin(x^2)\, dx$

6. $F(x) = \int_{x}^{4} (2 + \sqrt{u})^8\, du$

7. $h(x) = \int_{2}^{1/x} \sin^4 t\, dt$

8. $h(x) = \int_{1}^{\sqrt{x}} \frac{s^2}{s^2 + 1}\, ds$

9. $y = \int_{\tan x}^{17} \sin(t^4)\, dt$

10. $y = \int_{x^2}^{\pi} \frac{\sin t}{t}\, dt$

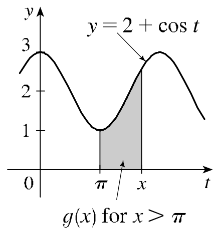
11. $y = \int_{0}^{5x+1} \frac{1}{u^2 - 5}\, du$

12. $y = \int_{-5}^{\sin x} t \cos(t^3)\, dt$

5.4 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. 

(a), (b) $2+\cos x$

2. $g'(x) = \left(x^2-1\right)^{20}$

3. $g'(x) = \sqrt{x^3+1}$

4. $g'(u) = \dfrac{1}{1+u^4}$

5. $g'(t) = \sin\left(t^2\right)$

6. $F'(x) = -\left(2+\sqrt{x}\right)^8$

7. $h'(x) = \dfrac{-\sin^4(1/x)}{x^2}$

8. $h'(x) = \dfrac{\sqrt{x}}{2(x+1)}$

9. $\dfrac{dy}{dx} = -\sin\left(\tan^4 x\right)\sec^2 x$

10. $\dfrac{dy}{dx} = -\dfrac{2\sin\left(x^2\right)}{x}$

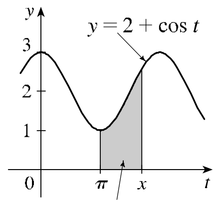
11. $\dfrac{dy}{dx} = \dfrac{5}{25x^2+10x-4}$

12. $\dfrac{dy}{dx} = \sin x\cos x\cos\left(\sin^3 x\right)$

5.4 SOLUTIONS

E Click here for exercises.

1.



$g(x)$ for $x > \pi$

(a) $g(x) = \int_\pi^x (2 + \cos t)\,dt \quad \Rightarrow \quad g'(x) = 2 + \cos x$

(b) $g(x) = \int_\pi^x (2 + \cos t)\,dt = [2t + \sin t]_\pi^x$

$= (2x + \sin x) - (2\pi + 0) = 2x + \sin x - 2\pi$

so $g'(x) = 2 + \cos x$.

2. $g(x) = \int_1^x (t^2 - 1)^{20}\,dt \quad \Rightarrow \quad g'(x) = (x^2 - 1)^{20}$

3. $g(x) = \int_{-1}^x \sqrt{t^3 + 1}\,dt \quad \Rightarrow \quad g'(x) = \sqrt{x^3 + 1}$

4. $g(u) = \displaystyle\int_\pi^u \frac{1}{1 + t^4}\,dt \quad \Rightarrow \quad g'(u) = \frac{1}{1 + u^4}$

5. $g(t) = \int_0^t \sin(x^2)\,dx \quad \Rightarrow \quad g'(t) = \sin(t^2)$

6. $F(x) = \int_x^4 (2 + \sqrt{u})^8\,du = -\int_4^x (2 + \sqrt{u})^8\,du \quad \Rightarrow$
$F'(x) = -(2 + \sqrt{x})^8$

7. Let $u = \dfrac{1}{x}$. Then $\dfrac{du}{dx} = -\dfrac{1}{x^2}$, so

$$\frac{d}{dx}\int_2^{1/x} \sin^4 t\,dt = \frac{d}{du}\int_2^u \sin^4 t\,dt \cdot \frac{du}{dx} = \sin^4 u \frac{du}{dx} = \frac{-\sin^4(1/x)}{x^2}$$

8. Let $u = \sqrt{x}$. Then $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$, so

$$h'(x) = \frac{d}{dx}\int_1^{\sqrt{x}} \frac{s^2}{s^2 + 1}\,ds = \frac{d}{du}\int_1^u \frac{s^2}{s^2 + 1}\,ds \cdot \frac{du}{dx} = \frac{u^2}{u^2 + 1}\frac{du}{dx} = \frac{x}{x + 1}\frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2(x + 1)}$$

9. Let $u = \tan x$. Then $\dfrac{du}{dx} = \sec^2 x$, so

$$\frac{d}{dx}\int_{\tan x}^{17} \sin(t^4)\,dt = -\frac{d}{dx}\int_{17}^{\tan x} \sin(t^4)\,dt = -\frac{d}{du}\int_{17}^u \sin(t^4)\,dt \cdot \frac{du}{dx} = -\sin(u^4)\frac{du}{dx} = -\sin(\tan^4 x)\sec^2 x$$

10. Let $u = x^2$. Then $\dfrac{du}{dx} = 2x$, so

$$\frac{dy}{dx} = \frac{d}{dx}\int_{x^2}^{\pi} \frac{\sin t}{t}\,dt = -\frac{d}{dx}\int_\pi^{x^2} \frac{\sin t}{t}\,dt = -\frac{d}{du}\int_\pi^u \frac{\sin t}{t}\,dt \cdot \frac{du}{dx} = -\frac{\sin u}{u} \cdot \frac{du}{dx} = -\frac{\sin(x^2)}{x^2} \cdot 2x = -\frac{2\sin(x^2)}{x}$$

11. Let $t = 5x + 1$. Then $\dfrac{dt}{dx} = 5$, so

$$\frac{d}{dx}\int_0^{5x+1} \frac{1}{u^2 - 5}\,du = \frac{d}{dt}\int_0^t \frac{1}{u^2 - 5}\,du \cdot \frac{dt}{dx} = \frac{1}{t^2 - 5}\frac{dt}{dx} = \frac{5}{25x^2 + 10x - 4}$$

12. Let $u = \sin x$. Then $\dfrac{du}{dx} = \cos x$, so

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{d}{du}\int_{-5}^u t\cos(t^3)\,dt \cdot \frac{du}{dx} = u\cos(u^3)\frac{du}{dx} = \sin x \cos(\sin^3 x)\cos x = \sin x \cos x \cos(\sin^3 x)$$