6.5 APPROXIMATE INTEGRATION

A Click here for answers.

S Click here for solutions.

1–3 ▪ Use (a) the Trapezoidal Rule and (b) Simpson's Rule to approximate the given integral with the specified value of n. (Round your answers to six decimal places.)

1. $\int_{-1}^{1} \sqrt{1 + x^3}\, dx, \quad n = 8$

2. $\int_{\pi/2}^{\pi} \frac{\sin x}{x}\, dx, \quad n = 6$

3. $\int_{0}^{\pi/4} x \tan x\, dx, \quad n = 6$

4–10 ▪ Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of n. (Round your answers to six decimal places.)

4. $\int_{0}^{1} e^{-x^2}\, dx, \quad n = 10$

5. $\int_{0}^{2} \frac{1}{\sqrt{1 + x^3}}\, dx, \quad n = 10$

6. $\int_{2}^{3} \frac{1}{\ln x}\, dx, \quad n = 10$

7. $\int_{0}^{1} \ln(1 + e^x)\, dx, \quad n = 8$

8. $\int_{0}^{1} x^5 e^x\, dx, \quad n = 10$

9. $\int_{0}^{1/2} \cos(e^x)\, dx, \quad n = 8$

10. $\int_{0}^{3} \frac{1}{1 + x^4}\, dx, \quad n = 6$

11. Use Simpson's Rule and the following data to estimate the value of the integral $\int_2^6 y\, dx$.

x	y	x	y
2.0	9.22	4.5	6.83
2.5	9.01	5.0	7.32
3.0	8.76	5.5	7.69
3.5	8.30	6.0	7.91
4.0	7.52		

12. The speedometer reading (v) on a car was observed at 1-minute intervals and recorded in the following chart. Use Simpson's Rule to estimate the distance traveled by the car.

t (min)	v (mi/h)	t (min)	v (mi/h)
0	40	6	56
1	42	7	57
2	45	8	57
3	49	9	55
4	52	10	56
5	54		

13. A log 10 meters long is cut at 1-meter intervals and its cross-sectional areas A (at a distance x from the end of the log) are listed in the following table. Use Simpson's Rule to estimate the volume of the log.

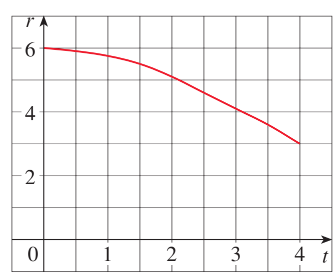
x (m)	A (m^2)	x (m)	A (m^2)
0	0.68	6	0.53
1	0.65	7	0.55
2	0.64	8	0.52
3	0.61	9	0.50
4	0.58	10	0.48
5	0.59		

14. (a) Use the Trapezoidal Rule and the following data to estimate the value of the integral $\int_1^{3.2} y\, dx$.

x	y	x	y
1.0	4.9	2.2	7.3
1.2	5.4	2.4	7.5
1.4	5.8	2.6	8.0
1.6	6.2	2.8	8.2
1.8	6.7	3.0	8.3
2.0	7.0	3.2	8.3

(b) If it is known that $-1 \leq f''(x) \leq 3$ for all x, estimate the error involved in the approximation in part (a).

15. Water leaked from a tank at a rate of $r(t)$ liters per hour, where the graph of r is as shown. Use Simpson's Rule to estimate the total amount of water that leaked out during the first four hours.



16. The table (supplied by Pacific Gas and Electric) gives the power consumption in megawatts in the San Francisco Bay Area from midnight to noon on September 19, 1996. Use Simpson's Rule to estimate the energy used during that time period. (Use the fact that power is the derivative of energy.)

t	P	t	P
0	4182	7	4699
1	3856	8	5151
2	3640	9	5514
3	3558	10	5751
4	3547	11	6044
5	3679	12	6206
6	4112		

6.5 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. (a) 1.913972 (b) 1.934766

2. (a) 0.481672 (b) 0.481172

3. (a) 0.189445 (b) 0.185822

4. (a) 0.746211 (b) 0.747131 (c) 0.746825

5. (a) 1.401435 (b) 1.402558 (c) 1.402206

6. (a) 1.119061 (b) 1.118107 (c) 1.118428

7. (a) 0.984120 (b) 0.983669 (c) 0.983819

8. (a) 0.409140 (b) 0.388849 (c) 0.395802

9. (a) 0.132465 (b) 0.132857 (c) 0.132727

10. (a) 1.098004 (b) 1.098709 (c) 1.109031

11. 31.94

12. 8.6 mi

13. 5.8 m^3

14. (a) 15.4 (b) 0.022

15. $19.4\overline{6}$ L

16. 54,730 megawatt-hours

6.5 SOLUTIONS

E Click here for exercises.

1. $f(x)=\sqrt{1+x^3}$, $\Delta x=\dfrac{1-(-1)}{8}=\dfrac{1}{4}$

(a) $T_8=\frac{0.25}{2}\left[f(-1)+2f\left(-\frac{3}{4}\right)+2f\left(-\frac{1}{2}\right)+\cdots+2f\left(\frac{1}{2}\right)+2f\left(\frac{3}{4}\right)+f(1)\right]$
≈ 1.913972

(b) $S_8=\frac{0.25}{3}\left[f(-1)+4f\left(-\frac{3}{4}\right)+2f\left(-\frac{1}{2}\right)+4f\left(-\frac{1}{4}\right)+2f(0)+4f\left(\frac{1}{4}\right)+2f\left(\frac{1}{2}\right)+4f\left(\frac{3}{4}\right)+f(1)\right]$
≈ 1.934766

2. $f(x)=\dfrac{\sin x}{x}$, $\Delta x=\dfrac{\pi-\pi/2}{6}=\dfrac{\pi}{12}$

(a) $T_6=\frac{\pi}{24}\left[f\left(\frac{\pi}{2}\right)+2f\left(\frac{7\pi}{12}\right)+2f\left(\frac{2\pi}{3}\right)+2f\left(\frac{3\pi}{4}\right)+2f\left(\frac{5\pi}{6}\right)+2f\left(\frac{11\pi}{12}\right)+f(\pi)\right]$
≈ 0.481672

(b) $S_6=\frac{\pi}{36}\left[f\left(\frac{\pi}{2}\right)+4f\left(\frac{7\pi}{12}\right)+2f\left(\frac{2\pi}{3}\right)+4f\left(\frac{3\pi}{4}\right)+2f\left(\frac{5\pi}{6}\right)+4f\left(\frac{11\pi}{12}\right)+f(\pi)\right]$
≈ 0.481172

3. $f(x)=x\tan x$, $\Delta x=\dfrac{\pi/4-0}{6}=\dfrac{\pi}{24}$

(a) $T_6=\frac{\pi}{48}\left[f(0)+2f\left(\frac{\pi}{24}\right)+2f\left(\frac{\pi}{12}\right)+\cdots+2f\left(\frac{5\pi}{24}\right)+f\left(\frac{\pi}{4}\right)\right]$
≈ 0.189445

(b) $S_6=\frac{\pi}{72}\left[f(0)+4f\left(\frac{\pi}{24}\right)+2f\left(\frac{\pi}{12}\right)+4f\left(\frac{\pi}{8}\right)+2f\left(\frac{\pi}{6}\right)+4f\left(\frac{5\pi}{24}\right)+f\left(\frac{\pi}{4}\right)\right]$
≈ 0.185822

4. $f(x)=e^{-x^2}$, $\Delta x=\dfrac{1-0}{10}=\dfrac{1}{10}$

(a) $T_{10}=\frac{1}{10\cdot 2}\left[f(0)+2f(0.1)+2f(0.2)+\cdots+2f(0.8)+2f(0.9)+f(1)\right]$
≈ 0.746211

(b) $M_{10}=\frac{1}{10}\left[f(0.05)+f(0.15)+f(0.25)+\cdots+f(0.75)+f(0.85)+f(0.95)\right]$
≈ 0.747131

(c) $S_{10}=\frac{1}{10\cdot 3}\left[f(0)+4f(0.1)+2f(0.2)+4f(0.3)+2f(0.4)+4f(0.5)+2f(0.6)+4f(0.7)+2f(0.8)+4f(0.9)+f(1)\right]$
≈ 0.746825

5. $f(x)=\dfrac{1}{\sqrt{1+x^3}}$, $\Delta x=\dfrac{2-0}{10}=\dfrac{1}{5}$

(a) $T_{10}=\frac{1}{5\cdot 2}\left[f(0)+2f(0.2)+2f(0.4)+\cdots+2f(1.6)+2f(1.8)+f(2)\right]$
≈ 1.401435

(b) $M_{10}=\frac{1}{5}\left[f(0.1)+f(0.3)+f(0.5)+\cdots+f(1.5)+f(1.7)+f(1.9)\right]$
≈ 1.402558

(c) $S_{10}=\frac{1}{5\cdot 3}\left[f(0)+4f(0.2)+2f(0.4)+4f(0.6)+2f(0.8)+4f(1)+2f(1.2)+4f(1.4)+2f(1.6)+4f(1.8)+f(2)\right]$
≈ 1.402206

6. $f(x)=\dfrac{1}{\ln x}$, $\Delta x=\dfrac{3-2}{10}=\dfrac{1}{10}$

(a) $T_{10}=\frac{1}{10\cdot 2}\left[f(2)+2f(2.1)+2f(2.2)+\cdots+2f(2.9)+f(3)\right]\approx 1.119061$

(b) $M_{10}=\frac{1}{10}\left[f(2.05)+f(2.15)+f(2.25)+\cdots+f(2.85)+f(2.95)\right]\approx 1.118107$

(c) $S_{10}=\frac{1}{10\cdot 3}\left[f(2)+4f(2.1)+2f(2.2)+4f(2.3)+2f(2.4)+4f(2.5)+2f(2.6)+4f(2.7)+2f(2.8)+4f(2.9)+f(3)\right]$
≈ 1.118428

7. $f(x)=\ln(1+e^x)$, $\Delta x=\dfrac{1-0}{8}=\dfrac{1}{8}$

(a) $T_8=\frac{1}{8\cdot 2}\left[f(0)+2f\left(\frac{1}{8}\right)+2f\left(\frac{1}{4}\right)+2f\left(\frac{3}{8}\right)+2f\left(\frac{1}{2}\right)+2f\left(\frac{5}{8}\right)+2f\left(\frac{3}{4}\right)+2f\left(\frac{7}{8}\right)+f(1)\right]$
≈ 0.984120

(b) $M_8=\frac{1}{8}\left[f\left(\frac{1}{16}\right)+f\left(\frac{3}{16}\right)+f\left(\frac{5}{16}\right)+\cdots+f\left(\frac{13}{16}\right)+f\left(\frac{15}{16}\right)\right]\approx 0.983669$

(c) $S_8=\frac{1}{8\cdot 3}\left[f(0)+4f\left(\frac{1}{8}\right)+2f\left(\frac{1}{4}\right)+4f\left(\frac{3}{8}\right)+2f\left(\frac{1}{2}\right)+4f\left(\frac{5}{8}\right)+2f\left(\frac{3}{4}\right)+4f\left(\frac{7}{8}\right)+f(1)\right]$
≈ 0.983819

8. $f(x)=x^5e^x$, $\Delta x=\dfrac{1-0}{10}=\dfrac{1}{10}$

(a) $T_{10}=\frac{1}{10\cdot 2}\left[f(0)+2f(0.1)+2f(0.2)+\cdots+2f(0.9)+f(1)\right]\approx 0.409140$

(b) $M_{10}=\frac{1}{10}\left[f(0.05)+f(0.15)+\cdots+f(0.85)+f(0.95)\right]\approx 0.388849$

(c) $S_{10}=\frac{1}{10\cdot 3}\left[f(0)+4f(0.1)+2f(0.2)+4f(0.3)+2f(0.4)+4f(0.5)+2f(0.6)+4f(0.7)+2f(0.8)+4f(0.9)+f(1)\right]$
≈ 0.395802

9. $f(x)=\cos(e^x)$, $\Delta x=\dfrac{1/2-0}{8}=\dfrac{1}{16}$

(a) $T_8=\frac{1}{32}\left[f(0)+2f\left(\frac{1}{16}\right)+2f\left(\frac{1}{8}\right)+\cdots+2f\left(\frac{7}{16}\right)+f\left(\frac{1}{2}\right)\right]\approx 0.132465$

(b) $M_8=\frac{1}{16}\left[f\left(\frac{1}{32}\right)+f\left(\frac{3}{32}\right)+\cdots+f\left(\frac{13}{32}\right)+f\left(\frac{15}{32}\right)\right]\approx 0.132857$

(c) $S_8=\frac{1}{48}\left[f(0)+4f\left(\frac{1}{16}\right)+2f\left(\frac{1}{8}\right)+4f\left(\frac{3}{16}\right)+2f\left(\frac{1}{4}\right)+4f\left(\frac{5}{16}\right)+2f\left(\frac{3}{8}\right)+4f\left(\frac{7}{16}\right)+f\left(\frac{1}{2}\right)\right]$
≈ 0.132727

10. $f(x)=\dfrac{1}{1+x^4}$, $\Delta x=\dfrac{3-0}{6}=\dfrac{1}{2}$

(a) $T_6=\frac{1}{2\cdot 2}\left[f(0)+2f(0.5)+2f(1)+2f(1.5)+2f(2)+2f(2.5)+f(3)\right]$
≈ 1.098004

(b) $M_6=\frac{1}{2}\left[f(0.25)+f(0.75)+f(1.25)+f(1.75)+f(2.25)+f(2.75)\right]$
≈ 1.098709

(c) $S_6=\frac{1}{2\cdot 3}\left[f(0)+4f(0.5)+2f(1)+4f(1.5)+2f(2)+4f(2.5)+f(3)\right]$
$=1.109031$

11. $\int_2^6 y\,dx\approx\frac{0.5}{3}\left[9.22+4(9.01)+2(8.76)+4(8.30)+2(7.52)+4(6.83)+2(7.32)+4(7.69)+7.91\right]$
≈ 31.94

12. $\Delta t=1$ min $=\frac{1}{60}$ h, so

distance $=\int_0^{1/6}v(t)\,dt$
$\approx\frac{1/60}{3}\left[40+4(42)+2(45)+4(49)+2(52)+4(54)+2(56)+4(57)+2(57)+4(55)+56\right]$
≈ 8.6 mi

13. By Simpson's Rule,

volume $=\int_0^{10}A(x)\,dx$
$\approx\frac{1}{3}\left[A(0)+4A(1)+2A(2)+4A(3)+2A(4)+4A(5)+2A(6)+4A(7)+2A(8)+4A(9)+A(10)\right]$
$=5.7\overline{6}\approx 5.8\text{ m}^3$

14. (a) $\int_1^{3.2}y\,dx\approx\frac{0.2}{2}\left[4.9+2(5.4)+2(5.8)+2(6.2)+2(6.7)+2(7.0)+2(7.3)+2(7.5)+2(8.0)+2(8.2)+2(8.3)+8.3\right]$
$=15.4$

(b) $-1\le f''(x)\le 3\ \Rightarrow\ |f''(x)|\le 3$, so use $K=3$, $a=1$, $b=3.2$, and $n=11$ in Theorem 3. So
$|E_T|\le\dfrac{3(3.2-1)^3}{12(11)^2}=0.022.$

15. By the Net Change Theorem, the amount of water leaked is equal to $\int_0^4 r(t)\,dt$. We use Simpson's Rule with $n=4$ and $\Delta t=1$ to estimate this integral:

$\int_0^4 r(t)\,dt\approx S_4=\frac{1}{1\cdot 3}\left[r(0)+4r(1)+2r(2)+4r(3)+r(4)\right]$
$\approx\frac{1}{3}\left[6+4(5.7)+2(5.1)+4(4.1)+3\right]$
$=\frac{1}{3}(58.4)=19.4\overline{6}$ L

16. By the Net Change Theorem, the energy used is equal to $\int_0^{12}P(t)\,dt$. We use Simpson's Rule with $n=12$ and $\Delta t=1$ to estimate this integral:

$\int_0^{12}P(t)\,dt\approx S_{12}$
$=\frac{1}{1\cdot 3}\left[P(0)+4P(1)+2P(2)+\cdots+2P(10)+4P(11)+P(12)\right]$
$\approx\frac{1}{3}\left[4182+4(3856)+2(3640)+4(3558)+2(3547)+4(3679)+2(4112)+4(4699)+2(5151)+4(5514)+2(5751)+4(6044)+6206\right]$
$=\frac{1}{3}(164{,}190)=54{,}730$ megawatt-hours