

5.5

A [Click here for answers.](#)

1-6 ■ Evaluate the integral by making the given substitution.

1. $\int x(x^2 - 1)^{99} dx, \quad u = x^2 - 1$

2. $\int \frac{x^2}{\sqrt{2+x^3}} dx, \quad u = 2+x^3$

3. $\int \sin 4x \, dx, \quad u = 4x$

4. $\int \frac{dx}{(2x+1)^2}, \quad u = 2x+1$

5. $\int \frac{x+3}{(x^2+6x)^2} dx, \quad u = x^2 + 6x$

6. $\int \sec a\theta \tan a\theta \, d\theta, \quad u = a\theta$

7–35 ■ Evaluate the indefinite integral.

7. $\int (2x + 1)(x^2 + x + 1)^3 dx$ **8.** $\int x^3(1 - x^4)^5 dx$

9. $\int \sqrt{x-1} \, dx$ 10. $\int \sqrt[3]{1-x} \, dx$

11. $\int x^3 \sqrt{2+x^4} dx$ 12. $\int x(x^2+1)^{3/2} dx$

13. $\int \frac{2}{(t+1)^6} dt$ 14. $\int \frac{1}{(1-3t)^4} dt$

15. $\int (1 - 2y)^{1.3} dy$ **16.** $\int \sqrt[5]{3 - 5y} dy$

17. $\int \cos 2\theta \, d\theta$ **18.** $\int \sec^2 3\theta \, d\theta$

19. $\int \frac{3x-1}{(3x^2-2x+1)^4} dx$ 20. $\int \frac{x}{\sqrt{x^2+1}} dx$

S [Click here for solutions.](#)

21. $\int \sin^3 x \cos x \, dx$

23. $\int t \sin(t^2) dt$

25. $\int \sec x \tan x \sqrt{1 + \sec x} \, dx$

27. $\int e^x \sin(e^x) dx$

29. $\int \frac{x+1}{x^2+2x} dx$

31. $\int x^3(1 - x^2)^{3/2} dx$

33. $\int \sin(2x + 3) \, dx$

35. $\int (\sin 3x - \sin 3x) dx$

36–43 ■ Evaluate the definite integral, if it exists.

36. $\int_0^1 \cos \pi t \, dt$

36. $\int_0^1 \cos \pi t \, dt$ **37.** $\int_0^{\pi/4} \sin 4t \, dt$

38. $\int_1^4 \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx$

38. $\int_1^4 \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx$ **39.** $\int_0^3 \frac{dx}{2x + 3}$

40. $\int_0^1 (2x - 1)^{100} dx$

40. $\int_0^1 (2x - 1)^{100} dx$ **41.** $\int_0^4 \sqrt{1 - 2x} dx$

42. $\int_0^1 (x^4 + x)^5(4x^3 + 1) dx$

42. $\int_0^1 (x^4 + x)^5 (4x^3 + 1) dx$ **43.** $\int_2^3 \frac{3x^2 - 1}{(x^3 - x)^2} dx$

44. Show that the area under the graph of $y = \sin \sqrt{x}$ from 0 to 4 is the same as the area under the graph of $y = 2x \sin x$ from 0 to 2.

5.5 ANSWERS**E** [Click here for exercises.](#)**S** [Click here for solutions.](#)

1. $\frac{1}{200} (x^2 - 1)^{100} + C$

2. $\frac{2}{3} \sqrt{2 + x^3} + C$

3. $-\frac{1}{4} \cos 4x + C$

4. $-\frac{1}{2(2x+1)} + C$

5. $-\frac{1}{2(x^2 + 6x)} + C$

6. $\frac{\sec a\theta}{a} + C$

7. $\frac{1}{4} (x^2 + x + 1)^4 + C$

8. $-\frac{1}{24} (1 - x^4)^6 + C$

9. $\frac{2}{3} (x - 1)^{3/2} + C$

10. $-\frac{3}{4} (1 - x)^{4/3} + C$

11. $\frac{1}{6} (2 + x^4)^{3/2} + C$

12. $\frac{1}{5} (x^2 + 1)^{5/2} + C$

13. $-\frac{2}{5(t+1)^5} + C$

14. $\frac{1}{9(1-3t)^3} + C$

15. $-\frac{(1-2y)^{2.3}}{4.6} + C$

16. $-\frac{1}{6} (3 - 5y)^{6/5} + C$

17. $\frac{1}{2} \sin 2\theta + C$

18. $\frac{1}{3} \tan 3\theta + C$

19. $-\frac{1}{6(3x^2 - 2x + 1)^3} + C$

20. $\sqrt{x^2 + 1} + C$

21. $\frac{1}{4} \sin^4 x + C$

22. $\frac{1}{3} \tan^3 \theta + C$

23. $-\frac{1}{2} \cos(t^2) + C$

24. $\frac{(1 + \sqrt{x})^{10}}{5} + C$

25. $\frac{2}{3} (1 + \sec x)^{3/2} + C$

26. $-\frac{1}{3} \sin(1 - t^3) + C$

27. $-\cos(e^x) + C$

28. $-\frac{1}{5} \cos^5 x + C$

29. $\frac{1}{2} \ln|x^2 + 2x| + C$

30. $\tan^{-1}(e^x) + C$

31. $\frac{1}{7} (1 - x^2)^{7/2} - \frac{1}{5} (1 - x^2)^{5/2} + C$

32. $2 \sin \sqrt{x} + C$

33. $-\frac{1}{2} \cos(2x + 3) + C$

34. $-\frac{1}{3} \sin(7 - 3x) + C$

35. $(\sin 3\alpha)x + \frac{1}{3} \cos 3x + C$

36. 0

37. $\frac{1}{2}$

38. $\frac{4\sqrt{2}}{3} - \frac{5\sqrt{5}}{12}$

39. $\frac{1}{2} \ln 3$

40. $\frac{1}{101}$

41. $-\frac{26}{3}$

42. $\frac{32}{3}$

43. $\frac{1}{8}$

5.5 SOLUTIONS



1. Let $u = x^2 - 1$. Then $du = 2x dx$, so

$$\begin{aligned}\int x(x^2 - 1)^{99} dx &= \int u^{99} \left(\frac{1}{2} du \right) = \frac{1}{2} \frac{u^{100}}{100} + C \\ &= \frac{1}{200} (x^2 - 1)^{100} + C\end{aligned}$$

2. $u = 2 + x^3$. Then $du = 3x^2 dx$, so

$$\begin{aligned}\int \frac{x^2 dx}{\sqrt{2 + x^3}} &= \int \frac{\frac{1}{3} du}{\sqrt{u}} = \frac{1}{3} \int u^{-1/2} du \\ &= \frac{1}{3} \frac{u^{1/2}}{1/2} + C = \frac{2}{3} \sqrt{2 + x^3} + C\end{aligned}$$

3. Let $u = 4x$. Then $du = 4 dx$, so

$$\begin{aligned}\int \sin 4x dx &= \int \sin u \left(\frac{1}{4} du \right) \\ &= \frac{1}{4} (-\cos u) + C = -\frac{1}{4} \cos 4x + C\end{aligned}$$

4. Let $u = 2x + 1$. Then $du = 2 dx$, so

$$\begin{aligned}\int \frac{dx}{(2x + 1)^2} &= \int \frac{(1/2) du}{u^2} = \frac{1}{2} \int u^{-2} du \\ &= -\frac{1}{2} u^{-1} + C = -\frac{1}{2(2x + 1)} + C\end{aligned}$$

5. Let $u = x^2 + 6x$. Then $du = 2(x + 3) dx$, so

$$\begin{aligned}\int \frac{x + 3}{(x^2 + 6x)^2} dx &= \frac{1}{2} \int \frac{du}{u^2} \\ &= \frac{1}{2} \int u^{-2} du = -\frac{1}{2} u^{-1} + C \\ &= -\frac{1}{2(x^2 + 6x)} + C\end{aligned}$$

6. Let $u = a\theta$. Then $du = a d\theta$, so

$$\begin{aligned}\int \sec a\theta \tan a\theta d\theta &= \int \sec u \tan u \left(\frac{1}{a} du \right) \\ &= \frac{1}{a} \sec u + C = \frac{\sec a\theta}{a} + C\end{aligned}$$

7. Let $u = x^2 + x + 1$. Then $du = (2x + 1) dx$, so

$$\begin{aligned}\int (2x + 1)(x^2 + x + 1)^3 dx &= \int u^3 du \\ &= \frac{1}{4} u^4 + C \\ &= \frac{1}{4} (x^2 + x + 1)^4 + C\end{aligned}$$

8. Let $u = 1 - x^4$. Then $du = -4x^3 dx$, so

$$\begin{aligned}\int x^3 (1 - x^4)^5 dx &= \int u^5 \left(-\frac{1}{4} du \right) \\ &= -\frac{1}{4} \left(\frac{1}{6} u^6 \right) + C \\ &= -\frac{1}{24} (1 - x^4)^6 + C\end{aligned}$$

9. Let $u = x - 1$. Then $du = dx$, so

$$\begin{aligned}\int \sqrt{x - 1} dx &= \int u^{1/2} du = \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{3} (x - 1)^{3/2} + C\end{aligned}$$

10. Let $u = 1 - x$. Then $du = -dx$, so

$$\begin{aligned}\int \sqrt[3]{x - 1} dx &= -\int u^{1/3} du = -\frac{3}{4} u^{4/3} + C \\ &= -\frac{3}{4} (1 - x)^{4/3} + C\end{aligned}$$

11. Let $u = 2 + x^4$. Then $du = 4x^3 dx$, so

$$\begin{aligned}\int x^3 \sqrt{2 + x^4} dx &= \int u^{1/2} \left(\frac{1}{4} du \right) \\ &= \frac{1}{4} \frac{u^{3/2}}{3/2} + C \\ &= \frac{1}{6} (2 + x^4)^{3/2} + C\end{aligned}$$

12. Let $u = x^2 + 1$. Then $du = 2x dx$, so

$$\begin{aligned}\int x(x^2 + 1)^{3/2} dx &= \int u^{3/2} \left(\frac{1}{2} du \right) \\ &= \frac{1}{2} \frac{u^{5/2}}{5/2} + C = \frac{1}{5} u^{5/2} + C \\ &= \frac{1}{5} (x^2 + 1)^{5/2} + C\end{aligned}$$

13. Let $u = t + 1$. Then $du = dt$, so

$$\begin{aligned}\int \frac{2}{(t + 1)^6} dt &= 2 \int u^{-6} du = -\frac{2}{5} u^{-5} + C \\ &= -\frac{2}{5(t + 1)^5} + C\end{aligned}$$

14. Let $u = 1 - 3t$. Then $du = -3 dt$, so

$$\begin{aligned}\int \frac{1}{(1 - 3t)^4} dt &= \int u^{-4} \left(-\frac{1}{3} du \right) \\ &= -\frac{1}{3} \left(\frac{u^{-3}}{-3} \right) + C = \frac{1}{9u^3} + C \\ &= \frac{1}{9(1 - 3t)^3} + C\end{aligned}$$

15. Let $u = 1 - 2y$. Then $du = -2 dy$, so

$$\begin{aligned}\int (1 - 2y)^{1.3} dy &= \int u^{1.3} \left(-\frac{1}{2} du \right) \\ &= -\frac{1}{2} \left(\frac{u^{2.3}}{2.3} \right) + C \\ &= -\frac{(1 - 2y)^{2.3}}{4.6} + C\end{aligned}$$

16. Let
- $u = 3 - 5y$
- . Then
- $du = -5 dy$
- , so

$$\begin{aligned}\int \sqrt[5]{3-5y} dy &= \int u^{1/5} \left(-\frac{1}{5} du\right) \\ &= -\frac{1}{5} \cdot \frac{5}{6} u^{6/5} + C \\ &= -\frac{1}{6} (3-5y)^{6/5} + C\end{aligned}$$

17. Let
- $u = 2\theta$
- . Then
- $du = 2 d\theta$
- , so

$$\int \cos 2\theta d\theta = \int \cos u \left(\frac{1}{2} du\right) = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2\theta + C.$$

18. Let
- $u = 3\theta$
- . Then
- $du = 3 d\theta$
- , so
- $\int \sec^2 3\theta d\theta =$

$$\int \sec^2 u \left(\frac{1}{3} du\right) = \frac{1}{3} \tan u + C = \frac{1}{3} \tan 3\theta + C.$$

19. Let
- $u = 3x^2 - 2x + 1$
- . Then
- $du = 2(3x - 1) dx$
- , so

$$\begin{aligned}\int \frac{3x-1}{(3x^2-2x+1)^4} dx &= \int u^{-4} \left(\frac{1}{2} du\right) \\ &= \frac{1}{2} \frac{u^{-3}}{-3} + C \\ &= -\frac{1}{6(3x^2-2x+1)^3} + C\end{aligned}$$

20. Let
- $u = x^2 + 1$
- . Then
- $du = 2x dx$
- , so

$$\begin{aligned}\int \frac{x dx}{\sqrt{x^2+1}} &= \int \frac{\frac{1}{2} du}{\sqrt{u}} \\ &= \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} (2u^{1/2}) + C \\ &= \sqrt{u} + C = \sqrt{x^2+1} + C\end{aligned}$$

21. Let
- $u = \sin x$
- . Then
- $du = \cos x dx$
- , so

$$\int \sin^3 x \cos x dx = \int u^3 du = \frac{1}{4} u^4 + C = \frac{1}{4} \sin^4 x + C$$

22. Let
- $u = \tan \theta$
- . Then
- $du = \sec^2 \theta d\theta$
- , so

$$\int \tan^2 \theta \sec^2 \theta d\theta = \int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} \tan^3 \theta + C$$

23. Let
- $u = t^2$
- . Then
- $du = 2t dt$
- , so

$$\begin{aligned}\int t \sin(t^2) dt &= \int \sin u \left(\frac{1}{2} du\right) \\ &= -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos(t^2) + C\end{aligned}$$

24. Let
- $u = 1 + \sqrt{x}$
- . Then
- $du = \frac{dx}{2\sqrt{x}}$
- , so

$$\begin{aligned}\int \frac{(1+\sqrt{x})^9}{\sqrt{x}} dx &= \int u^9 \cdot 2 du \\ &= 2 \frac{u^{10}}{10} + C \\ &= \frac{(1+\sqrt{x})^{10}}{5} + C\end{aligned}$$

25. Let
- $u = 1 + \sec x$
- . Then
- $du = \sec x \tan x dx$
- , so

$$\begin{aligned}\int \sec x \tan x \sqrt{1+\sec x} dx &= \int u^{1/2} du \\ &= \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{3} (1+\sec x)^{3/2} + C\end{aligned}$$

26. Let
- $u = 1 - t^3$
- . Then
- $du = -3t^2 dt$
- , so

$$\begin{aligned}\int t^2 \cos(1-t^3) dt &= \int \cos u \left(-\frac{1}{3} du\right) \\ &= -\frac{1}{3} \sin u + C \\ &= -\frac{1}{3} \sin(1-t^3) + C\end{aligned}$$

27. Let
- $u = e^x$
- . Then
- $du = e^x dx$
- , so

$$\begin{aligned}\int e^x \sin(e^x) dx &= \int \sin u du \\ &= -\cos u + C = -\cos(e^x) + C\end{aligned}$$

28. Let
- $u = \cos x$
- . Then
- $du = -\sin x dx$
- , so

$$\begin{aligned}\int \cos^4 x \sin x dx &= \int u^4 (-du) \\ &= -\frac{1}{5} u^5 + C = -\frac{1}{5} \cos^5 x + C\end{aligned}$$

29. Let
- $u = x^2 + 2x$
- . Then
- $du = 2(x+1) dx$
- , so

$$\begin{aligned}\int \frac{x+1}{x^2+2x} dx &= \int \frac{\frac{1}{2} du}{u} \\ &= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |x^2+2x| + C\end{aligned}$$

30. Let
- $u = e^x$
- . Then
- $du = e^x dx$
- , so

$$\begin{aligned}\int \frac{e^x}{e^{2x}+1} dx &= \int \frac{du}{u^2+1} \\ &= \tan^{-1} u + C = \tan^{-1}(e^x) + C\end{aligned}$$

31. Let
- $u = 1 - x^2$
- . Then
- $x^2 = 1 - u$
- and
- $2x dx = -du$
- , so

$$\begin{aligned}\int x^3 (1-x^2)^{3/2} dx &= \int (1-x^2)^{3/2} x^2 \cdot x dx \\ &= \int u^{3/2} (1-u) \left(-\frac{1}{2} du\right) \\ &= \frac{1}{2} \int (u^{5/2} - u^{3/2}) du \\ &= \frac{1}{2} \left[\frac{2}{7} u^{7/2} - \frac{2}{5} u^{5/2} \right] + C \\ &= \frac{1}{7} (1-x^2)^{7/2} - \frac{1}{5} (1-x^2)^{5/2} + C\end{aligned}$$

32. Let
- $u = \sqrt{x}$
- . Then
- $du = \frac{dx}{2\sqrt{x}}$
- , so

$$\begin{aligned}\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx &= \int \cos u \cdot 2 du \\ &= 2 \sin u + C = 2 \sin \sqrt{x} + C\end{aligned}$$

33. Let
- $u = 2x + 3$
- . Then
- $du = 2 dx$
- , so

$$\begin{aligned}\int \sin(2x+3) dx &= \int \sin u \left(\frac{1}{2} du\right) \\ &= -\frac{1}{2} \cos u + C \\ &= -\frac{1}{2} \cos(2x+3) + C\end{aligned}$$

34. Let
- $u = 7 - 3x$
- . Then
- $du = -3 dx$
- , so

$$\begin{aligned}\int \cos(7-3x) dx &= \int \cos u \left(-\frac{1}{3} du\right) \\ &= -\frac{1}{3} \sin u + C \\ &= -\frac{1}{3} \sin(7-3x) + C\end{aligned}$$

35. Let $u = 3x$. Then $du = 3 dx$, so

$$\begin{aligned}\int (\sin 3\alpha - \sin 3x) dx &= \int (\sin 3\alpha - \sin u) \frac{1}{3} du \\ &= \frac{1}{3} [(\sin 3\alpha)u + \cos u] + C \\ &= (\sin 3\alpha)x + \frac{1}{3} \cos 3x + C\end{aligned}$$

36. Let $u = \pi t$, so $du = \pi dt$. When $t = 0$, $u = 0$; when $t = 1$, $u = \pi$. Therefore,

$$\begin{aligned}\int_0^1 \cos \pi t dt &= \int_0^\pi \cos u \left(\frac{1}{\pi} du\right) \\ &= \frac{1}{\pi} [\sin u]_0^\pi = \frac{1}{\pi} (0 - 0) = 0\end{aligned}$$

37. Let $u = 4t$, so $du = 4 dt$. When $t = 0$, $u = 0$; when $t = \frac{\pi}{4}$, $u = \pi$. Therefore,

$$\begin{aligned}\int_0^{\pi/4} \sin 4t dt &= \int_0^\pi \sin u \left(\frac{1}{4} du\right) \\ &= -\frac{1}{4} [\cos u]_0^\pi = -\frac{1}{4} (-1 - 1) = \frac{1}{2}\end{aligned}$$

38. Let $u = 1 + \frac{1}{x}$, so $du = -\frac{dx}{x^2}$. When $x = 1$, $u = 2$; when $x = 4$, $u = \frac{5}{4}$. Therefore,

$$\begin{aligned}\int_1^4 \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx &= \int_2^{5/4} u^{1/2} (-du) \\ &= \int_{5/4}^2 u^{1/2} du = \left[\frac{2}{3} u^{3/2} \right]_{5/4}^2 \\ &= \frac{2}{3} \left(2\sqrt{2} - \frac{5\sqrt{5}}{8} \right) \\ &= \frac{4\sqrt{2}}{3} - \frac{5\sqrt{5}}{12}\end{aligned}$$

39. Let $u = 2x + 3$, so $du = 2 dx$. When $x = 0$, $u = 3$; when $x = 3$, $u = 9$. Therefore,

$$\begin{aligned}\int_0^3 \frac{dx}{2x+3} &= \int_3^9 \frac{\frac{1}{2} du}{u} \\ &= \left[\frac{1}{2} \ln u \right]_3^9 = \frac{1}{2} (\ln 9 - \ln 3) \\ &= \frac{1}{2} \ln \frac{9}{3} = \frac{1}{2} \ln 3 \quad (\text{or } \ln \sqrt{3})\end{aligned}$$

40. Let $u = 2x - 1$. Then $du = 2 dx$, so

$$\begin{aligned}\int_0^1 (2x-1)^{100} dx &= \int_{-1}^1 u^{100} \left(\frac{1}{2} du\right) \\ &= \int_0^1 u^{100} du \quad \left[\begin{array}{l} \text{since the integrand} \\ \text{is an even function} \end{array} \right] \\ &= \left[\frac{1}{101} u^{101} \right]_0^1 = \frac{1}{101}\end{aligned}$$

41. Let $u = 1 - 2x$. Then $du = -2 dx$, so

$$\begin{aligned}\int_0^{-4} \sqrt{1-2x} dx &= \int_1^9 u^{1/2} \left(-\frac{1}{2} du\right) \\ &= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} \Big|_1^9 \\ &= -\frac{1}{3} (27 - 1) = -\frac{26}{3}\end{aligned}$$

42. Let $u = x^4 + x$. Then $du = (4x^3 + 1) dx$, so

$$\begin{aligned}\int_0^1 (x^4 + x)^5 (4x^3 + 1) dx &= \int_0^2 u^5 du \\ &= \left[\frac{u^6}{6} \right]_0^2 = \frac{2^6}{6} = \frac{32}{3}\end{aligned}$$

43. Let $u = x^3 - x$. Then $du = (3x^2 - 1) dx$, so

$$\begin{aligned}\int_2^3 \frac{3x^2 - 1}{(x^3 - x)^2} dx &= \int_6^{24} \frac{du}{u^2} \\ &= \left[-\frac{1}{u} \right]_6^{24} = -\frac{1}{24} + \frac{1}{6} = \frac{1}{8}\end{aligned}$$

44. The area under the graph of $y = \sin \sqrt{x}$ from 0 to 4 is

$A_1 = \int_0^4 \sin \sqrt{x} dx$. The area under the graph of $y = 2x \sin x$ from 0 to 2 is

$$\begin{aligned}A_2 &= \int_0^2 2x \sin x dx \quad \left[\begin{array}{l} u = x^2, du = 2x dx, \\ \sqrt{u} = x \text{ for } 0 \leq x \leq 2 \end{array} \right] \\ &= \int_0^4 \sin \sqrt{u} du\end{aligned}$$

Since the integration variable is immaterial, $A_1 = A_2$.