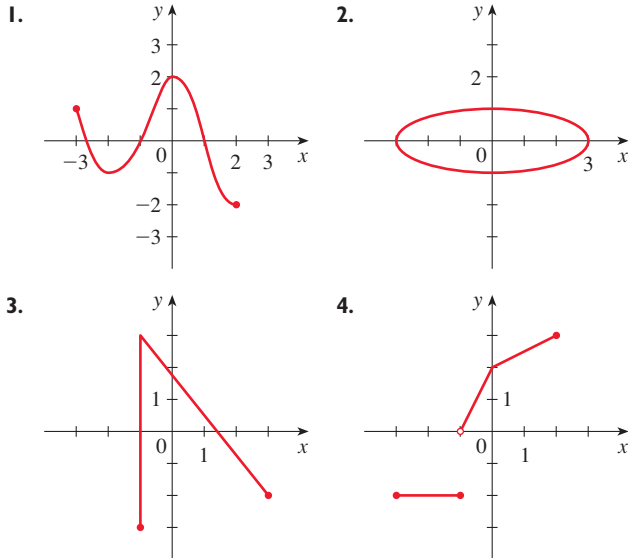


1.1 FUNCTIONS AND THEIR REPRESENTATIONS

A Click here for answers.

1–4 ■ Determine whether the curve is the graph of a function of x . If it is, state the domain and range of the function.



5. Temperature readings T (in $^{\circ}\text{F}$) were recorded every two hours from midnight to noon in Atlanta, Georgia, on March 18, 1996. The time t was measured in hours from midnight.

t	0	2	4	6	8	10	12
T	58	57	53	50	51	57	61

- (a) Use the readings to sketch a rough graph of T as a function of t .
 (b) Use the graph to estimate the temperature at 11 A.M.
6. The population P (in thousands) of San Jose, California, from 1984 to 1994 is shown in the table. (Midyear estimates are given.)

t	1984	1986	1988	1990	1992	1994
P	695	716	733	782	800	817

- (a) Draw a graph of P as a function of time.
 (b) Use the graph to estimate the population in 1991.
7. If $f(x) = 2x^2 + 3x - 4$, find $f(0)$, $f(2)$, $f(\sqrt{2})$, $f(1 + \sqrt{2})$, $f(-x)$, $f(x + 1)$, $2f(x)$, and $f(2x)$.
8. If $g(x) = x^3 + 2x^2 - 3$, find $g(0)$, $g(3)$, $g(-x)$, and $g(1 + h)$.

S Click here for solutions.

- 9–17** ■ Find the domain of the function.

9. $f(x) = \frac{x+2}{x^2-1}$ 10. $f(x) = \frac{x^4}{x^2+x-6}$
 11. $g(x) = \sqrt[4]{x^2-6x}$ 12. $h(x) = \sqrt[4]{7-3x}$
 13. $f(t) = \sqrt[3]{t-1}$ 14. $g(x) = \sqrt{x^2-2x-8}$
 15. $\phi(x) = \sqrt{\frac{x}{\pi-x}}$ 16. $\phi(x) = \sqrt{\frac{x^2-2x}{x-1}}$
 17. $f(t) = \sqrt{t^2+1}$

- 18–36** ■ Find the domain and sketch the graph of the function.

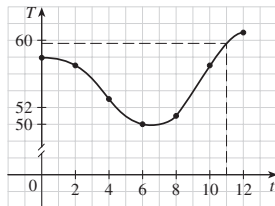
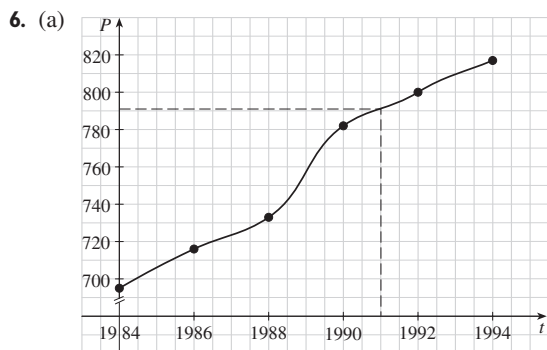
18. $f(x) = 3 - 2x$ 19. $f(x) = x^2 + 2x - 1$
 20. $g(x) = \sqrt{-x}$ 21. $g(x) = \sqrt{6-2x}$
 22. $h(x) = \sqrt{x^2-4}$ 23. $F(x) = \frac{1}{x}$
 24. $G(x) = |x| + x$ 25. $G(x) = |x| - x$
 26. $H(x) = |2x|$ 27. $f(x) = x/|x|$
 28. $H(x) = |2x - 3|$ 29. $f(x) = \frac{x^2-1}{x-1}$

30. $f(x) = \frac{x^2+5x+6}{x+2}$
 31. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$
 32. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x \leq 1 \\ -1 & \text{if } x > 1 \end{cases}$
 33. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$
 34. $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 1 & \text{if } |x| > 1 \end{cases}$
 35. $f(x) = \begin{cases} 1-x^2 & \text{if } x \leq 2 \\ 2x-7 & \text{if } x > 2 \end{cases}$
 36. $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 2 \\ \sqrt{x-2} & \text{if } x > 2 \end{cases}$

1.1 ANSWERS

E Click here for exercises.

1. Yes, $[-3, 2]$, $[-2, 2]$ 2. No
 3. No 4. Yes, $[-3, 2]$, $\{-2\} \cup (0, 3]$
 5. (a)

(b) 59°F 

(b) 791,000

7. $-4, 10, 3\sqrt{2}, 5 + 7\sqrt{2}, 2x^2 - 3x - 4, 2x^2 + 7x + 1, 4x^2 + 6x - 8, 8x^2 + 6x - 4$

8. $-3, 42, -x^3 + 2x^2 - 3, h^3 + 5h^2 + 7h$

9. $\{x \mid x \neq \pm 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

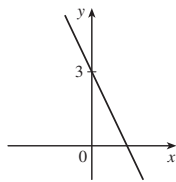
10. $\{x \mid x \neq -3, 2\} = (-\infty, -3] \cup [-3, 2] \cup [2, \infty)$

11. $\{x \mid x \leq 0 \text{ or } x \geq 6\} = (-\infty, 0] \cup [6, \infty)$

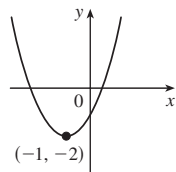
12. $(-\infty, \frac{7}{3}]$ 13. $(-\infty, \infty)$ 14. $(-\infty, -2] \cup [4, \infty)$

15. $[0, \pi)$ 16. $[0, 1) \cup [2, \infty)$ 17. $(-\infty, \infty)$

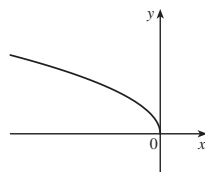
18. $(-\infty, \infty)$



19. $(-\infty, \infty)$

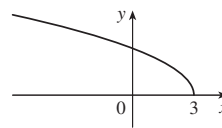


20. $(-\infty, 0]$

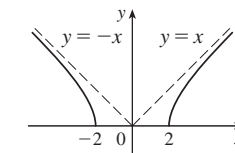


S Click here for solutions.

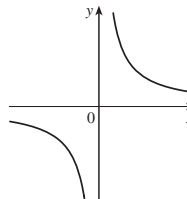
21. $(-\infty, 3]$



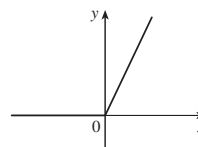
22. $(-\infty, -2] \cup [2, \infty)$



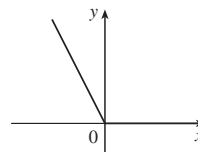
23. $\{x \mid x \neq 0\}$



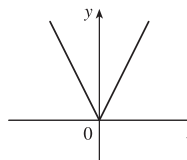
24. $(-\infty, \infty)$



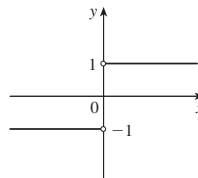
25. $(-\infty, \infty)$



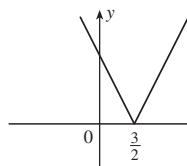
26. $(-\infty, \infty)$



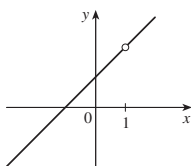
27. $(-\infty, 0) \cup (0, \infty)$



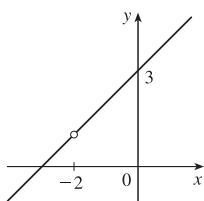
28. $(-\infty, \infty)$



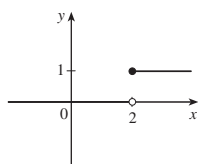
29. $(-\infty, 1) \cup (1, \infty)$



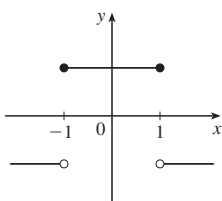
30. $(-\infty, -2) \cup (-2, \infty)$



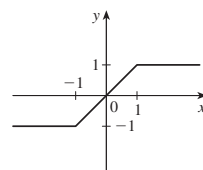
31. $(-\infty, \infty)$



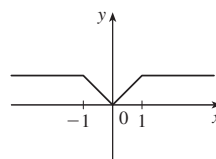
32. $(-\infty, \infty)$



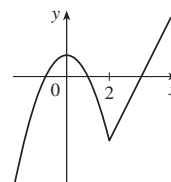
33. $(-\infty, \infty)$



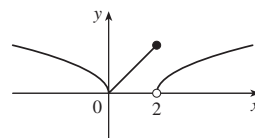
34. $(-\infty, \infty)$



35. $(-\infty, \infty)$



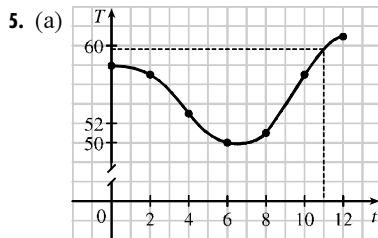
36. $(-\infty, \infty)$



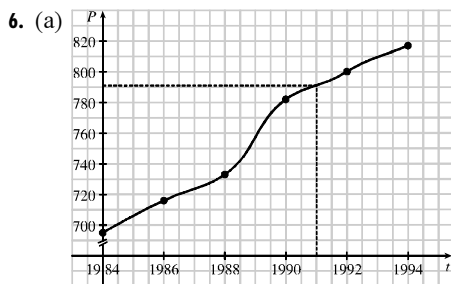
1.1 SOLUTIONS

[Click here for exercises.](#)

- Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-2, 2]$.
- No, the curve is not the graph of a function because a vertical line intersects the curve more than once and hence, the curve fails the Vertical Line Test.
- No, the curve is not the graph of a function since for $x = -1$ there are infinitely many points on the curve.
- Yes, the curve is the graph of a function with domain $[-3, 2]$ and range $\{-2\} \cup (0, 3]$.



(b) $T(11) \approx 59^\circ\text{F}$



(b) $P(1991) \approx 791,000$ people

- $f(x) = 2x^2 + 3x - 4$, so $f(0) = 2(0)^2 + 3(0) - 4 = -4$,
 $f(2) = 2(2)^2 + 3(2) - 4 = 10$,
 $f(\sqrt{2}) = 2(\sqrt{2})^2 + 3(\sqrt{2}) - 4 = 3\sqrt{2}$,
 $f(1 + \sqrt{2}) = 2(1 + \sqrt{2})^2 + 3(1 + \sqrt{2}) - 4$
 $= 2(1 + 2 + 2\sqrt{2}) + 3 + 3\sqrt{2} - 4$
 $= 5 + 7\sqrt{2}$
 $f(-x) = 2(-x)^2 + 3(-x) - 4 = 2x^2 - 3x - 4$,
 $f(x + 1) = 2(x + 1)^2 + 3(x + 1) - 4$
 $= 2(x^2 + 2x + 1) + 3x + 3 - 4$
 $= 2x^2 + 7x + 1$
 $2f(x) = 2(2x^2 + 3x - 4) = 4x^2 + 6x - 8$, and
 $f(2x) = 2(2x)^2 + 3(2x) - 4$
 $= 2(4x^2) + 6x - 4$
 $= 8x^2 + 6x - 4$

- $g(x) = x^3 + 2x^2 - 3$, so $g(0) = 0^3 + 2(0)^2 - 3 = -3$,
 $g(3) = 3^3 + 2(3)^2 - 3 = 42$,
 $g(-x) = (-x)^3 + 2(-x)^2 - 3 = -x^3 + 2x^2 - 3$, and
 $g(1 + h) = (1 + h)^3 + 2(1 + h)^2 - 3 = h^3 + 5h^2 + 7h$.

- $f(x) = \frac{x+2}{x^2-1}$ is defined for all x except when $x^2 - 1 = 0$
 $\Leftrightarrow x = 1$ or $x = -1$, so the domain is $\{x \mid x \neq \pm 1\}$.

- $f(x) = x^4 / (x^2 + x - 6)$ is defined for all x except when
 $0 = x^2 + x - 6 = (x + 3)(x - 2) \Leftrightarrow x = -3$ or 2 , so
the domain is $\{x \mid x \neq -3, 2\}$.

- $g(x) = \sqrt[4]{x^2 - 6x}$ is defined when
 $0 \leq x^2 - 6x = x(x - 6) \Leftrightarrow x \geq 6$ or $x \leq 0$, so the
domain is $(-\infty, 0] \cup [6, \infty)$.

- $h(x) = \sqrt[4]{7 - 3x}$ is defined when $7 - 3x \geq 0$ or $x \leq \frac{7}{3}$, so
the domain is $(-\infty, \frac{7}{3}]$.

- $f(t) = \sqrt[3]{t-1}$ is defined for every t , since every real
number has a cube root. The domain is the set of all real
numbers, $\mathbb{R}$.

- $g(x) = \sqrt{x^2 - 2x - 8}$ is defined when
 $0 \leq x^2 - 2x - 8 = (x - 4)(x + 2) \Leftrightarrow x \geq 4$ or
 $x \leq -2$, so the domain is $(-\infty, -2] \cup [4, \infty)$.

- $\phi(x) = \sqrt{\frac{x}{\pi - x}}$ is defined when $\frac{x}{\pi - x} \geq 0$. So either
 $x \leq 0$ and $\pi - x < 0$ ($\Leftrightarrow x > \pi$), which is impossible, or
 $x \geq 0$ and $\pi - x > 0$ ($\Leftrightarrow x < \pi$), and so the domain is
 $[0, \pi)$.

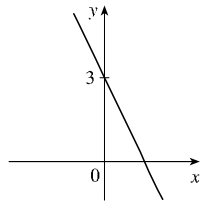
- $\phi(x) = \sqrt{\frac{x^2 - 2x}{x - 1}}$ is defined when
 $0 \leq \frac{x^2 - 2x}{x - 1} = \frac{x(x - 2)}{x - 1}$. Constructing a table:

Interval	x	$x - 1$	$x - 2$	$x(x - 2) / (x - 1)$
$x < 0$	—	—	—	—
$0 < x < 1$	+	—	—	+
$1 < x < 2$	+	+	—	—
$x > 2$	+	+	+	+

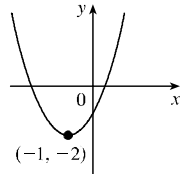
So the domain is $[0, 1) \cup [2, \infty)$.

- $f(t) = \sqrt{t^2 + 1}$ is defined for every t , since $t^2 + 1$ is always
positive. The domain is the set of all real numbers.

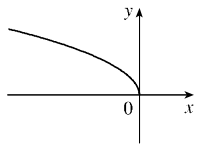
18. $f(x) = 3 - 2x$. Domain is $\mathbb{R}$.



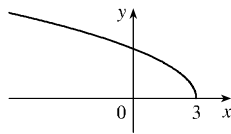
19. $f(x) = x^2 + 2x - 1 = (x^2 + 2x + 1) - 2 = (x + 1)^2 - 2$, so the graph is a parabola with vertex at $(-1, -2)$. The domain is $\mathbb{R}$.



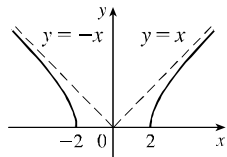
20. $g(x) = \sqrt{-x}$. The domain is $\{x \mid -x \geq 0\} = (-\infty, 0]$.



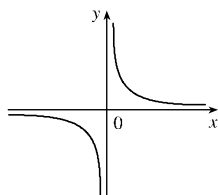
21. $g(x) = \sqrt{6 - 2x}$. The domain is $\{x \mid 6 - 2x \geq 0\} = (-\infty, 3]$.



22. $h(x) = \sqrt{x^2 - 4}$. Now $y = \sqrt{x^2 - 4} \Rightarrow y^2 = x^2 - 4 \Leftrightarrow x^2 - y^2 = 4$, so the graph is the top half of a hyperbola. The domain is $\{x \mid x^2 - 4 \geq 0\} = (-\infty, -2] \cup [2, \infty)$.



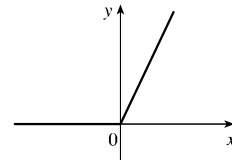
23. $F(x) = \frac{1}{x}$. The domain is $\{x \mid x \neq 0\}$.



24. $G(x) = |x| + x$. Since $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ we have

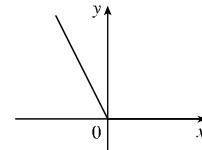
$$G(x) = \begin{cases} x + x & \text{if } x \geq 0 \\ -x + x & \text{if } x < 0 \end{cases} = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Domain is $\mathbb{R}$. Note that the negative x -axis is part of the graph of G .



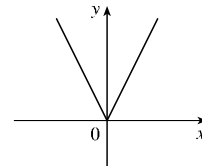
25. $G(x) = |x| - x = \begin{cases} 0 & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$

Domain is $\mathbb{R}$.



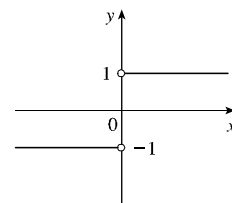
26. $H(x) = |2x| = \begin{cases} 2x & \text{if } 2x \geq 0 \\ -2x & \text{if } 2x < 0 \end{cases} = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$

Domain is $\mathbb{R}$.



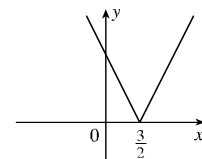
27. $f(x) = \frac{x}{|x|} = \begin{cases} x/x & \text{if } x > 0 \\ x/(-x) & \text{if } x < 0 \end{cases} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

Note that we did not use $x \geq 0$, because $x \neq 0$. Hence, the domain of f is $\{x \mid x \neq 0\}$.

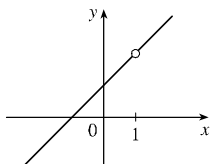


28. $H(x) = |2x - 3| = \begin{cases} 2x - 3 & \text{if } x \geq \frac{3}{2} \\ 3 - 2x & \text{if } x < \frac{3}{2} \end{cases}$

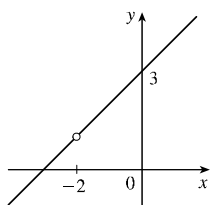
Domain is $\mathbb{R}$.



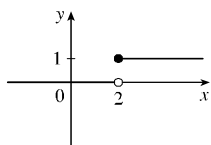
29. $f(x) = \frac{x^2 - 1}{x - 1} = \frac{(x + 1)(x - 1)}{x - 1}$, so for $x \neq 1$,
 $f(x) = x + 1$. Domain is $\{x \mid x \neq 1\}$.



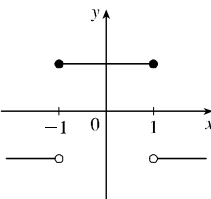
30. $f(x) = \frac{x^2 + 5x + 6}{x + 2} = \frac{(x + 3)(x + 2)}{x + 2}$, so for $x \neq -2$,
 $f(x) = x + 3$. Domain is $\{x \mid x \neq -2\}$. The hole in the graph can be found using the simplified function,
 $h(x) = x + 3$. $h(-2) = 1$ indicates that the hole has coordinates $(-2, 1)$.



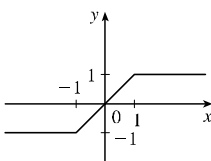
31. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$ Domain is $\mathbb{R}$.



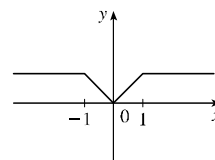
32. $f(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 1 \\ -1 & \text{if } x > 1 \text{ or } x < -1 \end{cases}$
Domain is $\mathbb{R}$.



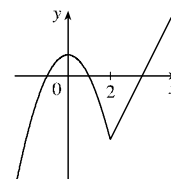
33. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$ Domain is $\mathbb{R}$.



34. $f(x) = \begin{cases} |x| & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \text{ or } x < -1 \end{cases}$ Domain is $\mathbb{R}$.



35. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 2 \\ 2x - 7 & \text{if } x > 2 \end{cases}$ Domain is $\mathbb{R}$.



36. $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 2 \\ \sqrt{x - 2} & \text{if } x > 2 \end{cases}$ Domain is $\mathbb{R}$.

