2.5 THE CHAIN RULE

A Click here for answers.

S Click here for solutions.

1–4 ■ Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative dy/dx.

1. $y = (x^2 + 4x + 6)^5$

2. $y = \tan 3x$

3. $y = \cos(\tan x)$

4. $y = \sqrt[3]{1 + x^3}$

5–29 ■ Find the derivative of the function.

5. $F(x) = (x^3 - 5x)^4$

6. $f(t) = (2t^2 + 6t + 1)^{-8}$

7. $g(x) = \sqrt{x^2 - 7x}$

8. $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4}$

9. $h(t) = \left(t - \dfrac{1}{t}\right)^{3/2}$

10. $y = \sin \dfrac{1}{x}$

11. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

12. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$

13. $F(y) = \left(\dfrac{y - 6}{y + 7}\right)^3$

14. $s(t) = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$

15. $f(z) = \dfrac{1}{\sqrt[5]{2z - 1}}$

16. $f(x) = \dfrac{x}{\sqrt{7 - 3x}}$

17. $y = \sqrt{1 + 2\tan x}$

18. $y = \sin^3 x + \cos^3 x$

19. $y = \sin^2(\cos kx)$

20. $y = \left(\sin \sqrt{x^2 + 1}\right)^{\sqrt{2}}$

21. $y = \cos^2(\cos x) + \sin^2(\cos x)$

22. $f(x) = [x^3 + (2x - 1)^3]^3$

23. $g(t) = \sqrt[4]{(1 - 3t)^4 + t^4}$

24. $y = \cos^2\left(\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}\right)$

25. $y = \sqrt{1 + \tan(x + (1/x))}$

26. $p(t) = \left[\left(1 + \dfrac{2}{t}\right)^{-1} + 3t\right]^{-2}$

27. $N(y) = \left(y + \sqrt[3]{y + \sqrt{2y - 9}}\right)^8$

28–34 ■ Find an equation of the tangent line to the curve at the given point.

28. $y = \dfrac{8}{\sqrt{4 + 3x}}, \quad (4, 2)$

29. $y = \sin x + \cos 2x, \quad (\pi/6, 1)$

30. $y = (x^3 - x^2 + x - 1)^{10}, \quad (1, 0)$

31. $y = \sqrt{x + (1/x)}, \quad (1, \sqrt{2})$

32. $y = \dfrac{x}{(3 - x^2)^5}, \quad (2, -2)$

33. $y = \cot^2 x, \quad (\pi/4, 1)$

34–37 ■ Find f' and state the domains of f and f'.

34. $f(x) = x^2 \sec^2 3x$

35. $f(x) = \sin\sqrt{2x + 1}$

36. $f(x) = \sqrt{\cos\sqrt{x}}$

37. $f(x) = \cos\sqrt{x} + \sqrt{\cos x}$

2.5 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $10(x^2+4x+6)^4(x+2)$ **2.** $3\sec^2 3x$

3. $-\sin(\tan x)\sec^2 x$ **4.** $\dfrac{x^2}{(1+x^3)^{2/3}}$

5. $F'(x)=4(x^3-5x)^3(3x^2-5)$

6. $f'(t)=-16(2t^2-6t+1)^{-9}(2t-3)$

7. $g'(x)=\dfrac{2x-7}{2\sqrt{x^2-7x}}$ **8.** $f'(t)=\dfrac{8(1-t)}{(t^2-2t-5)^5}$

9. $h'(t)=\frac{3}{2}(t-1/t)^{1/2}(1+1/t^2)$ **10.** $y'=-\dfrac{1}{x^2}\cos\dfrac{1}{x}$

11. $G'(x)=6(3x-2)^9(5x^2-x+1)^{11}(85x^2-51x+9)$

12. $g'(t)=12t(6t^2+5)^2(t^3-7)^3(9t^3+5t-21)$

13. $F'(y)=\dfrac{39(y-6)^2}{(y+7)^4}$

14. $s'(t)=\dfrac{1}{2}\left(\dfrac{t^3+1}{t^3+1}\right)^{-3/4}\dfrac{-3t^2}{(t^3-1)^2}$

15. $f'(z)=-\frac{2}{5}(2z-1)^{-6/5}$ **16.** $f'(x)=\dfrac{14-3x}{2(7-3x)^{3/2}}$

17. $y'=\dfrac{\sec^2 x}{\sqrt{1+2\tan x}}$ **18.** $y'=3\sin x\cos x(\sin x-\cos x)$

19. $y'=-k\sin kx\sin(2\cos kx)$

20. $y'=\sqrt{2}x\left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}-1}\dfrac{\cos\sqrt{x^2+1}}{\sqrt{x^2+1}}$

21. $y'=0$ **22.** $f'(x)=9[x^3+(2x-1)^3]^2(9x^2-8x+2)$

23. $g'(t)=[(1-3t)^4+t^4]^{-3/4}[t^3-3(1-3t^3)]$

24. $y'=\dfrac{2}{\sqrt{x}(1+\sqrt{x})^2}\sin\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cos\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

25. $y'=\dfrac{(x^2-1)\sec^2\left(x+\dfrac{1}{x}\right)}{2x^2\sqrt{1+\tan\left(x+\dfrac{1}{x}\right)}}$

26. $p'(t)=-2[(1+2/t)^{-1}+3t]^{-3}[2(t+2)^{-2}+3]$

27. $N'(y)=8\left(y+\sqrt[3]{y+\sqrt{2y-9}}\right)^7\left[1+\frac{1}{3}\left(y+\sqrt{2y-9}\right)^{-2/3}\left(1+\dfrac{1}{\sqrt{2y-9}}\right)\right]$

28. $y=-\frac{3}{16}x+\frac{11}{4}$ **29.** $y=-\frac{\sqrt{3}}{2}x+1+\frac{\sqrt{3}\pi}{12}$

30. $y=0$ **31.** $y=\sqrt{2}$

32. $y=39x-80$ **33.** $4x+y=\pi+1$

34. $f'(x)=2x\sec^2 3x\,(1+3x\tan 3x)$,
$\left\{x\mid x\neq(2n-2)\frac{\pi}{6},\ n \text{ an integer}\right\}$ (both f and f')

35. $f'(x)=\dfrac{\cos\sqrt{2x+1}}{\sqrt{2x+1}}$,
$\operatorname{dom}(f)=\left[-\frac{1}{2},\infty\right)$, $\operatorname{dom}(f')=\left(-\frac{1}{2},\infty\right)$

36. $f'(x)=-\dfrac{\sin\sqrt{x}}{4\sqrt{x}\sqrt{\cos\sqrt{x}}}$,
$\operatorname{dom}(f)=\{x\mid 0\leqslant x\leqslant\pi^2/4$ or
$[(4n-1)\pi/2]^2\leqslant x\leqslant[(4n+1)\pi/2]^2$
for some $n\in\{1,2,3,\ldots\}\}$,
$\operatorname{dom}(f')=\{x\mid 0<x<\pi^2/4$ or
$[(4n-1)\pi/2]^2<x<[(4n+1)\pi/2]^2$
for some $n\in\{1,2,3,\ldots\}\}$

37. $f'(x)=-\dfrac{\sin\sqrt{x}}{2\sqrt{x}}-\dfrac{\sin x}{2\sqrt{\cos x}}$,
$\operatorname{dom}(f)=\{x\mid 0\leqslant x\leqslant\pi/2$ or
$(4n-1)\pi/2\leqslant x\leqslant(4n+1)\pi/2$
for some $n=1,2,3,\ldots\}$,
$\operatorname{dom}(f')=\{x\mid 0<x<\pi/2$ or
$(4n-1)\pi/2<x<(4n+1)\pi/2$
for some $n=1,2,3,\ldots\}$

2.5 SOLUTIONS

E Click here for exercises.

1. Let $u = g(x) = x^2 + 4x + 6$ and $y = f(u) = u^5$. Then

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} = \left(5u^4\right)(2x+4) \\ &= 5\left(x^2+4x+6\right)^4(2x+4) \\ &= 10\left(x^2+4x+6\right)^4(x+2)\end{aligned}$$

2. Let $u = g(x) = 3x$ and $y = f(u) = \tan u$. Then
$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \left(\sec^2 u\right)(3) = 3\sec^2 3x.$

3. Let $u = g(x) = \tan x$ and $y = f(u) = \cos u$. Then
$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (-\sin u)\left(\sec^2 x\right) = -\sin(\tan x)\sec^2 x.$

4. Let $u = g(x) = 1 + x^3$ and $y = f(u) = u^{1/3}$. Then

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} = \tfrac{1}{3}u^{-2/3}\left(3x^2\right) \\ &= \left(1+x^3\right)^{-2/3}x^2 = \frac{x^2}{(1+x^3)^{2/3}}\end{aligned}$$

5. $F(x) = \left(x^3-5x\right)^4 \quad\Rightarrow$

$$\begin{aligned}F'(x) &= 4\left(x^3-5x\right)^3\frac{d}{dx}\left(x^3-5x\right) \\ &= 4\left(x^3-5x\right)^3\left(3x^2-5\right)\end{aligned}$$

6. $f(t) = \left(2t^2-6t+1\right)^{-8} \quad\Rightarrow$

$$\begin{aligned}f'(t) &= -8\left(2t^2-6t+1\right)^{-9}(4t-6) \\ &= -16\left(2t^2-6t+1\right)^{-9}(2t-3)\end{aligned}$$

7. $g(x) = \sqrt{x^2-7x} = \left(x^2-7x\right)^{1/2} \quad\Rightarrow$
$g'(x) = \frac{1}{2}\left(x^2-7x\right)^{-1/2}(2x-7) = \dfrac{2x-7}{2\sqrt{x^2-7x}}$

8. $f(t) = \dfrac{1}{(t^2-2t-5)^4} = \left(t^2-2t-5\right)^{-4} \quad\Rightarrow$
$f'(t) = -4\left(t^2-2t-5\right)^{-5}(2t-2) = \dfrac{8(1-t)}{(t^2-2t-5)^5}$

9. $h(t) = (t-1/t)^{3/2} \quad\Rightarrow$
$h'(t) = \frac{3}{2}(t-1/t)^{1/2}\left(1+1/t^2\right)$

10. $y = \sin\dfrac{1}{x} \quad\Rightarrow\quad y' = \cos\dfrac{1}{x}\left(-\dfrac{1}{x^2}\right) = -\dfrac{1}{x^2}\cos\dfrac{1}{x}$

11. $G(x) = (3x-2)^{10}\left(5x^2-x+1\right)^{12} \quad\Rightarrow$

$$\begin{aligned}G'(x) &= (3x-2)^{10}(12)\left(5x^2-x+1\right)^{11}(10x-1) \\ &\qquad + 10(3x-2)^9(3)\left(5x^2-x+1\right)^{12} \\ &= 6(3x-2)^9\left(5x^2-x+1\right)^{11} \\ &\qquad \left[2(3x-2)(10x-1)+5\left(5x^2-x+1\right)\right] \\ &= 6(3x-2)^9\left(5x^2-x+1\right)^{11}\left(85x^2-51x+9\right)\end{aligned}$$

12. $g(t) = \left(6t^2+5\right)^3\left(t^3-7\right)^4 \quad\Rightarrow$

$$\begin{aligned}g'(t) &= \left(6t^2+5\right)^3(4)\left(t^3-7\right)^3\left(3t^2\right) \\ &\qquad + 3\left(6t^2+5\right)^2(12t)\left(t^3-7\right)^4 \\ &= 12t\left(6t^2+5\right)^2\left(t^3-7\right)^3\left[t\left(6t^2+5\right)+3\left(t^3-7\right)\right] \\ &= 12t\left(6t^2+5\right)^2\left(t^3-7\right)^3\left(9t^3+5t-21\right)\end{aligned}$$

13. $F(y) = \left(\dfrac{y-6}{y+7}\right)^3 \quad\Rightarrow$

$$\begin{aligned}F'(y) &= 3\left(\frac{y-6}{y+7}\right)^2\frac{(y+7)(1)-(y-6)(1)}{(y+7)^2} \\ &= 3\left(\frac{y-6}{y+7}\right)^2\frac{13}{(y+7)^2} = \frac{39(y-6)^2}{(y+7)^4}\end{aligned}$$

14. $s(t) = \left(\dfrac{t^3+1}{t^3-1}\right)^{1/4} \quad\Rightarrow$

$$\begin{aligned}s'(t) &= \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4}\frac{3t^2\left(t^3-1\right)-\left(t^3+1\right)\left(3t^2\right)}{(t^3-1)^2} \\ &= \frac{1}{2}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4}\frac{-3t^2}{(t^3-1)^2}\end{aligned}$$

15. $f(z) = (2z-1)^{-1/5} \quad\Rightarrow$
$f'(z) = -\frac{1}{5}(2z-1)^{-6/5}(2) = -\frac{2}{5}(2z-1)^{-6/5}$

16. $f(x) = \dfrac{x}{\sqrt{7-3x}} \quad\Rightarrow$

$$\begin{aligned}f'(x) &= \frac{\sqrt{7-3x}-x\left(\frac{1}{2}\right)(7-3x)^{-1/2}(-3)}{7-3x} \\ &= \frac{1}{\sqrt{7-3x}}+\frac{3x}{2(7-3x)^{3/2}} \text{ or } \frac{14-3x}{2(7-3x)^{3/2}}\end{aligned}$$

17. $y = \sqrt{1+2\tan x} \quad\Rightarrow$
$y' = \frac{1}{2}(1+2\tan x)^{-1/2}2\sec^2 x = \dfrac{\sec^2 x}{\sqrt{1+2\tan x}}$

18. $y = \sin^3 x + \cos^3 x \quad \Rightarrow$

$$y' = 3\sin^2 x \cos x + 3\cos^2 x\,(-\sin x) = 3\sin x \cos x\,(\sin x - \cos x)$$

19. $y = \sin^2(\cos kx) \quad \Rightarrow$

$$y' = 2\sin(\cos kx)\cos(\cos kx)(-\sin kx)(k) = -k\sin kx \sin(2\cos kx)$$

20. $y = \left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}} \quad \Rightarrow$

$$y' = \sqrt{2}\left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}-1}\left(\cos\sqrt{x^2+1}\right)\left(\frac{1}{2}\right)\left(x^2+1\right)^{-1/2}(2x) = \sqrt{2}x\left(\sin\sqrt{x^2+1}\right)^{\sqrt{2}-1}\frac{\cos\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

21. $y = \cos^2(\cos x) + \sin^2(\cos x) = 1 \quad \Rightarrow \quad y' = 0$

22. $f(x) = \left[x^3 + (2x-1)^3\right]^3 \quad \Rightarrow$

$$f'(x) = 3\left[x^3+(2x-1)^3\right]^2\left[3x^2+3(2x-1)^2(2)\right] = 9\left[x^3+(2x-1)^3\right]^2\left[9x^2-8x+2\right]$$

23. $g(t) = \sqrt[4]{(1-3t)^4+t^4} \quad \Rightarrow$

$$g'(t) = \frac{1}{4}\left[(1-3t)^4+t^4\right]^{-3/4}\left[4(1-3t)^3(-3)+4t^3\right] = \left[(1-3t)^4+t^4\right]^{-3/4}\left[t^3-3\left(1-3t^3\right)\right]$$

24. $y = \cos^2\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right) \quad \Rightarrow$

$$y' = 2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)(-1)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\frac{(1+\sqrt{x})\left(-\frac{1}{2\sqrt{x}}\right)-(1-\sqrt{x})\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} = \frac{2}{\sqrt{x}\,(1+\sqrt{x})^2}\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$$

25. $y = \sqrt{1+\tan\left(x+\dfrac{1}{x}\right)} \quad \Rightarrow$

$$y' = \frac{1}{2\sqrt{1+\tan\left(x+\frac{1}{x}\right)}}\left[\sec^2\left(x+\frac{1}{x}\right)\right]\left(1-\frac{1}{x^2}\right) = \frac{\left(x^2-1\right)\sec^2\left(x+\frac{1}{x}\right)}{2x^2\sqrt{1+\tan\left(x+\frac{1}{x}\right)}}$$

26. $p(t) = \left[\left(1+\dfrac{2}{t}\right)^{-1}+3t\right]^{-2} \quad \Rightarrow$

$$p'(t) = -2\left[\left(1+\frac{2}{t}\right)^{-1}+3t\right]^{-3}\left[-\left(1+\frac{2}{t}\right)^{-2}\left(-\frac{2}{t^2}\right)+3\right] = -2\left[\left(1+\frac{2}{t}\right)^{-1}+3t\right]^{-3}\left[2(t+2)^{-2}+3\right]$$

27. $N(y) = \left(y+\sqrt[3]{y+\sqrt{2y-9}}\right)^8 = \left\{y+\left[y+(2y-9)^{1/2}\right]^{1/3}\right\}^8 \quad \Rightarrow$

$$N'(y) = 8\left(y+\sqrt[3]{y+\sqrt{2y-9}}\right)^7\left[1+\tfrac{1}{3}\left(y+\sqrt{2y-9}\right)^{-2/3}\left[1+\tfrac{1}{2}(2y-9)^{-1/2}(2)\right]\right] = 8\left(y+\sqrt[3]{y+\sqrt{2y-9}}\right)^7\left[1+\tfrac{1}{3}\left(y+\sqrt{2y-9}\right)^{-2/3}\left(1+\frac{1}{\sqrt{2y-9}}\right)\right]$$

28. $y = f(x) = \dfrac{8}{\sqrt{4+3x}} = 8(4+3x)^{-1/2} \quad \Rightarrow$
$f'(x) = 8\left(-\frac{1}{2}\right)(4+3x)^{-3/2}(3) = -12(4+3x)^{-3/2}$.
The slope of the tangent at $(4,2)$ is $f'(4) = -\frac{12}{64} = -\frac{3}{16}$
and its equation is $y-2 = -\frac{3}{16}(x-4)$ or $y = -\frac{3}{16}x+\frac{11}{4}$.

29. $y = f(x) = \sin x + \cos 2x \quad \Rightarrow$
$f'(x) = \cos x - 2\sin 2x$. The slope of the tangent at $\left(\frac{\pi}{6},1\right)$
is $f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - 2\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$ and its equation is
$y-1 = -\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)$ or $\sqrt{3}x+2y = 2+\frac{\sqrt{3}}{6}\pi$.

30. $y = f(x) = \left(x^3-x^2+x-1\right)^{10} \quad \Rightarrow$
$f'(x) = 10\left(x^3-x^2+x-1\right)^9\left(3x^2-2x+1\right)$. The
slope of the tangent at $(1,0)$ is $f'(1) = 0$ and its equation is
$y-0 = 0(x-1)$ or $y = 0$.

31. $y = f(x) = \sqrt{x+1/x} \quad \Rightarrow$
$f'(x) = \frac{1}{2}\left(x+\dfrac{1}{x}\right)^{-1/2}\left(1-\dfrac{1}{x^2}\right)$. The slope of the
tangent at $\left(1,\sqrt{2}\right)$ is $f'(1) = 0$ and its equation is
$y-\sqrt{2} = 0(x-1)$ or $y = \sqrt{2}$.

32. $y = f(x) = \dfrac{x}{(3-x^2)^5} \quad\Rightarrow$

$$f'(x) = \frac{(3-x^2)^5(1) - x(5)(3-x^2)^4(-2x)}{(3-x^2)^{10}} = \frac{9x^2+3}{(3-x^2)^6}$$

The slope of the tangent at $(2, -2)$ is $f'(2) = 39$ and its equation is $y + 2 = 39(x-2)$ or $y = 39x - 80$.

33. $y = f(x) = \cot^2 x \quad\Rightarrow$

$y' = 2\cot x\left(-\csc^2 x\right) = -2\cot x \csc^2 x$. The slope of the tangent at $\left(\frac{\pi}{4}, 1\right)$ is $f'\left(\frac{\pi}{4}\right) = -2(1)\left(\sqrt{2}\right)^2 = -4$ and its equation is $y - 1 = -4\left(x - \frac{\pi}{4}\right)$ or $4x + y = \pi + 1$.

34. $f(x) = x^2 \sec^2 3x \quad\Rightarrow$

$$f'(x) = 2x\sec^2 3x + x^2(2\sec 3x)(\sec 3x \tan 3x)(3) = 2x\sec^2 3x\,(1 + 3x\tan 3x)$$

Domain of f = domain of $f' = \{x \mid \cos 3x \neq 0\}$
$= \left\{x \mid x \neq (2n-1)\frac{\pi}{6},\ n \text{ an integer}\right\}$

35. $f(x) = \sin\sqrt{2x+1} \quad\Rightarrow$

$f'(x) = \cos\sqrt{2x+1}\left(\dfrac{1}{2\sqrt{2x+1}}\right)(2) = \dfrac{\cos\sqrt{2x+1}}{\sqrt{2x+1}}$.

$\mathrm{Dom}(f) = \{x \mid 2x+1 \geq 0\} = \left[-\frac{1}{2}, \infty\right)$.

$\mathrm{Dom}(f') = \{x \mid 2x+1 > 0\} = \left(-\frac{1}{2}, \infty\right)$.

36. $f(x) = \sqrt{\cos\sqrt{x}} \quad\Rightarrow$

$$f'(x) = \tfrac{1}{2}\left(\cos\sqrt{x}\right)^{-1/2}\left(-\sin\sqrt{x}\right)\left(\tfrac{1}{2}\right)x^{-1/2} = -\frac{\sin\sqrt{x}}{4\sqrt{x}\sqrt{\cos\sqrt{x}}}$$

Domain of $f = \{x \mid x \geq 0 \text{ and } \cos\sqrt{x} \geq 0\}$
$= \left\{x \mid 0 \leq x \leq \frac{\pi^2}{4} \text{ or } \left[(4n-1)\frac{\pi}{2}\right]^2 \leq x \leq \left[(4n+1)\frac{\pi}{2}\right]^2 \text{ for some } n \in \{1,2,3,\ldots\}\right\}$

Domain of $f' = \{x \mid x > 0 \text{ and } \cos\sqrt{x} > 0\}$
$= \left\{x \mid 0 < x < \frac{\pi^2}{4} \text{ or } \left[(4n-1)\frac{\pi}{2}\right]^2 < x < \left[(4n+1)\frac{\pi}{2}\right]^2 \text{ for some } n \in \{1,2,3,\ldots\}\right\}$

37. $\cos\sqrt{x} + \sqrt{\cos x} \quad\Rightarrow$

$$f'(x) = -\sin\sqrt{x}\,\tfrac{1}{2}x^{-1/2} + \tfrac{1}{2}(\cos x)^{-1/2}(-\sin x) = -\frac{\sin\sqrt{x}}{2\sqrt{x}} - \frac{\sin x}{2\sqrt{\cos x}}$$

Domain of $f = \{x \mid x \geq 0 \text{ and } \cos x \geq 0\}$
$= \left\{x \mid 0 \leq x \leq \frac{\pi}{2} \text{ or } (4n-1)\frac{\pi}{2} \leq x \leq (4n+1)\frac{\pi}{2} \text{ for some } n \in \{1,2,3,\ldots\}\right\}$

Domain of $f' = \{x \mid x > 0 \text{ and } \cos x > 0\}$
$= \left\{x \mid 0 < x < \frac{\pi}{2} \text{ or } (4n-1)\frac{\pi}{2} < x < (4n+1)\frac{\pi}{2} \text{ for some } n \in \{1,2,3,\ldots\}\right\}$