## 2.7 RELATED RATES

A Click here for answers.

S Click here for solutions.

1. If $xy = 1$ and $dx/dt = 4$, find $dy/dt$ when $x = 2$.
2. If $x^2 + 3xy + y^2 = 1$ and $dy/dt = 2$, find $dx/dt$ when $y = 1$.
3. A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1 $cm^3/min$. At what rate is the diameter decreasing when the diameter is 10 cm?

## 2.7 ANSWERS

E Click here for exercises.

S Click here for solutions.

**1.** $-1$

**2.** $-\frac{4}{3}$ (if $x = 0$), $-\frac{14}{3}$ (if $x = -3$)

**3.** $1/(50\pi)$ cm/min

## 2.7 SOLUTIONS

E Click here for exercises.

**1.** $xy = 1 \quad \Rightarrow \quad x\dfrac{dy}{dt} + y\dfrac{dx}{dt} = 0$. If $\dfrac{dx}{dt} = 4$ and $x = 2$, then $y = \frac{1}{2}$, so $\dfrac{dy}{dt} = -\dfrac{y}{x}\dfrac{dx}{dt} = -\frac{1/2}{2}\,(4) = -1$.

**2.** $x^2 + 3xy + y^2 = 1 \quad \Rightarrow$

$2x\dfrac{dx}{dt} + 3y\dfrac{dx}{dt} + 3x\dfrac{dy}{dt} + 2y\dfrac{dy}{dt} = 0 \quad \Rightarrow$

$\dfrac{dx}{dt} = -\dfrac{3x + 2y}{2x + 3y}\dfrac{dy}{dt}$. When $y = 1$, we have $x^2 + 3x = 0$

$\Rightarrow \quad x = 0$ or $-3$. If $\dfrac{dy}{dt} = 2$ and $x = 0$ and $y = 1$, then

$\dfrac{dx}{dt} = -\dfrac{3\,(0) + 2\,(1)}{2\,(0) + 3\,(1)}\,(2) = -\dfrac{4}{3}$. If $x = -3$, then

$\dfrac{dx}{dt} = -\dfrac{3\,(-3) + 2\,(1)}{2\,(-3) + 3\,(1)}\,(2) = -\dfrac{14}{3}$.

**3.** If the radius is $r$ and the diameter $x$, then $V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}x^3$

$\Rightarrow \quad -1 = \dfrac{dV}{dt} = \dfrac{\pi}{2}x^2\dfrac{dx}{dt} \quad \Rightarrow \quad \dfrac{dx}{dt} = -\dfrac{2}{\pi x^2}$. When $x = 10$, $\dfrac{dx}{dt} = -\dfrac{2}{\pi\,(100)} = -\dfrac{1}{50\pi}$. So the rate of decrease is $\dfrac{1}{50\pi}\ \dfrac{\text{cm}}{\text{min}}$.