9.1 PARAMETRIC CURVES

A Click here for answers.

S Click here for solutions.

1–15 ▪
(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a Cartesian equation of the curve.

1. $x = 2t + 4, \quad y = t - 1$

2. $x = 3 - t, \quad y = 2t - 3, \quad -1 \leqslant t \leqslant 4$

3. $x = 1 - 2t, \quad y = t^2 + 4, \quad 0 \leqslant t \leqslant 3$

4. $x = t^2, \quad y = 6 - 3t$

5. $x = 1 - t, \quad y = 2 + 3t$

6. $x = 2t - 1, \quad y = 2 - t, \quad -3 \leqslant t \leqslant 3$

7. $x = 3t^2, \quad y = 2 + 5t, \quad 0 \leqslant t \leqslant 2$

8. $x = 2t - 1, \quad y = t^2 - 1$

9. $x = 3\cos\theta, \quad y = 2\sin\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

10. $x = \cos^2\theta, \quad y = \sin\theta$

11. $x = e^t, \quad y = \sqrt{t}, \quad 0 \leqslant t \leqslant 1$

12. $x = e^t, \quad y = e^t$

13. $x = \cos^2 t, \quad y = \cos^4 t$

14. $x = \dfrac{1 - t^2}{1 + t^2}, \quad y = \dfrac{2t}{1 + t^2}$

15. $x = \dfrac{1 - t}{1 + t}, \quad y = t^2, \quad 0 \leqslant t \leqslant 1$

16–19 ▪
(a) Eliminate the parameter to find a Cartesian equation of the curve.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

16. $x = 2\cos\theta, \quad y = \frac{1}{2}\sin\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

17. $x = 2\cos\theta, \quad y = \sin^2\theta$

18. $x = \tan\theta + \sec\theta, \quad y = \tan\theta - \sec\theta, \quad -\pi/2 < \theta < \pi/2$

19. $x = \cos t, \quad y = \cos 2t$

20–23 ▪ Describe the motion of a particle with position (x, y) as t varies in the given interval.

20. $x = 4 - 4t, \quad y = 2t + 5, \quad 0 \leqslant t \leqslant 2$

21. $x = \tan t, \quad y = \cot t, \quad \pi/6 \leqslant t \leqslant \pi/3$

22. $x = 8t - 3, \quad y = 2 - t, \quad 0 \leqslant t \leqslant 1$

23. $x = \sin t, \quad y = \csc t, \quad \pi/6 \leqslant t \leqslant 1$

24–26 ▪ Graph x and y as functions of t and observe how x and y increase or decrease as t increases. Use these observations to make a rough sketch by hand of the parametric curve. Then use a graphing device to check your sketch.

24. $x = 3(t^2 - 3), \quad y = t^3 - 3t$

25. $x = \cos t, \quad y = \tan^{-1} t$

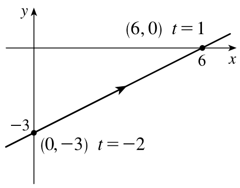
26. $x = t^4 - 1, \quad y = t^3 + 1$

9.1 ANSWERS

E Click here for exercises.

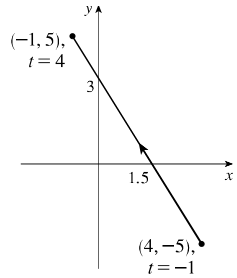
S Click here for solutions.

1. (a)



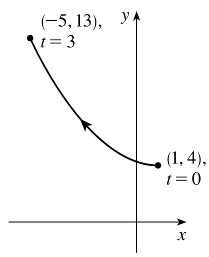
(b) $y = \frac{1}{2}x - 3$

2. (a)



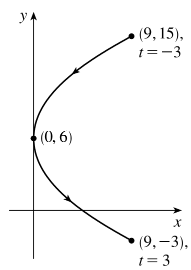
(b) $y = 3 - 2x$

3. (a)



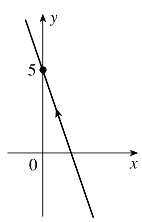
(b) $y = \frac{1}{4}(x-1)^2 + 4$

4. (a)



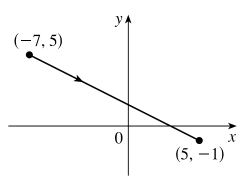
(b) $x = \frac{1}{9}(y-6)^2$

5. (a)



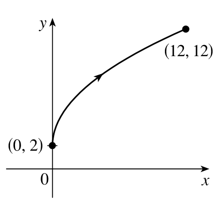
(b) $y = 5 - 3x$

6. (a)



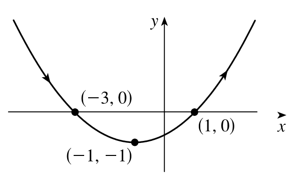
(b) $x + 2y = 3$,
$-7 \le x \le 5$

7. (a)



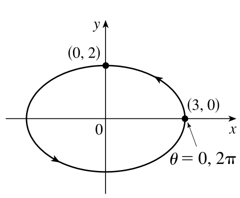
(b) $x = \frac{3}{25}(y-2)^2$,
$2 \le y \le 12$

8. (a)



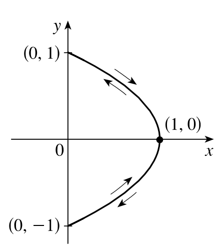
(b) $y + 1 = \frac{1}{4}(x+1)^2$

9. (a)



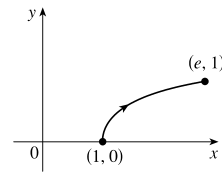
(b) $\frac{1}{9}x^2 + \frac{1}{4}y^2 = 1$

10. (a)



(b) $x + y^2 = 1$,
$-1 \le y \le 1$

11. (a)

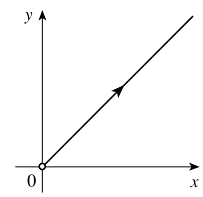


(b) $x = e^{y^2}$, $0 \le y \le 1$

Or:

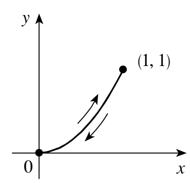
$y = \sqrt{\ln x}$, $1 \le x \le e$

12. (a)



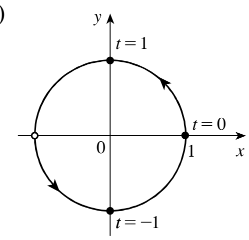
(b) $y = x$, $x > 0$

13. (a)



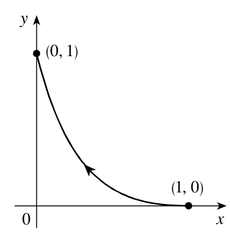
(b) $y = x^2$, $0 \le x \le 1$

14. (a)



(b) $x^2 + y^2 = 1$, $x \ne -1$

15. (a)



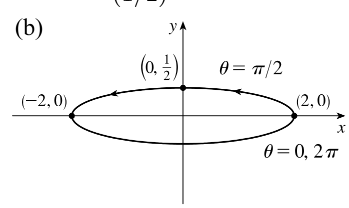
(b) $x = \dfrac{1-\sqrt{y}}{1+\sqrt{y}}$, $0 \le y \le 1$

Or:

$y = \left(\dfrac{1-x}{1+x}\right)^2$, $0 \le x \le 1$

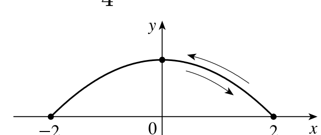
16. (a) $\dfrac{x^2}{2^2} + \dfrac{y^2}{(1/2)^2} = 1$

(b)



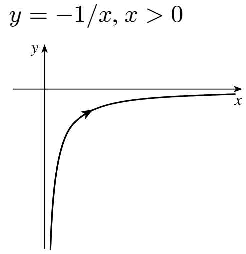
17. (a) $y = 1 - \dfrac{x^2}{4}$, $-2 \le x \le 2$

(b)



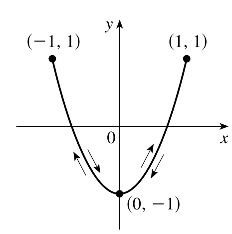
18. (a) $y = -1/x$, $x > 0$

(b)



19. (a) $y + 1 = 2x^2$,
$-1 \le x \le 1$

(b)



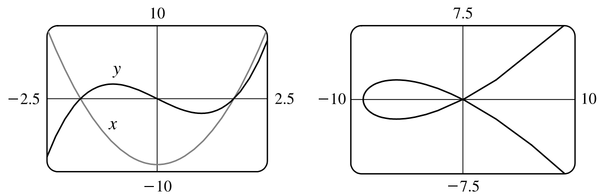
20. Moves along $y = -\frac{1}{2}x + 7$ from $(4, 5)$ to $(-4, 9)$

21. Moves along the first quadrant branch of $y = 1/x$ from $\left(\frac{1}{\sqrt{3}}, \sqrt{3}\right)$ to $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

22. Moves along $x + 8y = 13$ from $(-3, 2)$ to $(5, 1)$

23. Moves down the first quadrant branch of $xy = 1$ from $\left(\frac{1}{2}, 2\right)$ to $(\sin 1, \csc 1)$

24.



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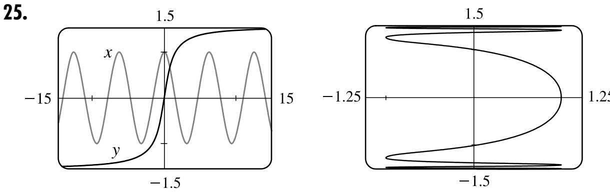

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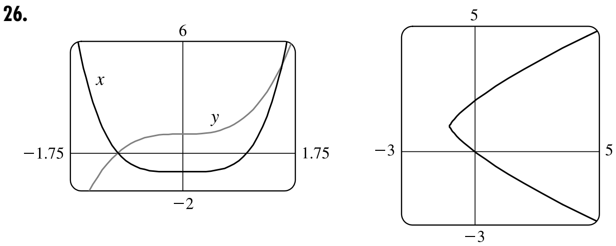
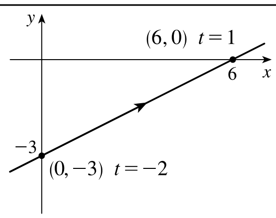


9.1 SOLUTIONS

E Click here for exercises.

1. (a) $x = 2t + 4$, $y = t - 1$

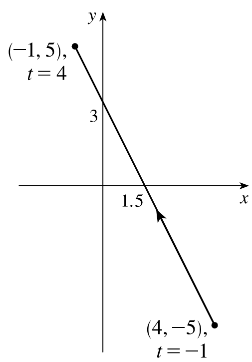
t	-3	-2	-1	0	1	2
x	-2	0	2	4	6	8
y	-4	-3	-2	-1	0	1



(b) $x = 2t + 4$, $y = t - 1 \Rightarrow$
$x = 2(y + 1) + 4 = 2y + 6$ or $y = \frac{1}{2}x - 3$

2. (a) $x = 3 - t$, $y = 2t - 3$, $-1 \le t \le 4$

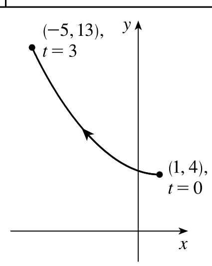
t	-1	0	1	2	3	4
x	4	3	2	1	0	-1
y	-5	-3	-1	1	3	5



(b) $x = 3 - t \Rightarrow t = 3 - x \Rightarrow$
$y = 2t - 3 = 2(3 - x) - 3 \Rightarrow y = 3 - 2x$

3. (a) $x = 1 - 2t$, $y = t^2 + 4$, $0 \le t \le 3$

t	0	1	2	3
x	1	-1	-3	-5
y	4	5	8	13



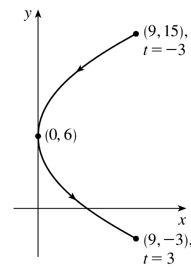
(b) $x = 1 - 2t \Rightarrow 2t = 1 - x \Rightarrow t = \dfrac{1 - x}{2} \Rightarrow$

$y = t^2 + 4 = \left(\dfrac{1 - x}{2}\right)^2 + 4 = \frac{1}{4}(x - 1)^2 + 4$ or

$y = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{17}{4}$

4. (a) $x = t^2$, $y = 6 - 3t$

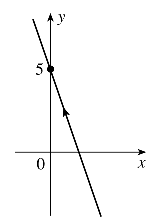
t	-3	-2	-1	0	1	2	3
x	9	4	1	0	1	4	9
y	15	12	9	6	3	0	-3



(b) $y = 6 - 3t \Rightarrow 3t = 6 - y \Rightarrow t = \dfrac{6 - y}{3} \Rightarrow$

$x = t^2 = \left(\dfrac{6 - y}{3}\right)^2 = \frac{1}{9}(y - 6)^2$

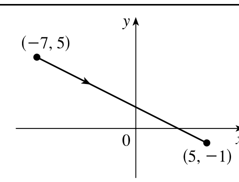
5. (a)



(b) $x = 1 - t$, $y = 2 + 3t \Rightarrow$
$y = 2 + 3(1 - x) = 5 - 3x$, so $3x + y = 5$

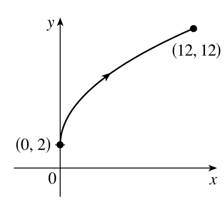
6. (a)

t	-3	-2	1	0	1	2
x	-7	-5	-3	-1	1	3
y	5	4	3	2	1	0



(b) $x = 2t - 1$, $y = 2 - t$, $-3 \le t \le 3 \Rightarrow$
$x = 2(2 - y) - 1 = 3 - 2y$, so $x + 2y = 3$, with
$-7 \le x \le 5$

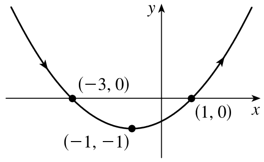
7. (a)



(b) $x = 3t^2$, $y = 2 + 5t$, $0 \le t \le 2 \Rightarrow$

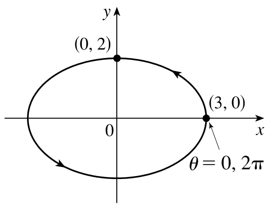
$x = 3\left(\dfrac{y - 2}{5}\right)^2 = \frac{3}{25}(y - 2)^2$, $2 \le y \le 12$

8. (a)



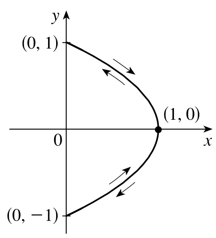
(b) $x = 2t - 1, y = t^2 - 1 \quad \Rightarrow \quad y = \left(\dfrac{x+1}{2}\right)^2 - 1$, so $y + 1 = \frac{1}{4}(x+1)^2$

9. (a)



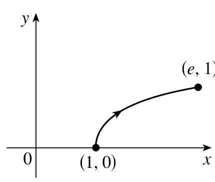
(b) $x = 3\cos\theta, y = 2\sin\theta, 0 \le \theta \le 2\pi \quad \Rightarrow$ $\left(\dfrac{x}{3}\right)^2 + \left(\dfrac{y}{2}\right)^2 = \cos^2\theta + \sin^2\theta = 1$, or $\frac{1}{9}x^2 + \frac{1}{4}y^2 = 1$

10. (a)



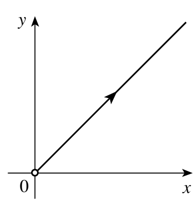
(b) $x = \cos^2\theta, y = \sin\theta \quad \Rightarrow$ $x + y^2 = \cos^2\theta + \sin^2\theta = 1, -1 \le y \le 1$

11. (a)



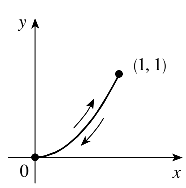
(b) $x = e^t, y = \sqrt{t} \quad \Rightarrow \quad x = e^{y^2}, 0 \le y \le 1$
Or: $y = \sqrt{\ln x}, 1 \le x \le e$

12. (a)



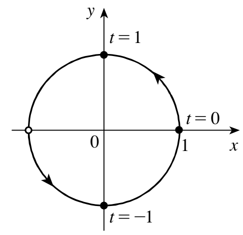
(b) $x = e^t, y = e^t \quad \Rightarrow \quad y = x, x > 0$

13. (a)



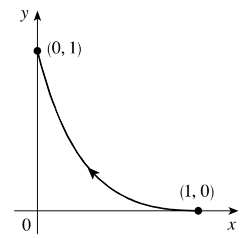
(b) $x = \cos^2 t, y = \cos^4 t \quad \Rightarrow \quad y = x^2, 0 \le x \le 1$

14. (a)



(b) $x = \dfrac{1-t^2}{1+t^2}, y = \dfrac{2t}{1+t^2} \quad \Rightarrow \quad x^2 + y^2 = 1, x \ne -1$

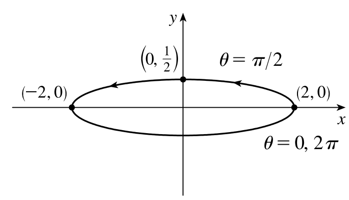
15. (a)



(b) $x = \dfrac{1-t}{1+t}, y = t^2, 0 \le t \le 1 \quad \Rightarrow \quad x = \dfrac{1-\sqrt{y}}{1+\sqrt{y}}$, $0 \le y \le 1$
Or: $y = \left(\dfrac{1-x}{1+x}\right)^2, 0 \le x \le 1$

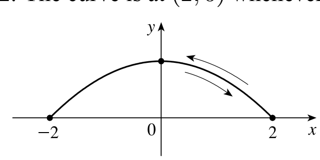
16. (a) $x = 2\cos\theta, y = \frac{1}{2}\sin\theta, 0 \le \theta \le 2\pi$.
$1 = \cos^2\theta + \sin^2\theta = \left(\dfrac{x}{2}\right)^2 + \left(\dfrac{y}{1/2}\right)^2$, so
$\dfrac{x^2}{2^2} + \dfrac{y^2}{(1/2)^2} = 1$.

(b)



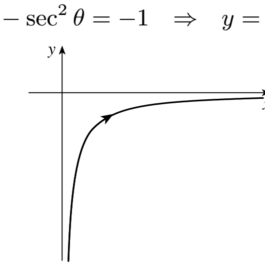
17. (a) $x = 2\cos\theta, y = \sin^2\theta$.
$1 = \cos^2\theta + \sin^2\theta = \left(\dfrac{x}{2}\right)^2 + y$, so $y = 1 - \dfrac{x^2}{4}$, $-2 \le x \le 2$. The curve is at $(2, 0)$ whenever $\theta = 2\pi n$.

(b)



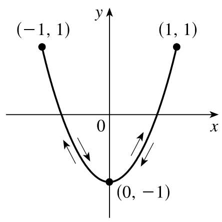
18. (a) $x = \tan\theta + \sec\theta, y = \tan\theta - \sec\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
$xy = \tan^2\theta - \sec^2\theta = -1 \quad \Rightarrow \quad y = -1/x, x > 0$.

(b)



19. (a) $x = \cos t$, $y = \cos 2t$.
$y = \cos 2t = 2\cos^2 t - 1 = 2x^2 - 1$, so $y + 1 = 2x^2$, $-1 \le x \le 1$.

(b)



20. $x = 4 - 4t$, $y = 2t + 5$, $0 \le t \le 2$.
$x = 4 - 2(2t) = 4 - 2(y - 5) = -2y + 14$, so the particle moves along the line $y = -\frac{1}{2}x + 7$ from $(4, 5)$ to $(-4, 9)$.

21. $x = \tan t$, $y = \cot t$, $\frac{\pi}{6} \le t \le \frac{\pi}{3}$. $y = 1/x$ for $\frac{1}{\sqrt{3}} \le x \le \sqrt{3}$. The particle moves along the first quadrant branch of the hyperbola $y = 1/x$ from $\left(\frac{1}{\sqrt{3}}, \sqrt{3}\right)$ to $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$.

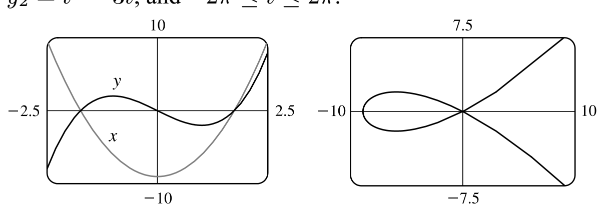
22. $x = 8t - 3$, $y = 2 - t$, $0 \le t \le 1 \quad \Rightarrow$
$x = 8(2 - y) - 3 = 13 - 8y$, so the particle moves along the line $x + 8y = 13$ from $(-3, 2)$ to $(5, 1)$.

23. $y = \csc t = 1/\sin t = 1/x$. The particle slides down the first quadrant branch of the hyperbola $xy = 1$ from $\left(\frac{1}{2}, 2\right)$ to $(\sin 1, \csc 1) \approx (0.84147, 1.1884)$ as t goes from $\frac{\pi}{6}$ to 1.

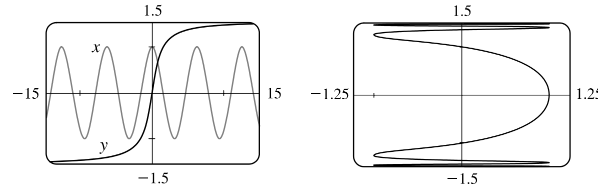
24. From the graphs, it seems that as $t \to -\infty$, $x \to \infty$ and $y \to -\infty$. So the point $(x(t), y(t))$ will move from far out in the fourth quadrant as t increases. At $t = -\sqrt{3}$, both x and y are 0, so the graph passes through the origin. After that the graph passes through the second quadrant (x is negative, y is positive), then intersects the x-axis at $x = -9$ when $t = 0$. After this, the graph passes through the third quadrant, going through the origin again at $t = \sqrt{3}$, and then as $t \to \infty$, $x \to \infty$ and $y \to \infty$. Note that for every point $(x(t), y(t)) = \left(3\left(t^2 - 3\right), t^3 - 3t\right)$, we can substitute $-t$ to get the corresponding point

$$\begin{aligned}(x(-t), y(-t)) &= \left(3\left[(-t)^2 - 3\right], (-t)^3 - 3(-t)\right) \\ &= (x(t), -y(t))\end{aligned}$$

and so the graph is symmetric about the x-axis. The first figure was obtained using $x_1 = t$, $y_1 = 3\left(t^2 - 3\right)$; $x_2 = t$, $y_2 = t^3 - 3t$; and $-2\pi \le t \le 2\pi$.



25. As $t \to -\infty$, $y \to -\frac{\pi}{2}$ and x oscillates between 1 and -1. Then, as t increases through 0, y increases while x continues to oscillate, and the graph passes through the origin. Then, as $t \to \infty$, $y \to \frac{\pi}{2}$ as x oscillates.



26. As $t \to -\infty$, $x \to \infty$ and $y \to -\infty$. The graph passes through the origin at $t = -1$, and then goes through the second quadrant (x negative, y positive), passing through the point $(-1, 1)$ at $t = 0$. As t increases, the graph passes through the point $(0, 2)$ at $t = 1$, and then as $t \to \infty$, both x and y approach ∞. The first figure was obtained using $x_1 = t$, $y_1 = t^4 - 1$; $x_2 = t$, $y_2 = t^3 + 1$; and $-2\pi \le t \le 2\pi$.