2.3 BASIC DIFFERENTIATION FORMULAS

A Click here for answers.

S Click here for solutions.

1–11 ■ Differentiate the function.

1. $f(x) = x^2 - 10x + 100$

2. $g(x) = x^{100} + 50x + 1$

3. $s(t) = t^3 - 3t^2 + 12t$

4. $F(x) = (16x)^3$

5. $H(s) = (s/2)^5$

6. $y = \sqrt{5x}$

7. $y = x^{4/3} - x^{2/3}$

8. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$

9. $y = x + \sqrt[5]{x^2}$

10. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}}$

11–12 ■ Find $f'(x)$. Compare the graphs of f and f' and use them to explain why your answer is reasonable.

11. $f(x) = 2x^2 - x^4$

12. $f(x) = x - 3x^{1/3}$

13. (a) By zooming in on the graph of $f(x) = x^{2/5}$, estimate the value of $f'(2)$.
(b) Use the Power Rule to find the exact value of $f'(2)$ and compare with your estimate in part (a).

14–16 ■ Find an equation of the tangent line to the curve at the given point. Illustrate by graphing the curve and the tangent line on the same screen.

14. $y = x + \dfrac{4}{x}$, $(2, 4)$

15. $y = x^{5/2}$, $(4, 32)$

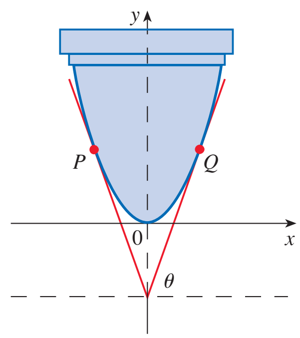
16. $y = x + \sqrt{x}$, $(1, 2)$

17. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

18. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

19. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

20. A manufacturer of cartridges for stereo systems has designed a stylus with parabolic cross-section as shown in the figure. The equation of a parabola is $y = 16x^2$, where x and y are measured in millimeters. If the stylus sits in a record groove whose sides make an angle of θ with the horizontal direction, where $\tan\theta = 1.75$, find the points of contact P and Q of the stylus with the groove.



21. The **normal line** to a curve C at a point P is, by definition, the line that passes through P and is perpendicular to the tangent line to C at P. Find an equation of the normal line to the curve $y = \sqrt[3]{x}$ at the point $(-8, -2)$. Sketch the curve and its normal line.

22. At what point on the curve $y = x^4$ does the normal line have slope 16?

2.3 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $f'(x) = 2x - 10$ **2.** $g'(x) = 100x^{99} + 50$

3. $s'(t) = 3t^2 - 6t + 12$ **4.** $F'(x) = 12{,}288x^2$

5. $H'(s) = \frac{5}{32}s^4$ **6.** $y' = \dfrac{\sqrt{5}}{2\sqrt{x}}$

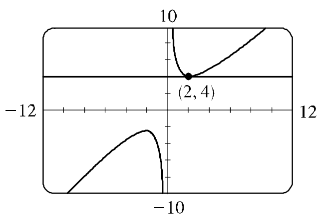
7. $y' = \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$ **8.** $y' = -\dfrac{B}{x^2} - 2\dfrac{C}{x^3}$

9. $y' = 1 + \dfrac{2}{5\sqrt[5]{x^3}}$ **10.** $v' = \frac{3}{2}\sqrt{x} - \dfrac{5}{2x^3\sqrt{x}}$

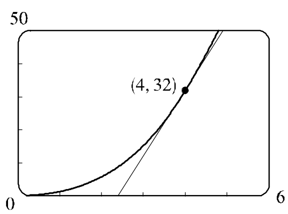
11. $4x - 4x^3$

12. $1 - x^{-2/3}$ **13.** (a) 0.264 (b) $2^{2/5}/5 \approx 0.263902$

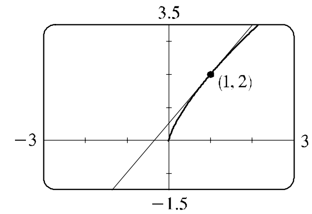
14. $y = 4$



15. $y = 20x - 48$



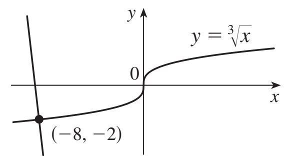
16. $y = \frac{3}{2}x + \frac{1}{2}$



17. $(1, 0), \left(-\frac{1}{3}, \frac{32}{27}\right)$ **18.** $x = \frac{1}{2}\left(1 \pm \sqrt{5}\right)$

19. $(4, 8)$ **20.** $\left(\pm\frac{7}{128}, \frac{49}{1024}\right)$

21. $12x + y + 98 = 0$



22. $\left(-\frac{1}{4}, \frac{1}{256}\right)$

2.3 SOLUTIONS

E Click here for exercises.

1. $f(x) = x^2 - 10x + 100 \quad \Rightarrow \quad f'(x) = 2x - 10$

2. $g(x) = x^{100} + 50x + 1 \quad \Rightarrow \quad g'(x) = 100x^{99} + 50$

3. $s(t) = t^3 - 3t^2 + 12t \quad \Rightarrow$
$s'(t) = 3t^{3-1} - 3\left(2t^{2-1}\right) + 12 = 3t^2 - 6t + 12$

4. $F(x) = (16x)^3 = 4096x^3 \quad \Rightarrow$
$F'(x) = 4096\left(3x^2\right) = 12{,}288x^2$

5. $H(s) = (s/2)^5 = s^5/2^5 = \frac{1}{32}s^5 \quad \Rightarrow$
$H'(s) = \frac{1}{32}\left(5s^{5-1}\right) = \frac{5}{32}s^4$

6. $y = \sqrt{5x} = \sqrt{5}x^{1/2} \quad \Rightarrow \quad y' = \sqrt{5}\left(\frac{1}{2}\right)x^{-1/2} = \dfrac{\sqrt{5}}{2\sqrt{x}}$

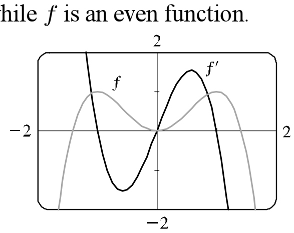
7. $y = x^{4/3} - x^{2/3} \quad \Rightarrow \quad y' = \frac{4}{3}x^{1/3} - \frac{2}{3}x^{-1/3}$

8. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2} = A + Bx^{-1} + Cx^{-2} \quad \Rightarrow$
$y' = -Bx^{-2} - 2Cx^{-3} = -\dfrac{B}{x^2} - 2\dfrac{C}{x^3}$

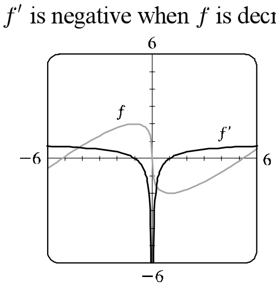
9. $y = x + \sqrt[5]{x^2} = x + x^{2/5} \quad \Rightarrow$
$y' = 1 + \frac{2}{5}x^{-3/5} = 1 + \dfrac{2}{5\sqrt[5]{x^3}}$

10. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}} = x^{3/2} + x^{-5/2} \quad \Rightarrow$
$v' = \frac{3}{2}x^{1/2} - \frac{5}{2}x^{-7/2} = \frac{3}{2}\sqrt{x} - \dfrac{5}{2x^3\sqrt{x}}$

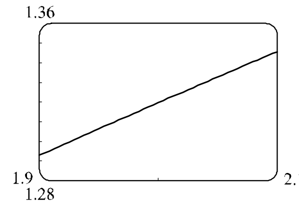
11. $f(x) = 2x^2 - x^4 \quad \Rightarrow \quad f'(x) = 4x - 4x^3$. Notice that $f'(x) = 0$ when f has a horizontal tangent and that f' is an odd function while f is an even function.



12. $f(x) = x - 3x^{1/3} \quad \Rightarrow$
$f'(x) = 1 - x^{-2/3} = 1 - 1/x^{2/3}$. Note that $f'(x) = 0$ when f has a horizontal tangent, f' is positive when f is increasing, and f' is negative when f is decreasing.



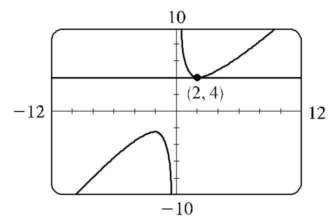
13. (a)



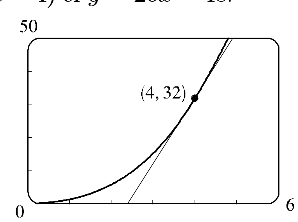
The endpoints of f in this graph are about $(1.9, 1.2927)$ and $(2.1, 1.3455)$. An estimate of $f'(2)$ is $\dfrac{1.3455 - 1.2927}{2.1 - 1.9} = \dfrac{0.0528}{0.2} = 0.264$.

(b) $f(x) = x^{2/5} \quad \Rightarrow \quad f'(x) = \frac{2}{5}x^{-3/5} = 2\Big/\left(5x^{3/5}\right)$.
$f'(2) = 2\Big/\left(5 \cdot 2^{3/5}\right) \approx 0.263902$.

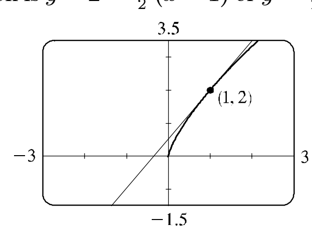
14. $y = f(x) = x + \dfrac{4}{x} \quad \Rightarrow \quad f'(x) = 1 - \dfrac{4}{x^2}$. So the slope of the tangent line at $(2, 4)$ is $f'(2) = 0$ and its equation is $y - 4 = 0$ or $y = 4$.



15. $y = f(x) = x^{5/2} \quad \Rightarrow \quad f'(x) = \frac{5}{2}x^{3/2}$. So the slope of the tangent line at $(4, 32)$ is $f'(4) = 20$ and its equation is $y - 32 = 20(x - 4)$ or $y = 20x - 48$.



16. $y = f(x) = x + \sqrt{x} \quad \Rightarrow \quad f'(x) = 1 + \frac{1}{2}x^{-1/2}$. So the slope of the tangent line at $(1, 2)$ is $f'(1) = 1 + \frac{1}{2}(1) = \frac{3}{2}$ and its equation is $y - 2 = \frac{3}{2}(x - 1)$ or $y = \frac{3}{2}x + \frac{1}{2}$.



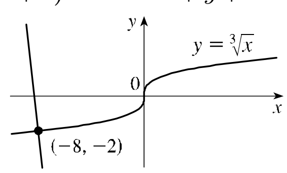
17. $y = x^3 - x^2 - x + 1$ has a horizontal tangent when $y' = 3x^2 - 2x - 1 = 0$. $(3x + 1)(x - 1) = 0 \quad \Leftrightarrow \quad x = 1$ or $-\frac{1}{3}$. Therefore, the points are $(1, 0)$ and $\left(-\frac{1}{3}, \frac{32}{27}\right)$.

18. $f(x) = 2x^3 - 3x^2 - 6x + 87$ has a horizontal tangent when $f'(x) = 6x^2 - 6x - 6 = 0 \quad \Leftrightarrow \quad x^2 - x - 1 = 0 \quad \Leftrightarrow$ $x = \frac{1 \pm \sqrt{5}}{2}$.

19. $y = x\sqrt{x} = x^{3/2} \quad \Rightarrow \quad y' = \frac{3}{2}\sqrt{x}$, so the tangent line is parallel to $3x - y + 6 = 0$ when $\frac{3}{2}\sqrt{x} = 3 \quad \Leftrightarrow \quad \sqrt{x} = 2$ $\Leftrightarrow \quad x = 4$. So the point is $(4, 8)$.

20. The sides of the groove must be tangent to the parabola $y = 16x^2$. $y' = 32x = 1.75$ when $x = \frac{1.75}{32} = \frac{7}{128}$, which implies that $y = 16\left(\frac{7}{128}\right)^2 = \frac{49}{1024}$. Therefore the points of contact are $\left(\pm\frac{7}{128}, \frac{49}{1024}\right)$.

21. $y = f(x) = \sqrt[3]{x} = x^{1/3} \quad \Rightarrow \quad f'(x) = \frac{1}{3}x^{-2/3}$, so the tangent line at $(-8, -2)$ has slope $f'(-8) = \frac{1}{12}$. The normal line has slope $-1/\left(\frac{1}{12}\right) = -12$ and equation $y + 2 = -12(x + 8) \quad \Leftrightarrow \quad 12x + y + 98 = 0$.



22. If the normal line has slope 16, then the tangent has slope $-\frac{1}{16}$, so $y' = 4x^3 = -\frac{1}{16} \quad \Rightarrow \quad x^3 = -\frac{1}{64} \quad \Rightarrow \quad x = -\frac{1}{4}$. The point is $\left(-\frac{1}{4}, \frac{1}{256}\right)$.