1.4 CALCULATING LIMITS

A Click here for answers.

S Click here for solutions.

1–5 ■ Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

1. $\lim_{x\to 4} (5x^2 - 2x + 3)$

2. $\lim_{x\to 3} (x^3 + 2)(x^2 - 5x)$

3. $\lim_{x\to -1} \dfrac{x-2}{x^2+4x-3}$

4. $\lim_{x\to 1} \left(\dfrac{x^4+x^2-6}{x^4+2x+3}\right)^2$

5. $\lim_{t\to -2} (t+1)^9(t^2-1)$

6–20 ■ Evaluate the limit, if it exists.

6. $\lim_{x\to -3} \dfrac{x^2-x+12}{x+3}$

7. $\lim_{x\to -3} \dfrac{x^2-x-12}{x+3}$

8. $\lim_{x\to -2} \dfrac{x+2}{x^2-x-6}$

9. $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-3x+2}$

10. $\lim_{h\to 0} \dfrac{(h-5)^2-25}{h}$

11. $\lim_{x\to -1} \dfrac{x^2-x-2}{x+1}$

12. $\lim_{x\to 1} \dfrac{x^2-x-2}{x+1}$

13. $\lim_{t\to 1} \dfrac{t^3-t}{t^2-1}$

14. $\lim_{x\to -1} \dfrac{x^2-x-3}{x+1}$

15. $\lim_{t\to 2} \dfrac{t^2+t-6}{t^2-4}$

16. $\lim_{t\to 0} \dfrac{\sqrt{2-t}-\sqrt{2}}{t}$

17. $\lim_{x\to 1} \left[\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right]$

18. $\lim_{x\to 2} \dfrac{\dfrac{1}{x}-\dfrac{1}{2}}{x-2}$

19. $\lim_{x\to 0} \dfrac{x}{\sqrt{1+3x}-1}$

20. $\lim_{x\to 2} \dfrac{x-\sqrt{3x-2}}{x^2-4}$

21. Prove that $\lim_{x\to 0^+} \sqrt{x}\cos^4 x = 0$.

22. Let

$$f(x) = \begin{cases} x^2 - 2x + 2 & \text{if } x < 1 \\ 3 - x & \text{if } x \geqslant 1 \end{cases}$$

(a) Find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$.
(b) Does $\lim_{x\to 1} f(x)$ exist?
(c) Sketch the graph of f.

23. Let

$$g(x) = \begin{cases} -x^3 & \text{if } x < -1 \\ (x+2)^2 & \text{if } x > -1 \end{cases}$$

(a) Find $\lim_{x\to 1^-} g(x)$ and $\lim_{x\to -1^+} g(x)$.
(b) Does $\lim_{x\to -1} g(x)$ exist?
(c) Sketch the graph of g.

24. Let $g(x) = [\![x/2]\!]$.
(a) Sketch the graph of g.
(b) Evaluate each of the following limits if it exists.

(i) $\lim_{x\to 1^+} g(x)$ (ii) $\lim_{x\to 1^-} g(x)$ (iii) $\lim_{x\to 1} g(x)$

(iv) $\lim_{x\to 2^+} g(x)$ (v) $\lim_{x\to 2^-} g(x)$ (vi) $\lim_{x\to 2} g(x)$

(b) For what values of a does $\lim_{x\to a} g(x)$ exist?

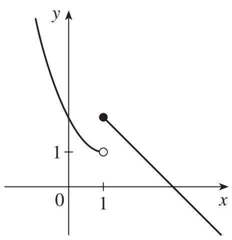
1.4 ANSWERS

E Click here for exercises.

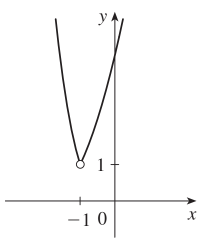
S Click here for solutions.

1. 75 **2.** -174 **3.** $\frac{1}{2}$ **4.** $\frac{4}{9}$ **5.** -3
6. Does not exist **7.** -7 **8.** $-\frac{1}{5}$ **9.** -3 **10.** -10
11. -3 **12.** -1 **13.** 1 **14.** Does not exist **15.** $\frac{5}{4}$
16. $-\sqrt{2}/4$ **17.** $\frac{1}{2}$ **18.** $-\frac{1}{4}$ **19.** $\frac{2}{3}$ **20.** $\frac{1}{16}$ **21.** 0

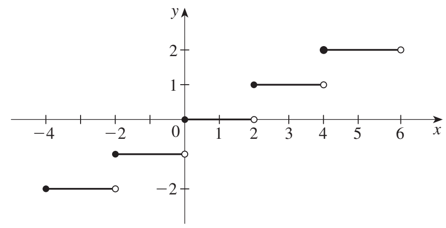
22. (a) 1, 2 (b) No (c)



23. (a) 1, 1 (b) Yes (c)



24. (a)



(b) (i) 0 (ii) 0 (iii) 0 (iv) 1 (v) 0
(vi) Does not exist
(c) All values except even integers

1.4 SOLUTIONS

E Click here for exercises.

1. $\lim\limits_{x\to 4}(5x^2-2x+3)$

$= \lim\limits_{x\to 4}5x^2 - \lim\limits_{x\to 4}2x + \lim\limits_{x\to 4}3$ (Limit Laws 2 & 1)

$= 5\lim\limits_{x\to 4}x^2 - 2\lim\limits_{x\to 4}x + 3$ (3 & 7)

$= 5(4)^2 - 2(4) + 3 = 75$ (9 & 8)

2. $\lim\limits_{x\to 3}(x^3+2)(x^2-5x)$

$= \lim\limits_{x\to 3}(x^3+2)\lim\limits_{x\to 3}(x^2-5x)$ (Limit Law 4)

$= \left(\lim\limits_{x\to 3}x^3 + \lim\limits_{x\to 3}2\right)\left(\lim\limits_{x\to 3}x^2 - 5\lim\limits_{x\to 3}x\right)$ (1, 2 & 3)

$= (3^3+2)(3^2-5\cdot 3)$ (9, 7 & 8)

$= 29(-6) = -174$

3. $\lim\limits_{x\to -1}\dfrac{x-2}{x^2+4x-3}$

$= \dfrac{\lim\limits_{x\to -1}(x-2)}{\lim\limits_{x\to -1}(x^2+4x-3)}$ (Limit Law 5)

$= \dfrac{\lim\limits_{x\to -1}x - \lim\limits_{x\to -1}2}{\lim\limits_{x\to -1}x^2 + 4\lim\limits_{x\to -1}x - \lim\limits_{x\to -1}3}$ (2, 1 & 3)

$= \dfrac{(-1)-2}{(-1)^2+4(-1)-3} = \dfrac{1}{2}$ (8, 7 & 9)

4. $\lim\limits_{x\to 1}\left(\dfrac{x^4+x^2-6}{x^4+2x+3}\right)^2$

$= \left[\dfrac{\lim\limits_{x\to 1}(x^4+x^2-6)}{\lim\limits_{x\to 1}(x^4+2x+3)}\right]^2$ (Limit Laws 6 & 5)

$= \left(\dfrac{\lim\limits_{x\to 1}x^4 + \lim\limits_{x\to 1}x^2 - \lim\limits_{x\to 1}6}{\lim\limits_{x\to 1}x^4 + 2\lim\limits_{x\to 1}x + \lim\limits_{x\to 1}3}\right)^2$ (1, 2 & 3)

$= \left(\dfrac{1^4+1^2-6}{1^4+2\cdot 1+3}\right)^2$ (9, 7 & 8)

$= \left(\frac{-4}{6}\right)^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

5. $\lim\limits_{t\to -2}(t+1)^9(t^2-1)$

$= \lim\limits_{t\to -2}(t+1)^9\lim\limits_{t\to -2}(t^2-1)$ (Limit Law 4)

$= \left[\lim\limits_{t\to -2}(t+1)\right]^9\lim\limits_{t\to -2}(t^2-1)$ (6)

$= \left[\lim\limits_{t\to -2}t + \lim\limits_{t\to -2}1\right]^9\left[\lim\limits_{t\to -2}t^2 - \lim\limits_{t\to -2}1\right]$ (1 & 2)

$= [(-2)+1]^9\left[(-2)^2-1\right] = -3$ (8, 7 & 9)

6. $\lim\limits_{x\to -3}\dfrac{x^2-x+12}{x+3}$ does not exist since $x+3\to 0$ but $x^2-x+12\to 24$ as $x\to -3$.

7. $\lim\limits_{x\to -3}\dfrac{x^2-x-12}{x+3} = \lim\limits_{x\to -3}\dfrac{(x+3)(x-4)}{x+3}$

$= \lim\limits_{x\to -3}(x-4) = -3-4 = -7$

8. $\lim\limits_{x\to -2}\dfrac{x+2}{x^2-x-6} = \lim\limits_{x\to -2}\dfrac{x+2}{(x-3)(x+2)}$

$= \lim\limits_{x\to -2}\dfrac{1}{x-3} = -\dfrac{1}{5}$

9. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-3x+2} = \lim\limits_{x\to 1}\dfrac{(x+2)(x-1)}{(x-2)(x-1)}$

$= \lim\limits_{x\to 1}\dfrac{x+2}{x-2} = \dfrac{1+2}{1-2} = -3$

10. $\lim\limits_{h\to 0}\dfrac{(h-5)^2-25}{h} = \lim\limits_{h\to 0}\dfrac{(h^2-10h+25)-25}{h}$

$= \lim\limits_{h\to 0}\dfrac{h^2-10h}{h} = \lim\limits_{h\to 0}(h-10)$

$= -10$

11. $\lim\limits_{x\to -1}\dfrac{x^2-x-2}{x+1} = \lim\limits_{x\to -1}\dfrac{(x+1)(x-2)}{x+1}$

$= \lim\limits_{x\to -1}(x-2) = -3$

12. $\lim\limits_{x\to 1}\dfrac{x^2-x-2}{x+1} = \dfrac{1^2-1-2}{1+1} = -1$

13. $\lim\limits_{t\to 1}\dfrac{t^3-t}{t^2-1} = \lim\limits_{t\to 1}\dfrac{t(t^2-1)}{t^2-1} = \lim\limits_{t\to 1}t = 1$

14. $\lim\limits_{x\to -1}\dfrac{x^2-x-3}{x+1}$ does not exist since as $x\to -1$, numerator $\to -1$ and denominator $\to 0$.

15. $\lim\limits_{t\to 2}\dfrac{t^2+t-6}{t^2-4} = \lim\limits_{t\to 2}\dfrac{(t+3)(t-2)}{(t+2)(t-2)} = \lim\limits_{t\to 2}\dfrac{t+3}{t+2} = \dfrac{5}{4}$

16. $\lim\limits_{t\to 0}\dfrac{\sqrt{2-t}-\sqrt{2}}{t} = \lim\limits_{t\to 0}\dfrac{\sqrt{2-t}-\sqrt{2}}{t}\cdot\dfrac{\sqrt{2-t}+\sqrt{2}}{\sqrt{2-t}+\sqrt{2}}$

$= \lim\limits_{t\to 0}\dfrac{-t}{t\left(\sqrt{2-t}+\sqrt{2}\right)}$

$= \lim\limits_{t\to 0}\dfrac{-1}{\sqrt{2-t}+\sqrt{2}} = -\dfrac{1}{2\sqrt{2}}$

$= -\dfrac{\sqrt{2}}{4}$

17. $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right) = \lim\limits_{x\to 1}\dfrac{(x+1)-2}{(x-1)(x+1)}$

$= \lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(x+1)}$

$= \lim\limits_{x\to 1}\dfrac{1}{x+1} = \dfrac{1}{2}$

18. $\lim\limits_{x\to 2}\dfrac{1/x-\frac{1}{2}}{x-2} = \lim\limits_{x\to 2}\dfrac{2-x}{2x(x-2)} = \lim\limits_{x\to 2}\dfrac{-1}{2x} = -\dfrac{1}{4}$

19. $\displaystyle \lim_{x\to 0} \frac{x}{\sqrt{1+3x}-1} = \lim_{x\to 0} \frac{x\left(\sqrt{1+3x}+1\right)}{\left(\sqrt{1+3x}-1\right)\left(\sqrt{1+3x}+1\right)}$

$\displaystyle = \lim_{x\to 0} \frac{x\left(\sqrt{1+3x}+1\right)}{3x}$

$\displaystyle = \lim_{x\to 0} \frac{\sqrt{1+3x}+1}{3}$

$\displaystyle = \frac{\sqrt{1}+1}{3} = \frac{2}{3}$

20. $\displaystyle \lim_{x\to 2} \frac{x-\sqrt{3x-2}}{x^2-4} = \lim_{x\to 2} \frac{\left(x-\sqrt{3x-2}\right)\left(x-\sqrt{3x-2}\right)}{\left(x^2-4\right)\left(x-\sqrt{3x-2}\right)}$

$\displaystyle = \lim_{x\to 2} \frac{x^2-3x+2}{\left(x^2-4\right)\left(x+\sqrt{3x-2}\right)}$

$\displaystyle = \lim_{x\to 2} \frac{(x-2)(x-1)}{(x-2)(x+2)\left(x+\sqrt{3x-2}\right)}$

$\displaystyle = \lim_{x\to 2} \frac{(x-1)}{(x+2)\left(x+\sqrt{3x-2}\right)}$

$\displaystyle = \frac{1}{4\left(2+\sqrt{4}\right)} = \frac{1}{16}$

21. $1 \le \cos x \le 1 \;\Rightarrow\; 0 \le \cos^4 x \le 1 \;\Rightarrow$ $0 \le \sqrt{x}\cos^4 x \le \sqrt{x}$. But $\displaystyle \lim_{x\to 0^+} 0 = \lim_{x\to 0^+} \sqrt{x} = 0$. So by the Squeeze Theorem, $\displaystyle \lim_{x\to 0^+} \sqrt{x}\cos^4 x = 0$.

22. (a) $\displaystyle \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(x^2-2x+2\right)$

$\displaystyle = \lim_{x\to 1^-} x^2 - 2\lim_{x\to 1^-} x + \lim_{x\to 1^-} 2$

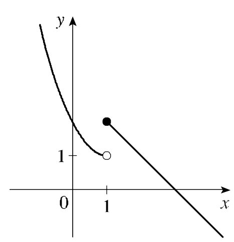
$= 1^2 - 2 + 2 = 1$

$\displaystyle \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3-x) = \lim_{x\to 1^+} 3 - \lim_{x\to 1^+} x$

$= 3 - 1 = 2$

(b) $\displaystyle \lim_{x\to 1} f(x)$ does not exist because $\displaystyle \lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$.

(c)

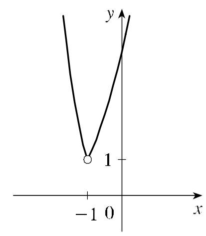


23. (a) $\displaystyle \lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} \left(-x^3\right) = -(-1)^3 = 1,$

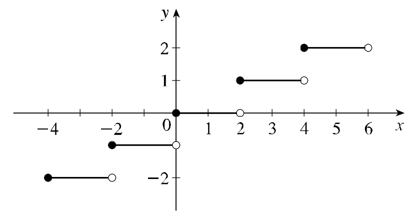
$\displaystyle \lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} (x+2)^2 = (-1+2)^2 = 1$

(b) By part (a), $\displaystyle \lim_{x\to -1} g(x) = 1$.

(c)



24. (a)



(b) (i) $\displaystyle \lim_{x\to 1^+} g(x) = 0$ since $[\![x/2]\!] = 0$ for $0 \le x < 2$.

(ii) $\displaystyle \lim_{x\to 1^-} g(x) = 0$ since $[\![x/2]\!] = 0$ for $0 \le x < 2$.

(iii) $\displaystyle \lim_{x\to 1} g(x) = 0$ since $[\![x/2]\!] = 0$ for $0 \le x < 2$.

(iv) $\displaystyle \lim_{x\to 2^+} g(x) = 1$ since $[\![x/2]\!] = 1$ for $2 \le x < 4$.

(v) $\displaystyle \lim_{x\to 2^-} g(x) = 0$ since $[\![x/2]\!] = 0$ for $0 \le x < 2$.

(vi) $\displaystyle \lim_{x\to 2} g(x)$ does not exist because $\displaystyle \lim_{x\to 2^+} g(x) \ne \lim_{x\to 2^-} g(x)$.

(c) $\displaystyle \lim_{x\to a} g(x)$ exists except when a is an even integer.