2.4 THE PRODUCT AND QUOTIENT RULES

A Click here for answers.

S Click here for solutions.

1–15 ■ Differentiate.

1. $h(x) = \dfrac{x + 2}{x - 1}$

2. $f(u) = \dfrac{1 - u^2}{1 + u^2}$

3. $G(s) = (s^2 + s + 1)(s^2 + 2)$

4. $g(x) = (1 + \sqrt{x})(x - x^3)$

5. $H(x) = (x^3 - x + 1)(x^{-2} + 2x^{-3})$

6. $y = \dfrac{3t - 7}{t^2 + 5t - 4}$

7. $y = \dfrac{4t + 5}{2 - 3t}$

8. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

9. $y = \dfrac{u^2 - u - 2}{u + 1}$

10. $f(x) = \dfrac{x^5}{x^3 - 2}$

11. $s = \sqrt{t}(t^3 - \sqrt{t} + 1)$

12–16 ■ Find an equation of the tangent line to the curve at the given point.

12. $y = x\sqrt{x}$, $(1, 1)$

13. $y = \dfrac{x}{x - 3}$, $(6, 2)$

14. $y = x + \dfrac{4}{x}$, $(2, 4)$

15. $y = x^{5/2}$, $(4, 32)$

16. $y = x + \sqrt{x}$, $(1, 2)$

2.4 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $h'(x) = -\dfrac{3}{(x-1)^2}$ **2.** $f'(u) = -\dfrac{4u}{(1+u^2)^2}$

3. $G'(s) = (2s+1)(s^2+2) + (s^2+s+1)(2s)$
$[= 4s^3 + 3s^2 + 6s + 2]$

4. $g'(x) = 1 - 3x^2 + \frac{3}{2}x^{1/2} - \frac{7}{2}x^{5/2}$

5. $H'(x) = 1 + x^{-2} + 2x^{-3} - 6x^{-4}$

6. $y' = \dfrac{-3t^2 + 14t + 23}{(t^2 + 5t - 4)^2}$ **7.** $y' = \dfrac{23}{(2-3t)^2}$

8. $y' = \frac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x\sqrt{x}}$ **9.** $y' = 1$

10. $f'(x) = \dfrac{2x^4(x^3-5)}{(x^3-2)^2}$

11. $s' = \frac{7}{2}t^{5/2} - 1 + \dfrac{1}{2\sqrt{t}}$ **12.** $y = \frac{3}{2}x - \frac{1}{2}$

13. $x + 3y = 12$ **14.** $y = 4$ **15.** $y = 20x - 48$

16. $y = \frac{3}{2}x + \frac{1}{2}$

2.4 SOLUTIONS

E Click here for exercises.

1. $h(x) = \dfrac{x+2}{x-1} \Rightarrow$

$$h'(x) = \frac{(x-1)(1)-(x+2)(1)}{(x-1)^2} = \frac{x-1-x-2}{(x-1)^2} = -\frac{3}{(x-1)^2}$$

2. $f(u) = \dfrac{1-u^2}{1+u^2} \Rightarrow$

$$f'(u) = \frac{(1+u^2)(-2u)-(1-u^2)(2u)}{(1+u^2)^2} = \frac{-2u-2u^3-2u+2u^3}{(1+u^2)^2} = -\frac{4u}{(1+u^2)^2}$$

3. $G(s) = (s^2+s+1)(s^2+2) \Rightarrow$

$$G'(s) = (s^2+s+1)(2s)+(s^2+2)(2s+1) = 2s^3+2s^2+2s+2s^3+s^2+4s+2 = 4s^3+3s^2+6s+2$$

4. $g(x) = (1+\sqrt{x})(x-x^3) = x-x^3+x^{3/2}-x^{7/2} \Rightarrow$

$g'(x) = 1-3x^2+\frac{3}{2}x^{1/2}-\frac{7}{2}x^{5/2}$

Another Method: Use the Product Rule.

5. $H(x) = (x^3-x+1)(x^{-2}+2x^{-3})$

$$= (x^3-x+1)(x^{-2})+(x^3-x+1)(2x^{-3}) = x-x^{-1}+x^{-2}+2-2x^{-2}+2x^{-3} = 2+x-x^{-1}-x^{-2}+2x^{-3}$$

$\Rightarrow \quad H'(x) = 1+x^{-2}+2x^{-3}-6x^{-4}$

Another Method: Use the Product Rule.

6. $y = \dfrac{3t-7}{t^2+5t-4} \Rightarrow$

$$y' = \frac{(t^2+5t-4)(3)-(3t-7)(2t+5)}{(t^2+5t-4)^2} = \frac{-3t^2+14t+23}{(t^2+5t-4)^2}$$

7. $y = \dfrac{4t+5}{2-3t} \Rightarrow$

$$y' = \frac{(2-3t)(4)-(4t+5)(-3)}{(2-3t)^2} = \frac{23}{(2-3t)^2}$$

8. $y = \dfrac{x^2+4x+3}{\sqrt{x}} = x^{3/2}+4x^{1/2}+3x^{-1/2} \Rightarrow$

$$y' = \frac{3}{2}x^{1/2}+4\left(\frac{1}{2}\right)x^{-1/2}+3\left(-\frac{1}{2}\right)x^{-3/2} = \frac{3}{2}\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{3}{2x\sqrt{x}}$$

Another Method: Use the Quotient Rule.

9. $y = \dfrac{u^2-u-2}{u+1} = \dfrac{(u-2)(u+1)}{u+1} = u-2$ for $u \neq -1$.

$y' = \dfrac{d}{du}(u-2) = 1$

10. $f(x) = \dfrac{x^5}{x^3-2} \Rightarrow$

$$f'(x) = \frac{(x^3-2)(5x^4)-x^5(3x^2)}{(x^3-2)^2} = \frac{2x^4(x^3-5)}{(x^3-2)^2}$$

11. $s = \sqrt{t}(t^3-\sqrt{t}+1) = t^{7/2}-t+t^{1/2} \Rightarrow$

$s' = \frac{7}{2}t^{5/2}-1+\dfrac{1}{2\sqrt{t}}$.

Another Method: Use the Product Rule.

12. $y = x\sqrt{x} = x^{3/2} \Rightarrow y' = \frac{3}{2}x^{1/2}$. At $(1,1)$, $y' = \frac{3}{2}$, and an equation of the tangent line is $y-1 = \frac{3}{2}(x-1)$, or $y = \frac{3}{2}x-\frac{1}{2}$.

13. $y = f(x) = \dfrac{x}{x-3} \Rightarrow$

$f'(x) = \dfrac{(x-3)1-x(1)}{(x-3)^2} = \dfrac{-3}{(x-3)^2}$. So the slope of the tangent line at $(6,2)$ is $f'(6) = -\frac{1}{3}$ and its equation is $y-2 = -\frac{1}{3}(x-6)$ or $x+3y = 12$.

14. $y = f(x) = x+\dfrac{4}{x} \Rightarrow f'(x) = 1-\dfrac{4}{x^2}$. So the slope of the tangent line at $(2,4)$ is $f'(2) = 0$ and its equation is $y-4 = 0$ or $y = 4$.

15. $y = f(x) = x^{5/2} \Rightarrow f'(x) = \frac{5}{2}x^{3/2}$. So the slope of the tangent line at $(4,32)$ is $f'(4) = 20$ and its equation is $y-32 = 20(x-4)$ or $y = 20x-48$.

16. $y = f(x) = x+\sqrt{x} \Rightarrow f'(x) = 1+\frac{1}{2}x^{-1/2}$. So the slope of the tangent line at $(1,2)$ is $f'(1) = 1+\frac{1}{2}(1) = \frac{3}{2}$ and its equation is $y-2 = \frac{3}{2}(x-1)$ or $y = \frac{3}{2}x+\frac{1}{2}$ or $3x-2y+1 = 0$.