2.1 DERIVATIVES AND RATES OF CHANGE

A Click here for answers. S Click here for solutions.

1. The point $P(4, 2)$ lies on the curve $y = \sqrt{x}$.
(a) If Q is the point $(x, \sqrt{x})$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:
(i) 5 (ii) 4.5 (iii) 4.1
(iv) 4.01 (v) 4.001 (vi) 3
(vii) 3.5 (viii) 3.9 (ix) 3.99
(x) 3.999
(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(4, 2)$.
(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(4, 2)$.

2. The point $P(0.5, 2)$ lies on the curve $y = 1/x$.
(a) If Q is the point $(x, 1/x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:
(i) 2 (ii) 1 (iii) 0.9
(iv) 0.8 (v) 0.7 (vi) 0.6
(vii) 0.55 (viii) 0.51 (ix) 0.45
(x) 0.49
(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 2)$.
(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 2)$.
(d) Sketch the curve, two of the secant lines, and the tangent line.

3. The point $P(1, 3)$ lies on the curve $y = 1 + x + x^2$.
(a) If Q is the point $(x, 1 + x + x^2)$, find the slope of the secant line PQ for the following values of x:
(i) 2 (ii) 1.5 (iii) 1.1
(iv) 1.01 (v) 1.001 (vi) 0
(vii) 0.5 (viii) 0.9 (ix) 0.99
(x) 0.999
(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(1, 3)$.
(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(1, 3)$.

4. The point $P(-1, 3)$ lies on the curve $y = 1 - 2x^3$.
(a) If Q is the point $(x, 1 - 2x^3)$, find the slope of the secant line PQ for the following values of x:
(i) -2 (ii) -1.5 (iii) -1.1
(iv) -1.01 (v) -1.001 (vi) 0
(vii) -0.5 (viii) -0.9 (ix) -0.99
(x) -0.999
(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(-1, 3)$.
(c) Using the slope from part (b), find the equation of the tangent line to the curve at $P(-1, 3)$.

5. The displacement (in meters) of a certain particle moving in a straight line is given by $s = t^2 + t$, where t is measured in seconds.
(a) Find the average velocity over the following time periods:
(i) [0, 2] (ii) [0, 1]
(iii) [0, 0.5] (iv) [0, 0.1]
(b) Find the instantaneous velocity when $t = 0$.
(c) Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities found in part (a).
(d) Draw the tangent line whose slope is the instantaneous velocity from part (b).

6. The experimental data in the table define y as a function of x.

x	0	1	2	3	4	5
y	2.6	2.0	1.1	1.3	2.1	3.5

(a) If P is the point (3, 1.3), find the slopes of the secant lines PQ when Q is the point on the graph with $x = 0, 1, 2, 4$, and 5.
(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.
(c) Use a graph of the function to estimate the slope of the tangent line at P.

7–10 ▪ Find an equation of the tangent line to the curve at the given point.

7. $y = 1 - 2x - 3x^2$, $(-2, -7)$

8. $y = 1/\sqrt{x}$, $(1, 1)$

9. $y = 1/x^2$, $\left(-2, \frac{1}{4}\right)$

10. $y = x/(1 - x)$, $(0, 0)$

11. (a) Find the slope of the tangent to the curve $y = 1/\sqrt{5 - 2x}$ at the point where $x = a$.
(b) Find equations of the tangent lines at the points (2, 1) and $\left(-2, \frac{1}{3}\right)$.
(c) Graph the curve and both tangents on a common screen.

12–18 ▪ Find $f'(a)$.

12. $f(x) = 1 + x - 2x^2$

13. $f(x) = x^3 + 3x$

14. $f(x) = \dfrac{x}{2x - 1}$

15. $f(x) = \dfrac{x}{x^2 - 1}$

16. $f(x) = \dfrac{2}{\sqrt{3 - x}}$

17. $f(x) = \sqrt{x - 1}$

18. $f(x) = \sqrt{3x + 1}$

19–24 ▪ Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

19. $\displaystyle\lim_{h \to 0} \frac{\sqrt{1 + h} - 1}{h}$

20. $\displaystyle\lim_{h \to 0} \frac{(2 + h)^3 - 8}{h}$

21. $\displaystyle\lim_{x \to 1} \frac{x^9 - 1}{x - 1}$

22. $\displaystyle\lim_{x \to 3\pi} \frac{\cos x + 1}{x - 3\pi}$

23. $\displaystyle\lim_{t \to 0} \frac{\sin\left(\frac{\pi}{2} + t\right) - 1}{t}$

24. $\displaystyle\lim_{x \to 0} \frac{3^x - 1}{x}$

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25. A function f is given by the data in the table. Find approximate values for $f'(x)$ when $x = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$, and 0.7.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	5.0	4.1	4.0	4.6	5.5	6.2	6.5	6.1	4.7

26. Let $C(t)$ be the amount of U.S. cash per capita in circulation at time t. The table, supplied by the Treasury Department, gives values of $C(t)$ as of June 30 of the specified year. Interpret and estimate the value of $C'(1980)$.

t	1960	1970	1980	1990
$C(t)$	\$177	\$265	\$571	\$1063

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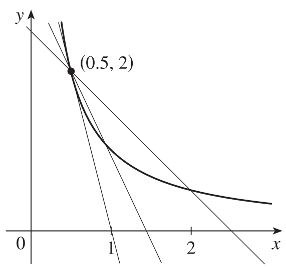
2.1 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. (a) (i) 0.236068 (ii) 0.242641 (iii) 0.248457 (iv) 0.249844 (v) 0.249984 (vi) 0.267949 (vii) 0.258343 (viii) 0.251582 (ix) 0.250156 (x) 0.250016
(b) $\frac{1}{4}$ (c) $y = \frac{1}{4}x + 1$

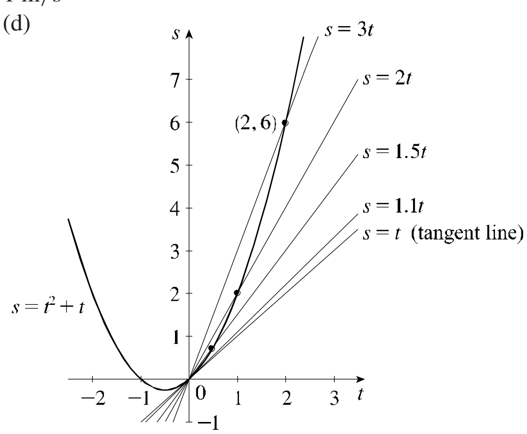
2. (a) (i) -1 (ii) -2 (iii) -2.222222 (iv) -2.5 (v) -2.857143 (vi) -3.333333 (vii) -3.636364 (viii) -3.921569 (ix) -4.444444 (x) -4.081633
(b) -4 (c) $y = -4x + 4$
(d)



3. (a) (i) 4 (ii) 3.5 (iii) 3.1 (iv) 3.01 (v) 3.001 (vi) 2 (vii) 2.5 (viii) 2.9 (ix) 2.99 (x) 2.999
(b) 3 (c) $y = 3x$

4. (a) (i) -14 (ii) -9.5 (iii) -6.62 (iv) -6.0602 (v) -6.006002 (vi) -2 (vii) -3.5 (viii) -5.42 (ix) -5.9402 (x) -5.994002
(b) -6 (c) $y = -6x - 3$

5. (a) (i) 3 m/s (ii) 2 m/s (iii) 1.5 m/s (iv) 1.1 m/s
(b) 1 m/s
(c), (d)

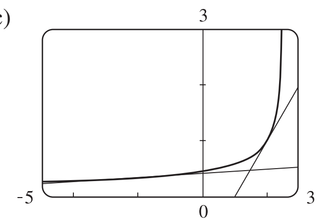


6. (a) $-0.43, -0.35, 0.2, 0.8, 1.1$ (b) 0.5 (c) 0.57

7. $y = 10x + 13$ **8.** $y = -\frac{1}{2}x + \frac{3}{2}$

9. $y = \frac{1}{4}x + \frac{3}{4}$ **10.** $y = x$

11. (a) $(5 - 2a)^{-3/2}$ (b) $y = x - 1,\ y = \frac{1}{27}x + \frac{11}{27}$
(c)



12. $1 - 4a$ **13.** $\dfrac{1}{2\sqrt{a - 1}}$ **14.** $-1/(2a - 1)^2$

15. $-\dfrac{a^2 + 1}{(a^2 - 1)^2}$ **16.** $1/(3 - a)^{3/2}$ **17.** $\dfrac{1}{2\sqrt{a - 1}}$

18. $\dfrac{3}{2\sqrt{3a + 1}}$ **19.** $f(x) = \sqrt{x},\ a = 1$

20. $f(x) = x^3,\ a = 2$ **21.** $f(x) = x^9,\ a = 1$

22. $f(x) = \cos x,\ a = 3\pi$ **23.** $f(x) = \sin x,\ a = \pi/2$

24. $f(x) = 3^x,\ a = 0$ **25.** $-5, 4, 8, 9, 5, -0.5, -8$

26. The rate at which the cash per capita in circulation is changing in dollars per year; \$39.90/year

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2.1 SOLUTIONS

E Click here for exercises.

1. For the curve $y = \sqrt{x}$ and the point $P(4, 2)$:

(a)

	x	Q	m_{PQ}
(i)	5	$(5, 2.236068)$	0.236068
(ii)	4.5	$(4.5, 2.121320)$	0.242641
(iii)	4.1	$(4.1, 2.024846)$	0.248457
(iv)	4.01	$(4.01, 2.002498)$	0.249844
(v)	4.001	$(4.001, 2.000250)$	0.249984
(vi)	3	$(3, 1.732051)$	0.267949
(vii)	3.5	$(3.5, 1.870829)$	0.258343
(viii)	3.9	$(3.9, 1.974842)$	0.251582
(ix)	3.99	$(3.99, 1.997498)$	0.250156
(x)	3.999	$(3.999, 1.999750)$	0.250016

(b) The slope appears to be $\frac{1}{4}$.

(c) $y - 2 = \frac{1}{4}(x - 4)$ or $y = \frac{1}{4}x + 1$.

2. For the curve $y = 1/x$ and the point $P(0.5, 2)$:

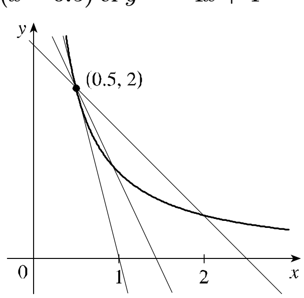
(a)

	x	Q	m_{PQ}
(i)	2	$(2, 0.5)$	-1
(ii)	1	$(1, 1)$	-2
(iii)	0.9	$(0.9, 1.111111)$	-2.222222
(iv)	0.8	$(0.8, 1.25)$	-2.5
(v)	0.7	$(0.7, 1.428571)$	-2.857143
(vi)	0.6	$(0.6, 1.666667)$	-3.333333
(vii)	0.55	$(0.55, 1.818182)$	-3.636364
(viii)	0.51	$(0.51, 1.960784)$	-3.921569
(ix)	0.45	$(0.45, 2.222222)$	-4.444444
(x)	0.49	$(0.49, 2.040816)$	-4.081633

(b) The slope appears to be -4.

(c) $y - 2 = -4(x - 0.5)$ or $y = -4x + 4$

(d)



3. For the curve $f(x) = 1 + x + x^2$ and the point $P(1, 3)$:

(a)

	x	Q	m_{PQ}
(i)	2	$(2, 7)$	4
(ii)	1.5	$(1.5, 4.75)$	3.5
(iii)	1.1	$(1.1, 3.31)$	3.1
(iv)	1.01	$(1.01, 3.0301)$	3.01
(v)	1.001	$(1.001, 3.003001)$	3.001
(vi)	0	$(0, 1)$	2
(vii)	0.5	$(0.5, 1.75)$	2.5
(viii)	0.9	$(0.9, 2.71)$	2.9
(ix)	0.99	$(0.99, 2.9701)$	2.99
(x)	0.999	$(0.999, 2.997001)$	2.999

(b) The slope appears to be 3.

(c) $y - 3 = 3(x - 1)$ or $y = 3x$

4. For the curve $y = 1 - 2x^3$ and the point $P(-1, 3)$:

(a)

	x	Q	m_{PQ}
(i)	-2	$(-2, 17)$	-14
(ii)	-1.5	$(-1.5, 7.75)$	-9.5
(iii)	-1.1	$(-1.1, 3.662)$	-6.62
(iv)	-1.01	$(-1.01, 3.060602)$	-6.0602
(v)	-1.001	$(-1.001, 3.006006)$	-6.006002
(vi)	0	$(0, 1)$	-2
(vii)	-0.5	$(-0.5, 1.25)$	-3.5
(viii)	-0.9	$(-0.9, 2.458)$	-5.42
(ix)	-0.99	$(-0.99, 2.940598)$	-5.9402
(x)	-0.999	$(-0.999, 2.994006)$	-5.994002

(b) The slope appears to be -6.

(c) $y - 3 = -6(x + 1)$ or $6x + y + 3 = 0$

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5. (a) The average velocity between times 0 and h is

$$\frac{s(h)-s(0)}{h} = \frac{h^2+h-0}{h} = h+1.$$

(i) $[0,2]$: $2+1=3$ m/s

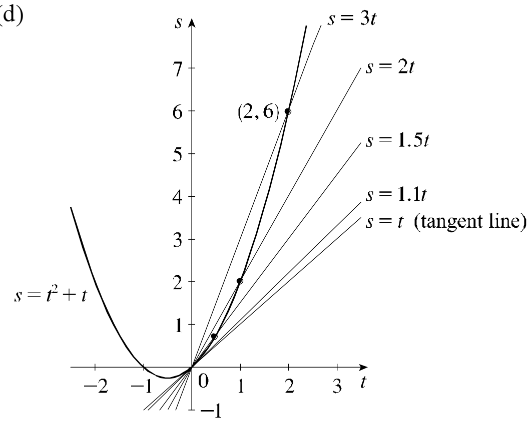
(ii) $[0,1]$: $1+1=2$ m/s

(iii) $[0,0.5]$: $0.5+1=1.5$ m/s

(iv) $[0,0.1]$: $0.1+1=1.1$ m/s

(b) As h approaches 0, the velocity approaches 1 m/s.

(c), (d)

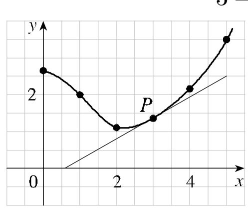


6. (a) Slopes of the secant lines:

x	m_{PQ}
0	$\frac{2.6-1.3}{0-3} \approx -0.43$
1	$\frac{2.0-1.3}{1-3} = -0.35$
2	$\frac{1.1-1.3}{2-3} = 0.2$
4	$\frac{2.1-1.3}{4-3} = 0.8$
5	$\frac{3.5-1.3}{5-3} = 1.1$

(b) We average the slopes of the two closest secant lines from part (a): $\frac{1}{2}(0.2+0.8)=0.5$.

(c) Using the points $(0.6, 0)$ and $(5, 2.5)$ from the graph, the slope of the tangent line at P is about $\frac{2.5-0}{5-0.6} \approx 0.57$.



7. Using (1),

$$\begin{aligned} m &= \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \\ &= \lim_{x\to -2}\frac{\left(1-2x-3x^2\right)-(-7)}{x-(-2)} \\ &= \lim_{x\to -2}\frac{-3x^2-2x+8}{x+2} \\ &= \lim_{x\to -2}\frac{(-3x+4)(x+2)}{x+2} \\ &= \lim_{x\to -2}(-3x+4) = 10 \end{aligned}$$

Thus, an equation of the tangent is $y+7=10(x+2)$ or $y=10x+13$.

Alternate Solution: Using (2),

$$\begin{aligned} m &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\ &= \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h} \\ &= \lim_{h\to 0}\frac{\left[1-2(-2+h)-3(-2+h)^2\right]-(-7)}{h} \\ &= \lim_{h\to 0}\frac{\left(-3h^2+10h-7\right)+7}{h} \\ &= \lim_{h\to 0}\frac{h(-3h+10)}{h} \\ &= \lim_{h\to 0}(-3h+10) = 10 \end{aligned}$$

8. Using (1),

$$\begin{aligned} m &= \lim_{x\to 1}\frac{1/\sqrt{x}-1}{x-1} = \lim_{x\to 1}\frac{-\left(\sqrt{x}-1\right)}{\sqrt{x}\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)} \\ &= \lim_{x\to 1}\frac{-1}{\sqrt{x}\left(\sqrt{x}+1\right)} = -\frac{1}{2} \end{aligned}$$

Thus, an equation of the tangent line is $y-1=-\frac{1}{2}(x-1)$ or $y=-\frac{1}{2}x+\frac{3}{2}$.

9. Using (1),

$$\begin{aligned} m &= \lim_{x\to -2}\frac{1/x^2-\frac{1}{4}}{x-(-2)} = \lim_{x\to -2}\frac{4-x^2}{4x^2(x+2)} \\ &= \lim_{x\to -2}\frac{(2-x)(2+x)}{4x^2(x+2)} = \lim_{x\to -2}\frac{2-x}{4x^2} = \frac{1}{4} \end{aligned}$$

Thus, an equation of the tangent line is $y-\frac{1}{4}=\frac{1}{4}(x+2)$ $\Rightarrow$ $y=\frac{1}{4}x+\frac{3}{4}$.

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10. Using (1),

$$m = \lim_{x\to 0} \frac{x/(1-x) - 0}{x - 0} = \lim_{x\to 0} \frac{x}{x(1-x)}$$

$$= \lim_{x\to 0} \frac{1}{1-x} = 1$$

Thus, an equation of the tangent line is $y - 0 = 1(x - 0)$ $\Rightarrow$ $y = x$.

11. (a) Using (1),

$$m = \lim_{x\to a} \frac{\dfrac{1}{\sqrt{5-2x}} - \dfrac{1}{\sqrt{5-2a}}}{x-a}$$

$$= \lim_{x\to a} \frac{\sqrt{5-2a} - \sqrt{5-2x}}{(x-a)\sqrt{5-2x}\sqrt{5-2a}}$$

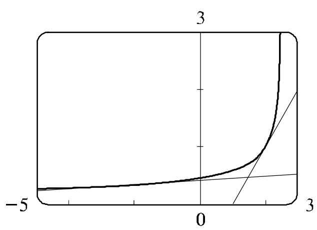
$$= \lim_{x\to a} \frac{2(x-a)}{(x-a)\sqrt{(5-2x)(5-2a)}\left(\sqrt{5-2a} + \sqrt{5-2x}\right)}$$

$$= \lim_{x\to a} \frac{2}{(x-a)\sqrt{(5-2x)(5-2a)}\left(\sqrt{5-2a} + \sqrt{5-2x}\right)}$$

$$= \frac{2}{2(5-2a)^{3/2}} = (5-2a)^{-3/2}$$

(b) At $(2, 1)$: $m = [5 - 2(2)]^{-3/2} = 1$ $\Leftrightarrow$ $y - 1 = 1(x - 2)$ $\Leftrightarrow$ $y = x - 1$.
At $\left(-2, \frac{1}{3}\right)$: $m = [5 - 2(-2)]^{-3/2} = \frac{1}{27}$ $\Leftrightarrow$ $y - \frac{1}{3} = \frac{1}{27}[x - (-2)]$ $\Leftrightarrow$ $y = \frac{1}{27}x + \frac{11}{27}$.

(c)



12.
$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$
$$= \lim_{h\to 0} \frac{1 + (a+h) - 2(a+h)^2 - \left(1 + a - 2a^2\right)}{h}$$
$$= \lim_{h\to 0} \frac{h - 4ah - 2h^2}{h} = \lim_{h\to 0} (1 - 4a - 2h)$$
$$= 1 - 4a$$

13.
$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$
$$= \lim_{h\to 0} \frac{(a+h)^3 + 3(a+h) - \left(a^3 + 3a\right)}{h}$$
$$= \lim_{h\to 0} \frac{3a^2h + 3ah^2 + h^3 + 3h}{h}$$
$$= \lim_{h\to 0} \left(3a^2 + 3ah + h^2 + 3\right)$$
$$= 3a^2 + 3 \frac{1}{\sqrt{a+h-1} + \sqrt{a-1}}$$
$$= \frac{1}{\sqrt{a-1} + \sqrt{a-1}} = \frac{1}{2\sqrt{a-1}}$$

14.
$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$
$$= \lim_{h\to 0} \frac{\dfrac{a+h}{2(a+h)-1} - \dfrac{a}{2a-1}}{h}$$
$$= \lim_{h\to 0} \frac{(a+h)(2a-1) - a(2a+2h-1)}{h(2a+2h-1)(2a-1)}$$
$$= \lim_{h\to 0} \frac{-h}{h(2a+2h-1)(2a-1)}$$
$$= \lim_{h\to 0} \frac{-1}{(2a+2h-1)(2a-1)}$$
$$= -\frac{1}{(2a-1)^2}$$

15.
$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$
$$= \lim_{h\to 0} \frac{\dfrac{a+h}{(a+h)^2 - 1} - \dfrac{a}{a^2 - 1}}{h}$$
$$= \lim_{h\to 0} \frac{(a+h)\left(a^2 - 1\right) - a\left(a^2 + 2ah + h^2 - 1\right)}{h\left(a^2 - 1\right)\left(a^2 + 2ah + h^2 - 1\right)}$$
$$= \lim_{h\to 0} \frac{h\left(-a^2 - 1 - ah\right)}{h\left(a^2 - 1\right)\left(a^2 + 2ah + h^2 - 1\right)}$$
$$= \lim_{h\to 0} \frac{-a^2 - 1 - ah}{\left(a^2 - 1\right)\left(a^2 + 2ah + h^2 - 1\right)}$$
$$= \frac{-a^2 - 1}{\left(a^2 - 1\right)\left(a^2 - 1\right)} = -\frac{a^2 + 1}{\left(a^2 - 1\right)^2}$$

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16.

$$\begin{aligned} f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\ &= \lim_{h\to 0}\frac{\dfrac{2}{\sqrt{3-(a+h)}}-\dfrac{2}{\sqrt{3-a}}}{h} \\ &= \lim_{h\to 0}\frac{2\left(\sqrt{3-a}-\sqrt{3-a-h}\right)}{h\sqrt{3-a-h}\sqrt{3-a}} \\ &= \lim_{h\to 0}\frac{2\left(\sqrt{3-a}-\sqrt{3-a-h}\right)}{h\sqrt{3-a-h}\sqrt{3-a}}\cdot\frac{\sqrt{3-a}+\sqrt{3-a-h}}{\sqrt{3-a}+\sqrt{3-a-h}} \\ &= \lim_{h\to 0}\frac{2\left[3-a-(3-a-h)\right]}{h\sqrt{3-a-h}\sqrt{3-a}\left(\sqrt{3-a}+\sqrt{3-a-h}\right)} \\ &= \lim_{h\to 0}\frac{2}{\sqrt{3-a-h}\sqrt{3-a}\left(\sqrt{3-a}+\sqrt{3-a-h}\right)} \\ &= \frac{2}{\sqrt{3-a}\sqrt{3-a}\left(2\sqrt{3-a}\right)}=\frac{1}{(3-a)^{3/2}} \end{aligned}$$

17.

$$\begin{aligned} f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\ &= \lim\frac{\sqrt{a+h-1}-\sqrt{a-1}}{h} \\ &= \lim_{h\to 0}\frac{\sqrt{a+h-1}-\sqrt{a-1}}{h}\cdot\frac{\sqrt{a+h-1}+\sqrt{a-1}}{\sqrt{a+h-1}+\sqrt{a-1}} \\ &= \lim_{h\to 0}\frac{(a+h-1)-(a-1)}{h\left(\sqrt{a+h-1}+\sqrt{a-1}\right)} \\ &= \lim_{h\to 0}\frac{1}{\sqrt{a+h-1}+\sqrt{a-1}} \\ &= \frac{1}{\sqrt{a-1}+\sqrt{a-1}}=\frac{1}{2\sqrt{a-1}} \end{aligned}$$

18.

$$\begin{aligned} f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\ &= \lim_{h\to 0}\frac{\sqrt{3(a+h)+1}-\sqrt{3a+1}}{h} \\ &= \lim_{h\to 0}\frac{\left(\sqrt{3a+3h+1}-\sqrt{3a+1}\right)\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)}{h\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)} \\ &= \lim_{h\to 0}\frac{(3a+3h+1)-(3a+1)}{h\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)} \\ &= \lim_{h\to 0}\frac{3}{\sqrt{3a+3h+1}+\sqrt{3a+1}}=\frac{3}{2\sqrt{3a+1}} \end{aligned}$$

19. By Equation 1, $\lim\limits_{h\to 0}\dfrac{\sqrt{1+h}-1}{h}=f'(1)$, where $f(x)=\sqrt{x}$.
Or: $f'(0)$, where $f(x)=\sqrt{1+x}$

20. By Equation 2, $\lim\limits_{h\to 0}\dfrac{(2+h)^3-8}{h}=f'(2)$, where $f(x)=x^3$.

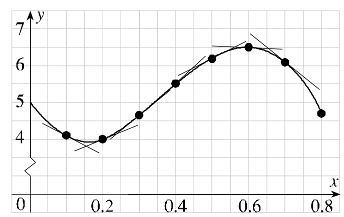
21. By Equation 5, $\lim\limits_{x\to 1}\dfrac{x^9-1}{x-1}=f'(1)$, where $f(x)=x^9$.

22. By Equation 5, $\lim\limits_{x\to 3\pi}\dfrac{\cos x+1}{x-3\pi}=f'(3\pi)$, where $f(x)=\cos x$.

23. By Equation 2, $\lim\limits_{t\to 0}\dfrac{\sin\left(\frac{\pi}{2}+t\right)-1}{t}=f'\left(\frac{\pi}{2}\right)$, where $f(x)=\sin x$.

24. By Equation 5, $\lim\limits_{x\to 0}\dfrac{3^x-1}{x}=f'(0)$, where $f(x)=3^x$.

25. We plot the points given by the data in the table, then sketch the rough shape of the curve. To estimate the derivative $f'(x)$, we draw the tangent line to the curve at x. It appears that $f'(0.1)\approx -5$, $f'(0.2)\approx 4$, $f'(0.3)\approx 8$, $f'(0.4)\approx 9$, $f'(0.5)\approx 5$, $f'(0.6)\approx -0.5$, and $f'(0.7)\approx -8$.



26. $C'(1980)$ is the rate of change of U.S. cash per capita in circulation with respect to time. To estimate the value of $C'(1980)$, we will average the difference quotients obtained using the years 1970 and 1990.

Let $A=\dfrac{C(1970)-C(1980)}{1970-1980}=\dfrac{265-571}{-10}=30.6$ and $B=\dfrac{C(1990)-C(1980)}{1990-1980}=\dfrac{1063-571}{10}=49.2$. Then

$$\begin{aligned} C'(1980) &= \lim_{t\to 1980}\frac{C(t)-C(1980)}{t-1980} \\ &\approx \frac{A+B}{2}=39.9 \text{ dollars per year} \end{aligned}$$

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